Magnetic helicity transport in the quiet Sun
by Brian Thomas Welsch

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University

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Brian Thomas Welsch

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ABSTRACT

The flux of magnetic helicity through the solar photosphere has implications in diverse areas of current solar research, including solar dynamo modelling and coronal heating. Other researchers have considered the flux of magnetic helicity from active regions; here, we do the same for quiet-sun magnetic fields. We derive a theoretical expression for the helicity flux in terms of the relative motions of separate flux elements, and the time evolution of the quadrupole moments of individual magnetic flux elements, summing both "mutual" and "self-helicity" contributions to get the total helicity flux. Using a tracking algorithm applied to high cadence, high resolution SOHO/MDI magnetograms, we determine the observed rate of helicity flux in the quiet sun and compare these measurements with our theoretical predictions.

CHAPTER 1

INTRODUCTION

'If it were not for its variable magnetic field, the Sun would have been a rather uninteresting star.' – E.N. Parker

Manifestations of Magnetic Fields in the Solar Photosphere

Observations of the Sun's visible surface, the photosphere, demonstrate the presence of magnetic fields over a range of spatial scales. The magnetic fields composing this continuous spectrum are usually lumped into two categories: "active region" fields, near the large end of the spectrum, and "quiet sun" fields, near the small end.

Both populations are thought to be composed of sub-resolution magnetic flux tubes, each with a typical magnetic field strength (or magnetic flux density) $|\mathbf{B}|$ on the order of 10^3 Gauss (Stenflo 1994), but the density (or "filling factor") of flux tubes in quiet-sun fields is much lower than in active-region fields. Consequently, average field strengths in quiet-sun fields at resolutions currently attainable are much lower than average field strengths in active region fields.

In addition to differences in average field strengths, comparisons of their typical sizes, spatial distributions, and temporal evolution highlight the differences between the two populations.

Concentrated active-region fields typically extend for tens of megameters (Mm) or more. These features are commonly referred to as sunspot fields, or sunspots. Typical active region fields often contain more that $\sim 10^{21}$ maxwells (1 Mx = 1 Gauss $\cdot \mathrm{cm}^2$) of unsigned magnetic flux, with average magnetic field strengths of the order of 1 kG. Because the presence of strong magnetic fields over a large area can noticeably affect the appearance of the Sun's visible surface, active-region fields are the oldest known manifestation of magnetism on the Sun: records of sunspot observations by Chinese observers exist as far back as A.D. 301.

Active-region fields are not always present in the photosphere, and they are not distributed uniformly over the surface at any time. The presence of active region fields oscillates with an 11-year period, known as the solar cycle, and active regions are observed only within about 35° heliocentric latitude of the Sun's equator. Bipolar active region fields have been observed "emerging" through the photosphere, beginning at relatively high latitudes near "solar minimum," when few active region fields are present, and emerging at progressively lower latitudes until, at "solar maximum," they emerge near the equator. Over an 11-year cycle, approximately 10^{24} Mx of active region flux may emerge (Parker 1984).

The fate of active-region fields after emergence is unclear: observations have demonstrated that active region flux diffuses over the photosphere, and that magnetic features associated with active region fields can erupt from the solar surface. And some researchers believe active region fields can also "submerge" beneath the photosphere. Bipolar active region fields are not randomly oriented: an imaginary line joining the centers of the opposite polarities would run roughly in an east-west direction. The western-most flux in each bipole lies in the direction of the Sun's rotation, and can thus be termed the "leading polarity" in each bipole. *Hale's law* states that the leading polarities within one hemisphere are the same, while the leading polarities in the different hemispheres are opposites, and, further, that the leading polarities flip sign every 11 years.

These and other "regular" properties of active region fields have led to a near consensus among solar physicists that active region fields are generated by a *solar dynamo* operating beneath the photosphere, probably near the base of the convection zone, at $\sim .7R_{\odot}$. The precise workings of this dynamo are, however, not understood at this time.

We now contrast these properties of active region fields with the properties of quiet sun fields. Typical quiet sun magnetic elements contain much less flux than active region fields ($\lesssim 10^{19}$ Mx) and have much weaker average field strengths, on the order of tens of gauss. Their smaller size prevented observations of these fields until the development of spectroscopic techniques over the previous century. In fact, only recently have high-resolution imaging techniques allowed solar physicists to begin characterizing the statistical properties of quiet-sun fields.

Few patterns are apparent in the spatial distribution and temporal evolution of quiet-sun magnetic fields. On the scale of the Sun's surface, their spatial distribution appears essentially uniform. At smaller scales, they are more common near the edges of *supergranules*, the large-scale (15-30 Mm) photospheric manifestation of upwellings in the convection zone. Observational studies demonstrating variations in quiet sun magnetic fields with the solar cycle are lacking; apparently, quiet sun fields come and go independently of the solar cycle. Using a lower-limit of 10^{18} Mx to define a quiet-sun magnetic flux element (Lin & Rimmele [1999] report lower flux fields, at the smallest spatial scales and fields strengths currently observable), rough estimates put $N \sim 10^4$ on the Sun's surface at any time (Hagenaar et al. 1999). Published estimates (Schrijver et al. 1997) suggest that all the flux in the quiet sun might be "cancelled" every $\sim$ 40 hours. If the fields replacing the cancelled fields are "new," and not just re-emerging fields that emerged and submerged at a previous time, then $\sim 10^{25}$ Mx of flux emerges in the quiet sun over an 11-year solar cycle, comparable to the whole-cycle active-region flux emergence rate.

The lack of readily discernable patterns in quiet sun fields' spatial and temporal variability have led some solar physicists to speculate that quiet sun fields are generated in a "surface dynamo" operating in the upper convection zone (Cattaneo 1999). Another hypothesis is that quiet sun fields are merely the "reprocessed" remnants of active region fields that underwent diffusion in the turbulent flows of the upper convection zone.

This work concerns itself with quiet sun fields, and, in particular, the flux of *magnetic helicity* in quiet sun fields.

What is Magnetic Helicity?

Magnetic helicity $\mathcal{H}_M$ is defined as

$$\mathcal{H}_M \equiv \int_V d^3x \, \mathbf{A} \cdot \mathbf{B} \, , \tag{1.1}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$. To be gauge-invariant, the volume of integration V must be bounded by a "magnetic surface" S, on which $\mathbf{B}$ has no component along $\hat{n}$, the normal to S, i.e., $\mathbf{B}\cdot\hat{n}|_S = 0$. This integral quantifies the linkages among all pairs of field lines; it is a measure of the topological complexity of a given magnetic field.

To build a mental concept corresponding to this rather abstract mathematical expression, it is helpful to consider the magnetic helicity in the special case of two thin magnetic flux tubes, of flux Φ_1 and Φ_2, immersed in field-free plasma. The vanishing divergence of the magnetic field, $\nabla \cdot \mathbf{B} = \mathbf{0}$, implies that each of these flux tubes forms a closed loop. We assume that field lines within each tube are untwisted, and, further, that flux tube 1 lies in a plane. In this idealized case, each flux tube can be envisioned as a closed loop of rope, with a sense of "direction" along each strand given by its magnetic field. Then, if the "ropes" are linked once, the integral in equation (1.1) is simple, and $\mathcal{H}_M = \pm 2\Phi_1\Phi_2$. The factor of 2 arises because the flux tubes are "mutually linked": if Φ_1 links Φ_2 once, Φ_2 must also link Φ_1 once.

The sign is determined by a "right hand rule" sense of linkage. We illustrate this idea by introducing the related concept of *crossings*: from any observer's point of view, two ropes that are linked will necessarily cross each other – one rope being "in front of" the other – at some points. (But ropes that cross each other are not necessarily linked!) Each crossing can be given a handedness: if the thumb of an observer's right hand points in the direction of rope 2 where rope 2 crosses rope 1, the crossing will be "right-handed" if the fingers of the right hand point in the same direction as rope 1, and "left-handed" if the fingers of the right hand point in the opposite direction to rope 1. Equivalently, crossings can be given a sign: a right-handed crossing is positive, while a left-handed crossing is negative. The linking number is the sum of all signed crossings the observer sees, divided by two.

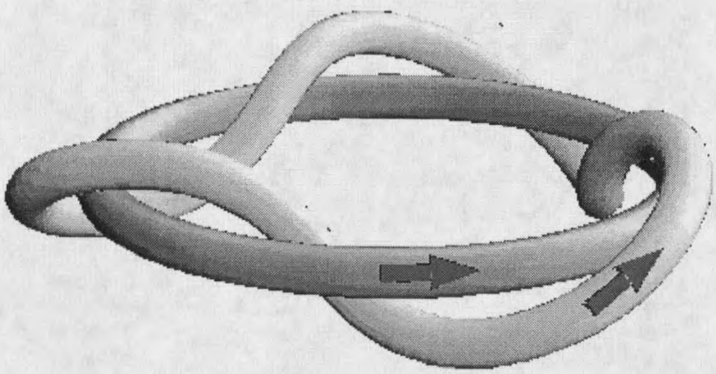

Figure 1.1: Two Linked Flux Tubes. In this case, the tubes have a linking number of -3 and mutual helicity of $-6\Phi_1\Phi_2$. *Illustration courtesy of M. Berger.*

If we had not assumed the field lines within each tube to be untwisted, the integral in equation (1.1) would also include "self" helicity contributions from the field-line linkages *within* each flux tube, in addition to the "mutual" helicity *between* the distinct flux tubes we computed above.

A drawing two flux tubes with a linking number of -3 and mutual helicity of $-6\Phi_1\Phi_2$ in figure 1.1 illustrates the concepts outlined here.

In many real-world situations, however, magnetic fields are not confined in flux tubes, and the integral in equation (1.1) in such cases can be non-trivial. But one property of fields that have non-zero magnetic helicity can still be intuitively understood: field lines in such cases have a *net* (as opposed to *gross*) *global* (as opposed to *local*) sense of *twist*.

More detailed discussions of magnetic helicity may be found in Moffatt (1978) and Berger & Field (1984).

In the magnetohydrodynamic approximation (Choudhuri 1998), the time evolution of a magnetic field is described by the induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad , \tag{1.2}$$

where η is the magnetic diffusivity $c^2/4\pi\sigma$, and σ is the electric conductivity.

In an *ideal* plasma, the conductivity is assumed infinite, meaning the magnetic diffusivity vanishes. In plasmas as hot as those in the Sun, this is a very reasonable approximation. Then the simplified induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad , \tag{1.3}$$

implies that the magnetic field is "frozen in" to the plasma, and the topology of field lines is invariant. Since the magnetic helicity quantifies the topological linkages in a given field, the helicity is invariant ($d\mathcal{H}_M/dt = 0$) if the topology is. In fact, in magnetized plasmas which are highly conductive, but not perfectly so, magnetic helicity is still approximately conserved, even though other ideal invariants, notably field line topology, are not (Taylor 1974). *Hence, even during episodes of magnetic reconnection, magnetic helicity is essentially not destroyed,* *only redistributed.*

Consequently, models meant to explain the evolution of solar magnetic fields must satisfy the constraint imposed by the approximate conservation of magnetic helicity. This has implications in many areas of current research in solar physics, including dynamo modeling, theories of coronal heating, research into flare triggers, the ultimate fate of active region flux, and *in situ* measurements of solar wind properties.

In the area of dynamo modelling, Boozer has pointed out in numerous papers (see his 1999 review) that the usual equations of a standard model for dynamos, called the mean-field dynamo theory, are inconsistent with the approximate conservation of magnetic helicity. He suggests that a correct treatment of the problem includes a helicity flux term, $\nabla \cdot \tilde{\mathbf{h}}$, where $2\tilde{\mathbf{h}}$ is the flux of magnetic helicity via the small-scale flow field. He also observes that two dynamos of equal but opposite helicity might form one "double dynamo" in the Sun. Presumeably, whatever helicity does not cancel in this way must be transported away from the Sun. Hence, a flux of helicity through the solar surface would be observed.

The flux of magnetic helicity might be related to coronal heating processes. Seehafer (Seehafer 1994) has suggested that coriolis-induced *kinetic helicity* in the flow field,

$$\mathcal{H}_K \equiv \int dV\, \mathbf{v} \cdot (\nabla \times \mathbf{v}), \tag{1.4}$$

can impart *current helicity*,

$$\mathcal{H}_C \equiv \int dV\, \mathbf{B} \cdot (\nabla \times \mathbf{B}), \tag{1.5}$$

to small-scale magnetic fields which initially contained no currents, $\mathbf{J} = c(\nabla \times \mathbf{B})/4\pi = 0$. Since fields with currents have more energy than current-free fields, or "potential" fields (so called because $\nabla \times \mathbf{B} = \mathbf{0}$ implies $\mathbf{B}$ can be written as the gradient of a scalar potential), the presence of current helicity implies the presence of free energy in the field. The presence of free energy in the field, in turn, has implications for coronal heating: the well-known "magnetic carpet" model of coronal heating (Schrijver et al. 1998), for instance, relies upon resistive dissipation of these currents to heat the corona. While general relationships between current and magnetic helicities are not known, Berger (1993) has demonstrated a correlation between the buildup of magnetic helicity and energy in one study. So the magnetic helicity flux through photosphere might be a proxy for the flux of energy through the photosphere.

In addition, the propagation of helicity (as a proxy for twist) in the solar atmosphere might have implications for triggering eruptive flares. Canfield & Reardon (1998) present intriguing observations that suggest reconnection might have been injecting twist into a magnetic flux system that later erupted from the solar surface in a process that might be termed "helicity pumping." Canfield and collaborators have pursued this line of inquiry by studying both twist propagation (Jibben & Canfield 2001) and pre-eruptive reconnection (Colman & Canfield 2001) in Hα data.

The presence of structure, or lack of it, in the flux of magnetic helicity at the photosphere might also bear on the question of the ultimate fate of active region flux. Observations clearly show active region magnetic fields diffusing spatially and weakening with time after they emerge (see, e.g., Leighton [1964]); but this diffusion and weakening could arise from flux submergence or eruption, or from "eddy diffusivity" in turbulent plasma flows, or from anomalous resistivity at the molecular level, or from other processes, or from combinations of these mechanisms. If the flux of magnetic helicity in smaller, quiet-sun fields were found to possess the same well-characterized latitudinal, longitudinal, and temporal dependencies as active region fields, one plausible interpretation of this result would be that quiet sun fluxes are the remnants of decayed active region fields.

Finally, the flux of magnetic helicity in the solar wind can be compared to observations of magnetic helicity flux from the Sun. At the largest scale, Bieber *et al.*(1987) have characterized the magnetic helicity in the Parker spiral, which they find to be largely insensitive to the distribution of magnetic fields on the Sun. On an intermediate scale ($\sim$ 1 AU), Rust & Kumar (1996) have investigated the expected flux of helicity in ejected flux ropes. DeVore used their results to estimate a whole-cycle solar wind helicity flux of 10^{46} Mx^2, which he asserts is in good agreement with *in situ* observations of the magnetic helicity flux near 1 AU (DeVore 2000). Regarding smaller scales, Smith's (1999) review of *in situ* observations of magnetic field structure in the solar wind near 1 AU reports minimal evidence for the presence of magnetic helicity, whether from solar "sources" or via helicity injection in the interplanetary medium between the Sun and 1 AU. Consequently, a measured non-zero flux of magnetic helicity through the Sun's surface at small scales might raise interesting questions about helicity transport above the photosphere.

Current Research in Solar Magnetic Helicity Transport

Various authors have considered the flux of magnetic helicity from active regions, using both theoretical and observational techniques. DeVore (2000) constructs models of bipolar active regions, and studies the injection of magnetic helicity through the photosphere into the corona from the shear introduced by a model differential rotation profile. He then extrapolates from these results to calculate a whole-Sun, whole-cycle flux of magnetic helicity from active regions. By combining the flux rope model of Burlaga et al. (1981) with the results of Webb

& Howard (1994), who studied the occurrence rates of and average magnetic flux in coronal mass ejections (CMEs), DeVore infers a rate of active-region helicity flux in the solar wind comparable to his theoretical rate of magnetic helicity injection. Further, he asserts that his rate is consistent with the *in situ* measurements reviewed by Smith (1999).

Chae (2001) uses local correlation tracking (LCT) of magnetic features in a sequence of line-of-sight magnetograms of an active region to measure the helicity flux through the photosphere into the corona in that active region. He finds that the temporal evolution of the helicity flux rate is highly structured, with occasional excursions well above the rate predicted by DeVore (2000), and, interestingly, a significant oscillation in the helicity flux, in the range of 10^{-4} Hz.

Green (2001) and collaborators have tracked an active region over several solar rotations and estimated both the helicity injected by differential rotation and the helicity ejected by eruptions. No attempt was made to track individual features in their analysis.

The Present Contribution to the Study of Solar Magnetic Helicity Transport

In this work, we estimate the helicity flux from the photosphere into the corona arising from the evolution of quiet-sun magnetic fields. We define *quiet-sun mag-*

netic fields as those arising from flux elements of approximately the characteristic flux size 3×10^{18}Mx, observed by Schrijver et al. (1997). These flux elements are isolated and can be approximated as point sources or sinks of magnetic flux.

By tracking individual magnetic flux elements and the temporal evolution of their structure, we can make an *observational* estimate of the quiet-sun helicity flux. The expression for the helicity flux can be written as the sum of two terms, the "mutual" and "self" helicity fluxes. The mutual helicity flux arises from the rotation of distinct flux elements about each other. The self helicity flux arises from the rotation of field lines in an individual flux element about each other.

The bulk of this thesis is devoted to telling the "stories" of these two helicity fluxes.

For comparison, we also compute theoretical values of the helicity flux. We assume the dominant source of systematic magnetic helicity injection is the differential rotation of the photosphere, and base our predicted helicity flux solely on this effect. Detailed calculations are presented below.

One other possible systematic source of helicity injection, the coriolis-induced kinetic helicity in the turbulent photospheric flow field discussed above, has not previously been observed, much less thoroughly characterized. Simple scaling arguments, however, suggest that it plays only a minor rôle in the dynamics of the turbulent solar flow field (see Appendix A).

In Chapter 2, we outline our theoretical approach to measuring the fluxes of mutual and self-helicities. In Chapter 3, we detail the procedures used to analyze data and calculate the helicity fluxes. In Chapter 4, we make theoretical predictions for the helicity flux, and in Chapter 5 we present the results of our measurements and compare them to our predictions. In Chapter 6, we conclude with a discussion of possible interpretations our results, and their implications.

CHAPTER 2

THEORY OF HELICITY TRANSPORT

Relative Helicity, and Its Flux

Magnetic fields above the solar photosphere can extend below the photosphere, so the normal component of the magnetic field at the surface S defined by the photosphere does not, in general, vanish:

$$\mathbf{B} \cdot \hat{n}|_S \neq 0 \ . \tag{2.1}$$

For this reason, the expression given in equation (1.1) for magnetic helicity,

$$\mathcal{H}_M = \int_V d^3x \ \mathbf{A} \cdot \mathbf{B} \ , \tag{2.2}$$

is *not* gauge-invariant. We expect that real physical systems, however, *are* gauge-invariant. Consequently, the physical interpretation of $\mathcal{H}_M$ defined this way is not clear in the solar case, and in other cases where the normal component of the magnetic field does not vanish on the boundary surface of the volume of integration.

In response, Berger and Field (1984) defined a guage-invariant quantity they termed the *relative helicity* of a magnetic field $\mathbf{B}$ and a vector potential for that field, $\mathbf{A}$. Their method uses the difference between the helicity computed using $\mathbf{B}$ and $\mathbf{A}$, and the helicity calculated using the current-free magnetic field, $\mathbf{B}_P$, that matches $(\mathbf{B}\cdot\hat{n})|_S$, the normal component of the magnetic field $\mathbf{B}$ on the boundary surface S, and $\mathbf{A}_P$, a vector potential for $\mathbf{B}_P$. Hence, $\mathbf{B}_P$ and $\mathbf{A}_P$ are used as "reference" fields. Finn & Antonsen (1985) showed that this approach is equivalent to defining the relative helicity as

$$\mathcal{H}_{M_R} \equiv \int_V d^3x\ (\mathbf{A}+\mathbf{A}_P)\cdot(\mathbf{B}-\mathbf{B}_P)\ . \tag{2.3}$$

Essentially, the contribution to the total helicity from fields outside the boundary surface S cancels in this expression (see also, e.g., Berger 1999).

We digress to comment on the relative helicity of non-potential fields. The "potential magnetic field" $\mathbf{B}_P$ gets its name from its lack of current, $\mathbf{J}_P = \nabla\times\mathbf{B}_P = 0$, which means the magnetic field can be represented using a scalar potential χ, usually as $\mathbf{B}_P = -\nabla\chi$. Then the vanishing divergence of any magnetic field means χ satisfies Laplace's equation,

$$0 = \nabla\cdot\mathbf{B}_P = -\nabla^2\chi\ , \tag{2.4}$$

which has a unique solution for a given boundary condition. Any field $\mathbf{B}$ which carries a current is, therefore, "non-potential." This fact, with the definition in equation (2.3), means that *a given field* $\mathbf{B}$ *has a nonzero relative helicity if and only if it carries a current.*

Taking the time derivative of their expression for the relative helicity, and using the induction equation, (1.3), Berger and Field (1984) and Berger (1984) have derived a "Poynting" theorem for the flux of relative helicity through a surface S,

$$\frac{d\mathcal{H}_{M_R}}{dt} = 2\oint_S da\Big((\mathbf{A}\cdot\mathbf{B})\mathbf{v} - (\mathbf{A}_P\cdot\mathbf{v})\mathbf{B}\Big)\cdot\hat{n} \quad . \tag{2.5}$$

In this work, we study the flux of helicity in Cartesian geometry, taking photosphere to be a plane surface S located at $z = 0$. In this case, $\hat{n} = \hat{z}$, and we may rewrite the expression above as

$$\frac{d\mathcal{H}_{M_R}}{dt} = 2\int_S da\Big(\underbrace{(\mathbf{A}\cdot\mathbf{B})v_z}_{\text{"advection"}} - \underbrace{(\mathbf{A}_P\cdot\mathbf{v})B_z}_{\text{"braiding"}}\Big) \quad . \tag{2.6}$$

Flux of Magnetic Helicity via Advection

The "advection" term corresponds to the emergence or submergence of fields already containing magnetic helicity across the photosphere. Since the magnetic vector potential cannot be measured, it is also impossible to measure the rate of magnetic helicity emergence directly.

Leka, et al. (1996), however, report observations of active-region magnetic flux, B_z, emerging through the photosphere carrying vertical current, J_z. These newly-emerged fields therefore possess non-zero current helicity (see equation [1.5]). Other observations also support the subphotospheric origin of observed current helicity in active-region fields observed at the photosphere, e.g., those of Pevstov et al. (1997). While the general relation between $\mathcal{H}_{M_R}$ and $\mathcal{H}_C$ is unclear, both are, in practice, commonly viewed as proxies for twist in fields. (In fact, sometimes the terms "magnetic helicity" and "current helicity" are used interchangeably, which is incorrect.) Hence, these observations have been taken to imply that active region fields containing nonzero *magnetic* helicity are advected into the corona (Chae 2001).

In this work, we are primarily concerned with quiet-sun magnetic fields, as opposed to active region magnetic fields. Unlike observations of active-region fields, observations of quiet-sun (Pevtsov & Longcope 2001) fluxes only tentatively support models in which small-scale magnetic fields emerge with a systematic bias in helicity. Hence, we ignore the contribution to the total helicity flux arising from the advection term.