High bandwidth optical coherent transient true-time delay
by Randy Ray Reibel

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University
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Abstract:
An approach to reaching high bandwidth optical coherent transient (OCT) true-time delay (TTD) is described and demonstrated in this thesis. Utilizing the stimulated photon echo process in rare-earth ion doped crystals, such as Tm3+:YAG, TTD of optical signals with bandwidths > 20 GHz and high time bandwidth products > 104 are possible. TTD regenerators using OCT’s have been demonstrated at low bandwidths (< 40 MHz) showing picosecond delay resolutions with microsecond delays. With the advent of high bandwidth chirped lasers and high bandwidth electro-optic phase modulators, OCT TTD of broadband optical signals is now possible in the multi-gigahertz regime.

To achieve this goal, several theoretical and technical aspects had to be explored. Theoretical discussions and numerical simulations are given using the Maxwell-Bloch equations with arbitrary phase. These simulations show good signal fidelity and high (60%) power efficiencies on echoes produced from gratings programmed with linear frequency chirps. New approaches for programming spectral gratings were also examined that utilized high bandwidth electro-optic modulators. In this technique, the phase modulation sidebands on an optical carrier are linearly chirped, creating an analog to the common linear frequency chirp. This approach allows multi-gigahertz true-time delay spectral grating programming. These new programming approaches are examined and characterized, both through simulation and experiment.

A high bandwidth injection locked amplifier, based on semiconductor diode lasers, had to be developed and characterized to boost optical powers from both electro-optic phase modulators as well as chirped lasers. The injection locking system in conjunction with acousto-optic modulators were used in high bandwidth TTD demonstrations in Tm3+:YAG. Ultimately, high bandwidth binary phase shift keyed probe pulses were used in a demonstration of broadband true-time delay at a data rate of 1 GBit/s. The techniques, theory, and demonstrations described in this thesis can also be applied to high bandwidth optical signal processing and arbitrary waveform generation using optical coherent transient phenomena.
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TRANSIENT TRUE-TIME DELAY

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This dissertation has been read by each member of the dissertation committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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R. B. R.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>1. INTRODUCTION TO HIGH BANDWIDTH OPTICAL COHERENT TRANSIENTS</td>
<td>1</td>
</tr>
<tr>
<td>Introduction To Research Topic</td>
<td>1</td>
</tr>
<tr>
<td>Overview of Thesis</td>
<td>10</td>
</tr>
<tr>
<td>2. THEORETICAL OVERVIEW OF OPTICAL COHERENT TRANSIENTS</td>
<td>14</td>
</tr>
<tr>
<td>Optical Coherent Transients And The Photon Echo</td>
<td>15</td>
</tr>
<tr>
<td>Properties Of OCT Media</td>
<td>15</td>
</tr>
<tr>
<td>Spectral Hole Burning</td>
<td>17</td>
</tr>
<tr>
<td>Spectral Gratings and The Photon Echo</td>
<td>19</td>
</tr>
<tr>
<td>Possible Methods of Solution For The Photon Echo</td>
<td>27</td>
</tr>
<tr>
<td>Exact Solutions</td>
<td>33</td>
</tr>
<tr>
<td>Fourier Transform Approximation</td>
<td>34</td>
</tr>
<tr>
<td>Bloch Equations Of Motion</td>
<td>35</td>
</tr>
<tr>
<td>Thin Crystal $\alpha L \ll 1$</td>
<td>35</td>
</tr>
<tr>
<td>Thick Crystal $\alpha L \gg 1$, Weak Pulses $\Theta \ll 1$</td>
<td>36</td>
</tr>
<tr>
<td>Thick Crystal $\alpha L \gg 1$, Area Theorem Approach</td>
<td>37</td>
</tr>
<tr>
<td>Maxwell-Bloch With Arbitrary Phase</td>
<td>39</td>
</tr>
<tr>
<td>Two Level Systems and the Bloch Equations</td>
<td>41</td>
</tr>
<tr>
<td>Maxwell-Bloch Derivation</td>
<td>47</td>
</tr>
<tr>
<td>Maxwell-Bloch Simulations</td>
<td>54</td>
</tr>
<tr>
<td>3. PRACTICAL CONSIDERATIONS FOR HIGH BANDWIDTH OCT PROGRAMMING</td>
<td>65</td>
</tr>
<tr>
<td>Material Considerations</td>
<td>66</td>
</tr>
<tr>
<td>OCT Programming Efficiencies and Considerations</td>
<td>68</td>
</tr>
<tr>
<td>Grating Efficiency vs. Bandwidth</td>
<td>70</td>
</tr>
<tr>
<td>Highly Efficient TTD Using Chirped Programming</td>
<td>75</td>
</tr>
<tr>
<td>Accumulation of Spectral Gratings</td>
<td>83</td>
</tr>
<tr>
<td>Spatial Gratings and Spatially Isolated Echoes</td>
<td>86</td>
</tr>
<tr>
<td>Coherent and Incoherent Saturation Effects</td>
<td>88</td>
</tr>
<tr>
<td>Optical Modulation and Detection Considerations</td>
<td>93</td>
</tr>
<tr>
<td>Phase Modulation And Spectral Filtering</td>
<td>94</td>
</tr>
<tr>
<td>Spectral Filtering and Efficient Detection</td>
<td>98</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4. BROADBAND INJECTION LOCKING AS AN AMPLIFIER</td>
<td>106</td>
</tr>
<tr>
<td>Semiconductor Diode Lasers And Injection Locking Theory</td>
<td>108</td>
</tr>
<tr>
<td>Experiments</td>
<td>117</td>
</tr>
<tr>
<td>Initial Injection Locking and Locking Regions</td>
<td>121</td>
</tr>
<tr>
<td>CW Phase Modulation</td>
<td>128</td>
</tr>
<tr>
<td>Broadband Operation (BPSK)</td>
<td>133</td>
</tr>
<tr>
<td>5. TEMPORALLY OVERLAPPED LINEAR FREQUENCY CHIRPED PULSES</td>
<td>138</td>
</tr>
<tr>
<td>Chirped Pulse Programming</td>
<td>139</td>
</tr>
<tr>
<td>Phase Matching Conditions</td>
<td>145</td>
</tr>
<tr>
<td>Low Bandwidth Demonstrations</td>
<td>147</td>
</tr>
<tr>
<td>Experimental Setup</td>
<td>147</td>
</tr>
<tr>
<td>Delay Versus Frequency Offset</td>
<td>148</td>
</tr>
<tr>
<td>Operating Conditions</td>
<td>155</td>
</tr>
<tr>
<td>Low Bandwidth Accumulation Experiments</td>
<td>159</td>
</tr>
<tr>
<td>Demonstrations Showing the Dynamics of Accumulation</td>
<td>160</td>
</tr>
<tr>
<td>Simulations Showing the Dynamics of Accumulation</td>
<td>163</td>
</tr>
<tr>
<td>High Bandwidth Experiments</td>
<td>172</td>
</tr>
<tr>
<td>6. LINEAR SIDEBAND CHIRPS AND THEIR APPLICATION</td>
<td>182</td>
</tr>
<tr>
<td>Linear Sideband Chirps</td>
<td>183</td>
</tr>
<tr>
<td>Spatial-Spectral Grating Programming</td>
<td>190</td>
</tr>
<tr>
<td>Phase Matching</td>
<td>195</td>
</tr>
<tr>
<td>Initial Demonstrations</td>
<td>198</td>
</tr>
<tr>
<td>High Bandwidth Single Laser Experiments</td>
<td>201</td>
</tr>
<tr>
<td>High Bandwidth Double Laser Experiments</td>
<td>210</td>
</tr>
<tr>
<td>Delay vs Frequency Offset and $T_2$</td>
<td>214</td>
</tr>
<tr>
<td>CW Amplitude Modulation</td>
<td>217</td>
</tr>
<tr>
<td>TTD of Broadband Data</td>
<td>228</td>
</tr>
<tr>
<td>7. SUMMARY</td>
<td>237</td>
</tr>
<tr>
<td>Maxwell-Bloch Theory and Simulations</td>
<td>239</td>
</tr>
<tr>
<td>Injection Locking</td>
<td>241</td>
</tr>
<tr>
<td>Temporally Overlapped LFC's</td>
<td>242</td>
</tr>
<tr>
<td>Linear Sideband Chirp Programming</td>
<td>243</td>
</tr>
<tr>
<td>Future Research Directions</td>
<td>243</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>247</td>
</tr>
<tr>
<td>APPENDIX A - DYNAMICS OF BROADBAND ACCUMULATED GRATINGS</td>
<td>248</td>
</tr>
<tr>
<td>APPENDIX B - NUTATION OF STIMULATED PHOTON ECHOES</td>
<td>255</td>
</tr>
</tbody>
</table>
APPENDIX C – ARBITRARY WAVEFORM GENERATION
USING LINEAR SIDEBAND CHIRPS .............................................................. 279
REFERENCES CITED .................................................................................. 296
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Expected and measured inverse chirp rates for echoes from figure 43 along with RMS deviations</td>
<td>149</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The operational design of how a phased array system works</td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>(a) Shows a typical Gaussian shaped inhomogeneously broadened transition made up of individual Lorentzian lines from atoms at different frequencies. (b) The transition after being illuminated by a laser with frequency $f_L$. The lower trace shows a hole in the transition, where the electrons have been transferred to the excited state, upper trace.</td>
<td>18</td>
</tr>
<tr>
<td>3.</td>
<td>Experimental demonstration of the two pulse photon echo (2PE) and the stimulated photon echo (SPE). In this figure, the temporal pulse widths were $\tau = 100$ ns and $\tau_2 = 1100$ ns. Experiment performed by the author.</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>(a) Shows the two programming pulses, FWHM $\approx 1$ ns, $\tau_2 = 10$ ns. (b) Shows the resulting power spectrum. (c) The expected output as calculated using non-causal linear filter theory. One can see the SPE and the non-causal virtual echo, or VE.</td>
<td>24</td>
</tr>
<tr>
<td>5.</td>
<td>Quadratic fits predicted from linear filter theory (solid and labeled with “Q”) and numerical solutions (dashed and labeled with “MB”) for the 2PE and the SPE power efficiencies. Here the quadratic solution was valid only for $\alpha L \ll 1$. The numerical solutions were found using a full Maxwell-Bloch treatment of the medium and give a more realistic picture of the true behavior of the system. Here $\Theta_1 = \Theta_2 = \Theta_3 = \pi/2$.</td>
<td>29</td>
</tr>
<tr>
<td>6.</td>
<td>A diagram showing different approaches to a predictive tool for photon echoes. Each pulse in a sequence should be treated with this “roadmap”. One asks the specific questions about the pulse, slowly traveling through the “roadmap”, marking down the specific conditions, and equations along the way. Once an output is reached an equation for the polarization, $P$, should have been determined as well as an equation for the output field in terms of that $P$. Solution types can be mixed, see text for discussion.</td>
<td>31</td>
</tr>
</tbody>
</table>
7. A screen shot of the Maxwell-Bloch simulator’s graphical user interface used to enter parameters and start simulations.......................... 55

8. A simulated echo sequence showing the 2PE and the SPE for an $\alpha L = 1.4$................................................................. 56

9. The population grating, stored in the $r_3$ component of the Bloch vector versus detuning at time $5 \mu s$ from the above echo sequence. The top plot shows the grating for an $\alpha L \approx .3$, and the bottom plot shows the grating for an $\alpha L \approx .3$ .................................................................. 57

10. The field amplitude for various times and absorption length of the first two pulses and the 2PE ................................................ 58

11. The field amplitude for various times and absorption length of the probe pulse and the SPE ...................................................... 59

12. The simulated power efficiencies for 2PE (o’s) and SPE (x’s) are shown versus absorption length. The solid line is the quadratic analytic solution with a coefficient of 1/4. Here $\tau_1 \gg \tau_2 = \tau_3$, and the pulse areas were $\Theta_1 = 0.01 \pi$, $\Theta_2 = 0.5 \pi$, and $\Theta_3 = 0.5 \pi$ ............................................ 61

13. The input and output intensities for a chirped pulse sequence from a crystal with $\alpha L = 1.4$. Here the echo efficiency is $\sim 20\%$......................... 64

14. The $r_3$ component of the Bloch vector as a function of the detuning. This shows the spectral grating created by two linear frequency chirped pulses. Here $\alpha L = 1.4$ and $t = 5 \mu s$ ................................................. 64

15. The atomic levels of interest for this thesis for the rare-earth ion doped crystal Tm$^{3+}$:YAG ........................................................................ 68

16. Data points showing simulated peak echo height versus bandwidth for constant programming powers. Here $\tau_C = 4 \mu s$ and $\delta$ is varied to keep $\tau_D$ constant. The solid line is calculated using the analytic functional dependence of $1/B^2$ (normalized). The dashed line is a plot of the analytic dependence of a brief pulse programming versus bandwidth and is normalized to the first point in the $1/B^2$ sequence for contrast. The triangles are simulated sequences for a constant $\tau_C/B$ ratio with efficiencies given by the right hand axis.............................................. 73
17. Illustrations of each of the simulated input sequences studied for this thesis. In (a) a data storage configuration is shown. This configuration has shown better than unit efficiencies (see text) and is simulated here for comparison with new arbitrary phase Maxwell-Bloch simulator. (b) A brief programming pulse scheme for programming TTD echo sequences. (c) A linear frequency chirped programming scheme for producing TTD echo sequences.

18. Power efficiencies for the data storage programming scheme shown in figure 17 (a). Here $\Theta_1 = \Theta_P = \pi$ and range from 0.1 to 0.7 $\pi$. The first and third pulses were Guassian pulses with FWHM's of 0.1 ns. Here, the data pulse was also Guassian and had a FWHM of 0.4 ns and $\Theta_2 = 0.01\pi$.

19. Power efficiencies for the brief pulse TTD programming scheme shown in figure 17 (b). Here $\Theta_1 = \Theta_2 = \pi$ and range from 0.1 to 0.9 $\pi$. The first and second pulses were Guassian with FWHM's of 0.1 ns. The data pulse was also Guassian and had a FWHM of 0.2 ns and $\Theta_P = 0.05\pi$.

20. Power efficiencies for the linear frequency chirped pulse TTD programming scheme shown in figure 17 (c). Here $\Omega_1 = \Omega_2 = \Omega$ and ranges from 5 GRad/s to 19 GRad/s. The programming bandwidth was $B = 50$ GHz and $\tau_C = 1$ ns. The data pulse was Guassian and had a FWHM of 0.04 ns and $\Theta_P = 0.05\pi$.

21. A simulation of bi-phase, amplitude modulated data undergoing highly efficient TTD. The input is the dotted line. The solid line is the output at $\alpha L = 3.06$ where the efficiency is close to 60%. This is more approximately twice the efficiency from the brief pulse programming scheme. Here the bits had FWHM's = 0.04 ns and $\Theta_{bit} = 0.02\pi$. The programming chirps had $\pi/2$ equivalent $\Omega = 12$ GRad/s, $B=50$ GHz, and $\tau_C = 1$ ns.

22. The typical propagation directions for an SPE experiment in the box-car configuration. Here 1, 2, and 3 represent the directions of pulses 1, 2 and 3, and $e$ represents the direction for which the echo would propagate after the crystal.

23. An optical pulse that has experienced Rabi oscillations.
24. Echoes from various strength quasi-continuous optical probe pulses. The echo shows the nutational effects that are described with theory and experiment in appendix 2 ................................................................. 92

25. The power spectrum of a simulated CW phase modulated signal on a carrier .............................................................................................................. 96

26. The reflection function of a fabry-perot etalon that is 3 mm thick measured using a frequency scan of a diode laser ........................................ 100

27. The strength of the intensity modulation at the modulation frequency for various detunings and modulation frequencies for experimental phase modulated signals ................................................................. 102

28. Simulated strengths of the intensity modulation at the modulation frequency for various detunings and modulation frequencies using linear filter theory. Frequency axes units are in the FWHM of the filter ...... 103

29. The operational premise for a laser injection locking system ............ 109

30. The simulated optical spectra for an injection locked slave laser with a free running frequency of 200 GHz showing the standard period doubling route to chaos with $\Omega_o = 0$, and (a) $\xi = 0.0013$, (b) $\xi = 0.0027$, (c) $\xi = 0.0077$, (d) $\xi = 0.0183$ ................................................................. 117

31. Optical injection locking setup using a fiber coupled integrated optics phase modulator. See text for discussion ................................................. 119

32. Optical spectra of (a) master laser, (b) slave laser before injection locking, and (c) slave laser after injection locking ........................................... 122

33. Optical spectrum of the different regions on the period doubling route to chaos showing (a) Stable locking, (b) undamped relaxation oscillations, (c) period doubled relaxation oscillations, and (d) the chaotic region .............................................................................................................. 124
34. Observed regions of injection locking versus the detuning, \( \Delta \) and the injection ratio, \( \zeta \) for a \( P_{\text{out}} = 88 \) mW. The symbols represent observations of the boundaries between different regions while the lines and shading are there to guide the eye. The various regions are: stable locking (S), undamped relaxation oscillations (P1), period doubled relaxation oscillations (P2), chaotic regions (C), four wave mixing (4W), multi-longitudinal mixing (M), period four relaxation oscillations (P4), an undefined region of both chaos and relaxation oscillations (U), as well as unlocked regions ....................................................... 127

35. (Left) CW phase modulation of the master laser with modulation frequencies of (a) 1 GHz, and (b) 3 GHz. (Right) Plot of the peak powers vs. modulation frequency for the carrier and sidebands of the phase modulated master. At each modulation frequency the rf power was adjusted to achieve roughly a 2:1 ratio of carrier to sidebands ...... 129

36. Injection locked peak powers vs. modulation frequency for (a) lower frequency first order sideband, (b) carrier, and (c) upper frequency first order sideband. The different power levels correspond to gains of \( A = 14, 18, 20, 22 \) and 23 dB .......................................................... 130

37. The residual amplitude modulation for both the phase modulated master and the injection locked signal. A deviation at around 3 GHz is apparent ................................................................. 133

38. Bits 1 through 15 of the delayed-self-heterodyne injection locked outputs of BPSK data at (a) 2 Gbit/s, (b) 3 Gbit/s, and (c) 6 Gbit/s. The expected output is shown as dotted lines. To the right of the data sequence is the eye diagram for the total 256 bit test sequence .... 137

39. Input sequences and expected echo for (a) Two LFC’s separated by a delay \( \tau_{21} \) (b) Two temporally overlapped LFC’s. Solid lines represent amplitude and dashed lines represent frequency ........................................... 140

40. Diagram of a typical temporally overlapped linear frequency chirped pulse experiment .......................................................... 146

41. Echoes from a probe pulse shown for various frequency offsets from \( \delta = 3 \) MHz to 19.8 MHz in steps of 1.2 MHz. Here \( B = 40 \) MHz and \( \tau_C = 3 \) \( \mu \)s .......................................................... 150
42. Simulated programming and probe pulses and the echo output for various frequency offsets, $\delta = 0.66$ MHz to 19.785 MHz in steps of 2.125 MHz. Here $\tau_c = 3 \mu s$ and $B = 40$ MHz ........................................ 150

43. Measured echo delays vs. frequency offset for several different $\tau_c$'s with linear fits using (5.1). The points for $\tau_c = 30$ and 100 $\mu s$ demonstrate the reduced chirp rate requirements .......................................... 151

44. (Top) Data points and a best fit line for the time delay versus frequency offset of the $3 \mu s$ data shown in the previous figure. Here the method to measure the time delay was a Gaussian fit to the echo peaks. (Bottom) The residuals of the above data points giving an RMS = 1.6 ns ............................................................ 152

45. An example of a true time delayed data sequence (right x30). Here $\tau_c = 100 \mu s$ with a programmed time delay of 625 ns. The data sequence (left) is 101011001 at a data rate of 20 Mbit/s ..................................... 154

46. The simulated $r_3$ component of the Bloch vector versus detuning for a single LFC pulse. Notice that the chirp undergoes chirping oscillations. Here $B = 40$ GHz, $\tau_c = 1$ ns, and $\alpha L = 0.3$ ................................ 155

47. The simulated $r_3$ component of the Bloch vector versus detuning for several different temporally separate LFC pulses. Notice both the rapid oscillations (the spectral grating with period $1/\tau_D$ and the chirp envelope. Here $B = 40$ GHz, $\tau_c = 1$ ns and $\alpha L = 0.3$ .......................... 156

48. The simulated $r_3$ component of the Bloch vector versus detuning for temporally overlapped LFC pulses. The two plots show two different frequency offsets, and show that the spectral envelopes of the spectral grating changes leading to the possibility of intensity fluctuations in the echo. Here $B = 40$ GHz, $\tau_c = 5$ mus and $\alpha L = 0.3$ .......................... 157

49. Experimental echo intensities plotted versus delay time for (a) TBP = 40, (b) TBP = 120, and (c) TBP = 200. For TBP < 120, a periodic intensity fluctuation is observed .................................................................. 158

50. Echo power efficiencies vs. programming number (lower axis) or time (upper axis) for a frequency stabilized Ti:Sapphire laser system locked to a spectral hole. The different plots are various programming strengths (Rabi frequency, $\Omega$) as shown in the legend. Here $\tau_c = 1 \mu s$, $B = 40$ MHz, $\tau_D = 250$ ns, and $\tau_r = 31 \mu s$ .......................... 164
51. Echo intensities vs. programming number (lower axis) or time (upper axis) using the injection locked ECDL laser system. The different plots are various programming strengths (Rabi frequency, $\Omega$) as shown in the legend. Here $\tau_c = 1\mu s$, $\tau_d = 125\mu s$, and $\tau_r = 46\mu s$ .................. 165

52. Simulated accumulation sequences with $\tau_C = 1\mu s$, $\tau_D = 250\mu s$, and $\tau_r = 31\mu s$. The echo power efficiencies are plotted versus programming number (lower axis) or time (upper axis). The different lines represent different programming strengths (Rabi frequency, $W$) with (a) showing programming Rabi frequencies from $\Omega = 0.3$ MRad/s to 1.2 MRad/s in steps of 0.1 MRad/s, and (b) showing programming Rabi frequencies from $\Omega = 1.3$ MRad/s to 1.8 MRad/s in steps of 0.1 MRad/s, then to 2.8 MRad/s in steps of 0.2 MRad/s ...................... 166

53. Grating accumulation sequences for $\Omega = 0.5$ MRad/s with $\tau_C = 1\mu s$, $\tau_D = 250\mu s$, and $\tau_r = 31\mu s$. The $r_3$ components are plotted versus detuning and programming number .......................... 169

54. Grating accumulation sequences for $\Omega = 1.2$ MRad/s with $\tau_C = 1\mu s$, $\tau_D = 250\mu s$, and $\tau_r = 31\mu s$. The $r_3$ components are plotted versus detuning and programming number .......................... 169

55. Grating accumulation sequences for $\Omega = 2.8$ MRad/s with $\tau_C = 1\mu s$, $\tau_D = 250\mu s$, and $\tau_r = 31\mu s$. The $r_3$ components are plotted versus detuning and programming number .......................... 170

56. Overall gratings versus detuning at $N = 900$ for (a) $\Omega = 0.5$ MRad/s programming, (b) $\Omega = 1.2$ MRad/s programming, and (c) $\Omega = 2.8$ MRad/s programming pulses .......................... 172

57. Diagram of the high bandwidth CECDL experiment utilizing temporally overlapped linear frequency chirped pulses ......................... 174

58. A low intensity LFC optical probe pulse before (dashed line) and after (solid line) transmission through the medium with an $\alpha L = 1.4$. Here $\tau_c = 15$ ns and $B = 15$ GHz, giving a chirp rate, $\gamma = 1$ GHz/ns ........ 176

59. (a) This figure shows the transmission of a low intensity 2.4 GHz LFC probing a 2.0 GHz TTD grating programmed with the TOLFC method. The large oscillation on the transmission is a frequency oscillation corresponding to a grating period $1/\tau_D$ where $\tau_D = 0.22\mu s$. (b) Shows these oscillations in greater detail .......................... 177
60. This figure shows echo intensity of a 50 ns probe pulse versus the optical frequency of the probe pulse (data points). The solid line shows the expected position of the programmed TTD grating .............. 178

61. This figure examines the experimental echo peak heights as a function of grating bandwidth created with the CECDL laser system. Here the programming power and \( \tau_C \) are kept constant, \( \tau_C = 4 \mu s \) and the offset frequency is kept at \( \delta = 20 \text{ MHz} \). The solid line is a fit using a functional dependence of \( 1/B^2 \) according to eqn. (3.6) .................... 181

62. This figure examines the experimental echo delay times versus the optical frequency of the 50 ns probe pulse. The echo delay time can be seen to decrease indicating non-linearities in the LFC pulse .............. 181

63. An example of \( r_3 \) of a linear sideband chirped pulse after passing through the medium. Here the bandwidth of the LSC is 20 GHz and the start frequency is 25 GHz. Note the start of the second order chirps at approximately 60 GHz ................................................. 185

64. A plot of the functional dependence of \( J_1(\beta)^2/J_2(\beta)^2 \) (right hand axis) as well as the square of \( J_1(\beta) \) and \( J_2(\beta) \). The maximum of \( J_1(\beta)^2/J_2(\beta)^2 \) occurs at 0 and an acceptable level must be chosen ...... 186

65. (a) A chirping sine wave used as a drive voltage. (b) A digital approximation to the chirping sine wave................................. 188

66. (top) The power spectrum of a square wave phase chirp on a carrier. (bottom) The power spectrum of a regular phase chirp on a carrier. In each case the chirps have \( \tau_C = 1 \mu s, B = 0.2 \text{ GHz}, \beta = \pi/2, \) and a 10 GHz carrier ....................................................................................... 189

67. The effect of two temporally overlapped frequency offset linear sideband chirped pulses incident upon the medium. Here the \( r_3 \) component of the Bloch vector is plotted as a function of frequency .......... 192

68. Programming and probe pulses created by separate laser sources. Here the programming pulses are LSC pulses and the probe pulse frequency \( f_{L2} \) is centered on the up-shifted first order grating ......................... 193
69. An input pulse diagram for a single laser experiment. This experiment utilizes the same laser to create the programming and probe pulses. Here the probe pulses have a CW phase modulation that pushes first order sidebands to the center frequency of the grating allowing echoes to be produced ................................................................. 195

70. A simulation showing the output of three separately detuned probe pulses. The detunings are -32 MHz, 0 MHz and +32 MHz for the first, second and third pulse respectively .................................................. 199

71. An experimental demonstration of LSC programming, showing the output of three separately detuned probe pulses. The detunings are -12 MHz, 0 MHz and +12 MHz for the first, second and third pulse respectively ................................................................. 201

72. The experimental setup for the single laser, collinear programming and probing method ................................................................. 203

73. Example echoes from the single laser LSC programming technique. Here the echoes are shown for various frequency offsets with a 20 MHz bandwidth detection limit. See text for discussion ..................... 205

74. Echo outputs observed on the fast detector showing the beat frequency of the spatially overlapped echoes at \(2f_m = 2.85\ \text{GHz}\) for various \(\delta\)'s ........................................ 206

75. An overview of how to use the single laser method to create data pulses discussed in the text ................................................................. 208

76. A 500 MBit/s amplitude modulated echo sequence (lower) created by modulating \(\beta\) in eqn. (6.1) as described in the text. The expected bit sequence is shown as the upper trace ................................................................. 210

77. The experimental setup for the high bandwidth two laser experiments ................................................................. 212

78. Various echoes produced from a temporally brief (50 ns) probe pulse. Here various \(\delta\)'s were chosen to give several different \(\tau_D\)'s .......... 215

79. Experimentally determined delay times versus frequency offset (triangles) for the echoes in 78. The solid line is a linear fit to the data giving an RMS deviation of 0.45 ns. This RMS deviation was found from the residuals plotted in the lower portion of the figure ............ 217
(a) Shows a spectral grating programmed with two LFC's. (b) Shows a phase modulated probe signal situated such that only the carrier and higher frequency sidebands will be diffracted from the grating in (a), producing an amplitude modulated echo at the programmed delay.

A plot of several output echoes from a CW phase modulated probe with $f_m = 0.5$ GHz for various frequency offsets.

The relative delay of the first peak in the amplitude modulated echoes from figure 81 as a function of frequency offset (circles). The solid line is a linear fit to the data. The expected slope is $71.42 \mu s/\text{MHz}$. The residuals are plotted in the lower figure from which an RMS deviation of 52 ps was found.

An example of the power spectrum of the amplitude modulated data in figure 81 (circles). The theoretical shape of this power spectrum is also plotted (solid line).

The phase of the Fourier transformed data from figure 81 plotted against the relative delay (circles). A linear fit to the data is shown (solid line). The expected slope is 3.14 rad/\text{ns}. The residuals are also shown and give an RMS deviation of 0.14 radians.

A plot of several output echoes from a CW phase modulated probe with $f_m = 1.0$ GHz for various frequency offsets.

The phase of the Fourier transformed data from figure 85 plotted against the relative delay (circles). A linear fit to the data is shown (solid line). The expected slope is 6.28 rad/\text{ns}. The RMS deviation was calculated to be 0.38 radians from the residuals plotted in the lower portion.

A 1 GBit/s BPSK modulated probe pulse (lower) is shown with the heterodyned echo signal (upper). The echo signal has the expected 80 ns delay.

Zoom of the delayed 1 GBit/s BPSK echo signal from figure 87 is compared to the expected echo signal. There are no bit errors in the echo sequence.

A sequence of heterodyned BPSK echoes with $\Gamma_D = 666$ MBits/s is shown for various $\delta$'s.
90. A 10\,\mu s long heterodyned BPSK echo with $\Gamma_D = 666$ MBits/s. Here the detector's lower bandwidth is 10 MHz producing the high pass filtered appearance .......................................................... 234

91. Several different 200 ns sections of the echo output in figure 90. No bit errors are observable for the echo output ................................................. 236

92. (a) Three echo pulses created from a single probe pulse. (b) Arbitrary heights produced on the three echoes ......................................................... 281
ABSTRACT

An approach to reaching high bandwidth optical coherent transient (OCT) true-time delay (TTD) is described and demonstrated in this thesis. Utilizing the stimulated photon echo process in rare-earth ion doped crystals, such as Tm³⁺:YAG, TTD of optical signals with bandwidths > 20 GHz and high time bandwidth products > 10⁴ are possible. TTD regenerators using OCT's have been demonstrated at low bandwidths (< 40 MHz) showing picosecond delay resolutions with microsecond delays. With the advent of high bandwidth chirped lasers and high bandwidth electro-optic phase modulators, OCT TTD of broadband optical signals is now possible in the multi-gigahertz regime.

To achieve this goal, several theoretical and technical aspects had to be explored. Theoretical discussions and numerical simulations are given using the Maxwell-Bloch equations with arbitrary phase. These simulations show good signal fidelity and high (60%) power efficiencies on echoes produced from gratings programmed with linear frequency chirps. New approaches for programming spectral gratings were also examined that utilized high bandwidth electro-optic modulators. In this technique, the phase modulation sidebands on an optical carrier are linearly chirped, creating an analog to the common linear frequency chirp. This approach allows multi-gigahertz true-time delay spectral grating programming. These new programming approaches are examined and characterized, both through simulation and experiment.

A high bandwidth injection locked amplifier, based on semiconductor diode lasers, had to be developed and characterized to boost optical powers from both electro-optic phase modulators as well as chirped lasers. The injection locking system in conjunction with acousto-optic modulators were used in high bandwidth TTD demonstrations in Tm³⁺:YAG. Ultimately, high bandwidth binary phase shift keyed probe pulses were used in a demonstration of broadband true-time delay at a data rate of 1 GBit/s. The techniques, theory, and demonstrations described in this thesis can also be applied to high bandwidth optical signal processing and arbitrary waveform generation using optical coherent transient phenomena.
CHAPTER 1

INTRODUCTION TO HIGH BANDWIDTH OPTICAL COHERENT TRANSIENTS

The goal of this thesis is to describe and demonstrate an approach to reaching high bandwidth optical coherent transient (OCT) true-time delay. True-time delay (TTD) is useful in a number of potential applications including phased array antenna systems, arbitrary waveform generation and correlators. This chapter presents an introduction to this research topic including a brief overview of stimulated photon echoes (SPE). The potential usefulness of photon echoes in these high bandwidth systems is examined and an overview of how to reach these bandwidths is shown. An overall description and the organization of this thesis is also given.

Introduction To Research Topic

In Merriam-Webster’s Online Collegiate Dictionary the word echo has the definition: *the repetition of a sound caused by reflection of sound waves* [1]. Most all of us are familiar with the effect of an acoustic or reflection echo. Whether we were in a large auditorium, outside near a building or inside some room, an echo is the repetition of our words caused by a reflection of the sound waves from some surface. Echoes are unique in that they mimic or repeat the words of the person who uttered them although often the echo is fainter or a less intense sound. The amount of time
it takes before an echo reaches an observer is directly proportional to the distance the observer is from the surface that has reflected the sound. Thus by changing the distance to the reflecting surface an echo's time delay can be varied. In much the same way, a photon echo, generated from an OCT process, follows these traits, however, the waveform is no longer an acoustic wave but an electromagnetic wave. The photon echo is not created by reflection from a surface, it is instead created by a complex physical process in an optically absorbing material. In certain situations the photon echo waveform will exactly mimic the original electromagnetic waveform, however with less intensity and a variable time delay. These characteristics are similar to the acoustic echo and are what gives rise to the name photon echo. But the photon echo is much more than a simple reflection from a surface and the OCT processes that govern it are truly unique and can be used as a powerful optical processing tool for a variety of applications.

One such application for the photon echo is as a component that can control, steer and adaptively beamform phased array antennas. Phased array antennas, steered with conventional electronics, currently perform a variety of tasks and find application to both military and civilian markets. These systems make use of the wavelike properties of electromagnetic radiation to "steer" beams without ever physically moving the antenna. The significant benefits of such systems over conventional radar systems were quickly noticed and several phased array radar systems became operational in the 1960's [2]. The operational premise of such a system is shown in figure 1. These
arrays can be used to transmit or receive a variety of waveforms. The array antenna is a collection of RF emitters, each emitting its individual electromagnetic field. In the far field, the electromagnetic fields of each emitter sum to form the overall beam pattern from the antenna array.

Figure 1. The operational design of how a phased array system works.
This overall beam pattern in the far field is known as the antenna beam lobe. This lobe can be controlled or steered utilizing the wave-nature of electromagnetic fields. If the time delay from each emitter to a spatial location away from the array can be controlled, the signal at that spatial location can be made to constructively or destructively interfere with the field produced at that location from other emitters. In the far field, the delay imparted to the electromagnetic fields from each emitter allows the beam lobe to be steered as shown in figure I. Thus, the time delay of the electrical signals becomes the key component to the operation of a phased array radar. In narrow band (nearly single frequency) antenna arrays, the time delay of an electrical signal need not be true time delay. Instead, the electrical signal can be simply phase shifted. A single frequency sinusoidal waveform that is phase shifted is equivalent to one that is time delayed. Thus, simple electronic phase shifting devices could be used to steer the antenna, assuming the waveform is close to a single frequency sinusoidal waveform (narrow band) or, in other words, has a low fractional bandwidth (bandwidth divided by carrier). As the bandwidth deviates from narrowband an equal phase shift no longer translates directly to an equal time delay of the whole waveform. Essentially, different frequency components getting the same phase shift from the phase shifter, have a time delay, $\tau_D$, that can be written as

$$\tau_D(\omega_m) = \phi/\omega_m.$$  

(1.1)

Here $\phi$ is the constant phase shift imparted by the phase shifting device, and $\omega_m$ is
a frequency component of the broadband signal. If the fractional bandwidth of the signal is large, one can see that different $\omega_n$'s will experience different time delays, $\tau_D$. This causes signals with significant bandwidths to behave improperly when steered with phase shifting devices. This imparts what is known as beam squint on the main lobe of the radiation field emitted from the array elements. Since delay and angle are related in a phased array, beam squint essentially means the steering of different frequency components of the signal into different angular directions. This angular spread, $\Delta \theta$, is given by

$$\Delta \theta = -\frac{B}{\omega_{RF}} \tan(\theta)$$ (1.2)

Here, $B$ is the bandwidth of the signal, $\omega_{RF}$ is the carrier frequency of the radar (giving a fractional bandwidth $B/\omega_{RF}$), and $\theta$ is the steering angle of the main lobe [3]. As can be seen, the angular deviation is a linear function of the fractional bandwidth, giving large angular deviations of frequency components for large fractional bandwidths. It should be noted that the angular size of the antenna beam lobe gets smaller with more antenna elements. Thus, if a large phased array radar (with high fractional bandwidths) uses phase shifters to control the time delay, beam squint will be a significant problem [4].

In order for a system of antenna elements to properly steer signals with large fractional bandwidths, the elements must impart a true-time delay (TTD) on the signals. TTD means that each frequency component of the signal for a given emitter gets the same time delay rather than the same phase shift. Thus, eqn. (1.1) can be
rearranged as

$$\phi(\omega_m) = \omega_m \tau_D.$$  \hspace{1cm} (1.3)

In a TTD system it is $\tau_D$ that typically remains constant (per element), and thus the phase shift needed will change according to the given frequency component, $\omega_m$. Producing broadband TTD waveforms is the main emphasis of this thesis.

There have been many proposals for optical TTD control of an RF signal. These include but are not limited to delaying optical signals via different optical delay lines or creating delays by frequency shifting the optical carrier in a highly dispersive optical fiber [3]. In the first approach, several different techniques have been envisioned but all follow the same principle. That principle is that one would choose the time delay for the optical signal by switching the optical pulse into an appropriate delay line. One suggestion is to use several different length optical fibers to give various different delays. Unfortunately, as the number of array elements grows, the switching network for such a system becomes increasingly complex as does the amount of fiber needed. For the second approach, simply by changing the frequency of the optical carrier a different delay can be chosen because the index of refraction for that frequency has changed. In this case, it is the number of distinctive frequency channels that can be achieved by the optical source that limits the resolution of the delays to less than 1 part in 1000 [3]. Along with this, a tunable diode laser and fiber must be used for each element of the array. Again, this results in an increasing complexity and cost as the number of array elements is increased. These systems also have no ability to
control the phase, number, or weight of these delays, and thus adaptive beamforming or jammer nulling can not be done with such systems.

It has been suggested that the SPE process can be used to create the TTD needed to avoid beam squint in a phased array radar or communication system [5]. Instead of using an electronic phase shifter, the photon echo would be used to create TTD. The properties of OCT's are such that the photon echo can theoretically delay signals with tens of gigahertz bandwidths over delays of microseconds with picosecond resolutions. The delays can be programmed on the fly or preprogrammed. The storage capabilities of OCT's enable several different delays to be stored spatially in the OCT medium providing more than a million different delays all within a single compact crystal. This along with the ability to individually control the delay of each frequency component makes OCT's an attractive basis for steering a phased array antenna, especially ones requiring adaptive beamforming and jammer nulling.

In order to create a stimulated photon echo, an inhomogeneously broadened absorber (IBA), such as a rare-earth ion doped crystal, is used. Because of the special properties of IBA's, a time delay between two incident brief programming pulses is stored as a spectral grating within the medium. This grating can be probed sometime later by another incident brief pulse. The medium is then coherently stimulated and produces a stimulated photon echo with the delay that was stored in the spectral grating. In the linear regime, this process is not limited to just temporally brief probe pulses. Any arbitrarily shaped incident probe pulse can be delayed assuming
the bandwidth of the spectral grating is larger than the bandwidth of the probe.

OCT's have previously been demonstrated as TTD devices giving microsecond delay times and picosecond resolutions [6]. Various methods including brief pulse programming and chirped pulse programming have been explored to produce TTD spectral gratings [7]. However, almost all previous demonstrations were over a limited bandwidth (typically 40 MHz) and did not reach the ideal goal of showing TTD with multi-gigahertz bandwidths. This thesis extends TTD demonstrations to the gigahertz bandwidth, along with developing more practical OCT techniques to program multi-gigahertz TTD.

Programming and probing broadband spectral gratings is not an easy task. In order to program an efficient spectral grating, a significant amount of optical energy must be transferred to the medium. Unfortunately, if brief pulses are used to program these broadband gratings, their temporal lengths must be extremely short (on the order of 100 picoseconds) to achieve large bandwidths. Producing these types of brief pulses is not easy, requiring the use of mode-locked lasers with amplification. These systems are impractical because of their cost, power inefficiencies and size. And, even though powerful pulsed lasers exist with tremendous power per pulse (~GW), producing efficient broadband spectral gratings with these lasers has proven challenging. The intensities required are near or exceed the damage threshold of OCT crystals. Processes such as accumulation of spectral gratings, where less powerful programming pulses are repeatedly applied to the medium, must be used in order to produce
efficient gratings.

Another way to produce a spectral grating in an OCT media is through the use of linear frequency chirped pulses [7]. Linear frequency chirps (LFC's) ramp their instantaneous frequency linearly as a function of time. Through the proper choice of chirp bandwidth and chirp duration, a power limited laser can program much more efficient TTD spectral gratings, as compared to brief pulses from the same laser. High bandwidth (> 20 GHz) LFC's are now a possibility due to the recent advances in chirped external cavity diode lasers (CECDL's) [8, 9].

Broadband signals are also inherently harder to detect because of noise issues. The level of thermal noise detected increases as the square root of the bandwidth. Thus, this thermal noise limits the detection of broadband echoes. Poor echo efficiencies also contribute to these detection problems. Recently, photon echoes with greater than unit efficiency have been suggested and observed for certain situations using optically thick samples [10, 11]. Unfortunately, the direct results from these research efforts are not applicable to TTD. This thesis examines the efficiencies of photon echoes for the situation of TTD for both brief pulse programming and LFC programming in optically thick crystals. Another problem is that these broadband RF signals must be imparted to the optical carrier, TTD, and subsequently converted to RF signals. The TTD optical signals must be detected in a fashion that allows detection of the encoded broadband signals. Various methods of modulation and detection are explored. It is also shown that the spectral grating itself can be utilized in making more efficient
detection methods for optically encoded broadband signals.

Another reason previous demonstrations were limited to low bandwidths (40 MHz) was the lack of suitable high bandwidth modulators to create probe signals. Advancements in high bandwidth, affordable electro-optic modulators (EOM's) makes the encoding of high bandwidth electronic signals onto optical carriers easier. Unfortunately, at the wavelength with which most of our work is done (793 nm) these EOM's are optical power limited due to photorefractive damage. The maximum output powers are 1 to 2 orders of magnitude lower than that required to produce detectable broadband photon echoes. In order to overcome this power limitation, a suitable amplifier for high bandwidth signals was developed using EOM's and an injection locked amplifier.

Overview of Thesis

This thesis details the possible approaches for creating high bandwidth TTD along with the problems and solutions. As stated above TTD applications such as phased array radars and communication systems will benefit from this research. But, as with any type of science, there are other unexpected benefits from the research and other applications to which this research can be applied. The desire to bring OCT's into the high bandwidth regime is a continuing struggle, one that is ongoing with new developments and twists everyday. This thesis is a snapshot of the past three years detailing the steps and barriers that had to be overcome before high bandwidth TTD
could be realized. What follows is a brief description of the chapters found within this thesis.

In chapter 2, the theoretical framework needed for this thesis is developed and presented. This includes a detailed overview of the stimulated photon echo process and the linear filter theory that predicts the dynamics of this process in certain regimes. An outline of other approaches to predictive solutions is given for various conditions. Chapter 2 also includes a discussion on the Maxwell-Bloch equations including arbitrary phases of the field. This mathematical approach, combined with simulation, allows quantitative predictions of the OCT phenomena covered in this thesis. Specifically, high efficiency TTD is studied for thick crystals utilizing linear frequency and linear phase chirps. Aspects such as delay resolution and signal fidelity are also examined with this simulator.

Chapter 3 presents the important practical considerations for reaching high bandwidth TTD using the stimulated photon echo process. A discussion of phase modulated signals created from high bandwidth EOM’s is given. Spectral filtering effects caused by the medium as well as programmed spectral gratings are examined for these high bandwidth signals. It is shown that detection schemes utilizing these spectral filters can be more efficient than conventional detection schemes. Experimental parameters and material details are discussed as well as problems related to temporally long probe pulses. Simulations are performed for optically thick samples, showing that expected echo efficiencies using chirped programming can reach 60%.
The chapter also examines the aspects of accumulation and continuous programming and continuous processing, which are relevant to this thesis.

Chapter 4 shows how to amplify broadband signals from the power limited EOM's using an injection locking system. The needed theoretical framework for injection locking is detailed as well as experimental demonstrations of the amplifier. The benefits of such a system include ease of use and gains of more than 20 dB. However, using injection locking with semiconductor diode lasers has inherent challenges that will be discussed.

Through experimental demonstrations, problems related to high bandwidth linear frequency chirped programming were uncovered that called for modifying the previous approach to linear frequency chirp programming. Chapter 5 presents the modification of the linear frequency chirped programming method. The modification was to temporally overlap two frequency offset linear frequency chirps. Both low bandwidth demonstrations and high bandwidth demonstrations (using a CECDL) are presented. The tuning linearity of this method is examined as well as the efficiency of programmed gratings for TTD versus bandwidth scaling. Results from accumulation using this new method are also presented.

Finally, a novel technique for programming TTD gratings into an OCT medium using the broadband EOM's is presented in chapter 6. By using an EOM in conjunction with the injection locked amplifier, one can create multi-gigahertz gratings in the OCT media. This novel technique relies upon linear sideband chirping (LSC) and in
chapter 6 this approach is detailed. Experimental TTD results of both data and CW waveforms, which are delayed over several hundreds of nanoseconds, are presented. Data rates of 1 Gbit/s and bandwidths of 1 GHz are achieved. The tuning linearity and the resolution of this method are also analyzed.
CHAPTER 2

THEORETICAL OVERVIEW OF OPTICAL COHERENT TRANSIENTS

A basic overview of OCT's followed by the development of the coupled Maxwell-Bloch equations is given in this chapter. The basic operation of OCT's in the linear regime (energetically weak pulses) can be understood from a simple conceptual framework using a Fourier transform approach. In optically thin media, one can invoke the undepleted pump approximation that assumes the output electric field is proportional to the polarization of the thin medium. Using this approximation, the output electric field is a linear transform of the input pulses. While this is useful in describing the output from the medium, the assumptions invoked mean working in a regime of poor power efficiencies. By including the effects of propagation, where the polarization acts back on the field, highly efficient photon echoes can be created in optically thick media [10, 12, 13, 11]. To do this, the Maxwell wave equation must be used in conjunction with the optical Bloch equations that describe the dynamics of the medium. Expressions that can be solved analytically for certain cases have been shown [13] for the coupled Maxwell-Bloch equations (assuming energetically weak pulses or temporally brief pulses). The analytic solutions found were compared with direct numerical integrations of the Maxwell-Bloch equations and were found to be in good agreement [13]. However, the previous approaches to solving the propagation
effects of the medium did not allow for arbitrary phase and frequency for the input pulses [10, 12, 13]. The phase can not be ignored for pulses that are linearly chirped such as those used in this thesis. Here, a derivation for the Maxwell-Bloch equations is given for arbitrary phase and frequency. These equations are then used as the basis for a Maxwell-Bloch simulator that can predict the output of pulses with arbitrary phase and frequency in an optically thick regime. This allows analysis of the echo power efficiency for linear frequency chirped programming pulses.

**Optical Coherent Transients And The Photon Echo**

**Properties Of OCT Media**

In order to begin a discussion on the photon echo, it helps to understand some of the basic properties of the medium in which photon echoes can take place. Photon echoes have been observed in various kinds of media ranging from inorganic rare-earth-ion-doped crystals, such as Tm$^{3+}$:YAG [14], to gases of heated materials, such as barium or sodium heated in an oven [15], and even to amorphous systems of large organic molecules [16]. All of these materials, as varied as their physical makeup is, contain similar physical properties, which allow them to produce the photon echo.

First, like any optical phenomena, there must be an optical transition between two atomic levels in the material of interest. This allows a given atom within the medium to become excited, by absorbing a photon. Once an atom is in an excited state, it can leave the excited state through either spontaneous emission or stimulated
emission. The spontaneous emission decay rate $\Gamma_e$ gives rise to an exponential decay of the upper state with lifetime, $T_1 = 1/2\pi \Gamma_e$. The upper state lifetime is important in the photon echo process, as it determines the lifetime of the upper state spectral grating.

For the photon echo process, there is a fundamental limit to the amount of time delay that can be created. This limit arises due to the coherence lifetime or irreversible dephasing time in the crystal. This lifetime is given as $T_2 = 1/\pi \Gamma_H$, where $\Gamma_H$ is the homogeneous linewidth. In an inorganic crystal, this individual atomic linewidth is created due to perturbations such as lattice phonon coupling, nuclear and electron spin couplings as well as the overall population decay rate. It is known as the “homogeneous” linewidth because the broadening is experienced equally by all atoms within the medium. For demonstrations in Tm$^{3+}$:YAG, $\Gamma_H$ is mostly dominated by phonon coupling and is thus heavily dependent upon temperature [17]. Typical homogeneous linewidths for rare-earth ion doped crystals used in this thesis are tens of kilohertz, but have been measured to be as narrow as 122 Hz [18].

Another broadening mechanism, called inhomogeneous broadening, exists within these crystals. Due to imperfections within a rare-earth ion doped crystal, the local environment of a given ion can be perturbed resulting in a different resonant frequency for the ion. Since the perturbations add randomly, the inhomogeneous line typically has a Gaussian lineshape. Because the inhomogeneous broadening plays such a critical role in OCT phenomena these materials are often referred to as inhomogeneously
broadened absorbers (IBA's).

An OCT medium is effectively a frequency selective absorber of optical energy. To first order (weak intensities), interaction of the medium with the optical field is known as Beer's Law and the intensity, \( I \), of the light at a given point in the medium is given by [19]

\[
I = I_0 e^{-\alpha z}.
\]  

Here, \( I_0 \), represents the intensity of the input electric field (\( z=0 \)), \( z \) is the coordinate of propagation in the medium, and \( \alpha \) is known as the absorption coefficient. Because the amount of light absorbed from the transmitted field depends upon the interaction length, \( L \), a particular crystal can be described by the unitless parameter known as the absorption length, \( \alpha L \). The absorption length is an important parameter. The smaller \( \alpha L \) is the less important propagation effects become, allowing them to be ignored for small enough \( \alpha L \) (undepleted pump approximation). However, for \( \alpha L \) on the order of 1, propagation effects become apparent and must be included in a full analysis of the system.

**Spectral Hole Burning**

In a crystal with a large inhomogeneous linewidth, the ions within \( \Gamma_H \) of each other can be accessed by a narrowband laser without disturbing the other ions within the inhomogeneous linewidth. A laser, tuned to a specific energy within the inhomogeneous band, excites ions with that frequency to the excited state and burns a hole
in the ground state population. Figure 2 (a) shows the Guassian shaped inhomogeneously broadened transition of width $\Gamma_L$ being made up of individual Lorentzian lines with width $\Gamma_H$, each of which represent a collection of ions with equal transition frequencies (within $\pm \Gamma_H/2$). In figure 2 (b), both the ground state population and the excited state population are shown after a narrowband laser with frequency $f_L$ illuminates the crystal.

Figure 2. (a) Shows a typical Guassian shaped inhomogeneously broadened transition made up of individual Lorentzian lines from atoms at different frequencies. (b) The transition after being illuminated by a laser with frequency $f_L$. The lower trace shows a hole in the transition, where the electrons have been transferred to the excited state, upper trace.
This process is known as spectral hole burning. Essentially, a hole is created in the absorption band of the medium at a specific frequency. Hole burning has been put forward as a possible way to store information and has been studied extensively in various types of media [20]. Theoretically, the number of holes that can be burned into an ideal IBA medium is $\Gamma_r/\Gamma_H$ and is often referred to as the time-bandwidth product (TBP) of the IBA. TBP's from $10^4 - 10^8$ are possible in rare-earth ion doped crystals [18]. Though practical limitations will likely keep the upper limit of the TBP to be less than $10^6$.

**Spectral Gratings and The Photon Echo**

In 1964, N. A. Kurnit, I. D. Abella and S. R. Hartmann made the first observation of photon echoes [21]. Figure 3 shows two types of photon echoes; the two pulse photon echo, 2PE, and the stimulated photon echo, SPE. This figure shows the first two pulses, $P_1$ and $P_2$. They are separated by a time delay, $\tau_{21} = t_2 - t_1$. Here $t_i$ indicates the arrival time at the front of the medium for the i'th pulse. At a time $\tau_{21}$ after the second pulse, the 2PE occurs. Essentially $P_1$ creates coherences within the medium. These coherences begin to dephase with respect to each other due to their individual atomic frequencies. Then $P_2$ acts on the medium and flips the phases of these coherences. The coherences then begin to rephase and at $\tau_{21}$ after $P_2$ produce the 2PE.

If another pulse of light is incident upon the medium after $P_1$ and $P_2$, such as
P_3, another echo, the SPE, occurs at \( \tau_{21} \) after P_3. In this figure, the temporal pulse widths were \( \tau_p = 100 \) ns and \( \tau_{21} = 1100 \) ns. The first two pulses, P_1 and P_2, will be referred to as the programming pulses, because they program the time delay, \( \tau_{21} \), into the medium, and P_3 is the probe pulse.

![Figure 3. Experimental demonstration of the two pulse photon echo (2PE) and the stimulated photon echo (SPE). In this figure, the temporal pulse widths were \( \tau_p = 100 \) ns and \( \tau_{21} = 1100 \) ns. Experiment performed by the author.](image)

As was stated above, the OCT medium has the ability to record the frequency information of incident light pulses. This happens because the light coherently excites the atoms from the ground state into the excited state. The spectral distribution of the pulse contains more energy in certain frequencies than in others. Those frequencies with more energy tend to excite more atoms, whereas those with less energy excite less atoms. This process has the effect of imprinting the spectral distribution of the pulse into the relative atomic populations of the medium. Thus energy has been transferred from the pulse to the medium. The OCT medium can be thought of as
a storage system for the power spectrum of the programming sequence, $P_1$ and $P_2$. After programming, this absorption spectrum acts as a linear frequency-domain filter on any other input pulses, assuming certain conditions are met. It is assumed that the pulses are energetically weak. The pulse area, $\Theta$, of a pulse, is defined as

$$\Theta = \frac{\mu_{12}}{\hbar} \int E_0(t) dt.$$  \hspace{1cm} (2.2)$$

Here $\mu_{12}$ is the dipole moment and $E_0$ is the slowly varying envelope of the electric field. If a pulse is energetically weak, the pulse area is small ($\Theta \ll 1$). Along with this, the system is taken to be optically thin ($\alpha L \ll 1$), allowing the undepleted pump approximation. With these assumptions, the output from the OCT system follows linear filter theory.

Under the linear filter theory, the output of the system is a combination of the input response, $f(t)$, with how the system was programmed, $h(t)$. In the case described above, the system was programmed with two incident optical pulses. The inhomogeneously broadened crystal stores the power spectrum of these combined pulses provided $\tau_{21} < T_2$. If the incident optical fields are defined by $E_1(t)$ and $E_2(t)$, the power spectrum of these two pulses is

$$\tilde{H}(\omega) = |\tilde{E}_1(\omega)|^2 + |\tilde{E}_2(\omega)|^2 + \tilde{E}_1^*(\omega)\tilde{E}_2(\omega) + \tilde{E}_1(\omega)\tilde{E}_2^*(\omega).$$  \hspace{1cm} (2.3)$$

Here $\tilde{E}_i(\omega)$ represents the electric field of the $i$'th pulse in the frequency domain. The first two terms are the power spectra of just the individual pulses themselves. The third and fourth terms represent the spectral interference of the two pulses. These
two terms are analogous to the interference terms encountered in spatial holography.

The important contribution from the power spectrum to the stimulated photon echo is the first cross term and thus the power spectrum is rewritten as

\[ \tilde{H}(\omega) = \tilde{E}_1^*(\omega)\tilde{E}_2(\omega) , \]  

(2.4)

where the other terms are ignored. In the linear filter theory, \( \tilde{H}(\omega) \) is known as the frequency-domain response function. The Fourier transform of a function is defined as

\[ \tilde{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt . \]  

(2.5)

In subsequent sections the Fourier transform can be identified by the "\( \omega \)" in the argument. The inverse Fourier transform is then

\[ f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{F}(\omega) e^{-i\omega t} d\omega . \]  

(2.6)

In the linear filter theory, the output from the programmed system, defined \( g(t) \), is related to the input of the system, \( f(t) \), by the relation

\[ g(t) = \int_{-\infty}^{\infty} h(t-\tau)f(\tau) d\tau . \]  

(2.7)

The function \( h(t) \) is called the time-domain response function. The function, \( f(t) \), represents the third optical pulse, \( E_3(t) \) (or other subsequent pulses), incident upon the OCT medium. The function, \( g(t) \), can be examined in the frequency-domain by taking the Fourier transform giving

\[ \tilde{G}(\omega) = \tilde{H}(\omega) \tilde{F}(\omega) . \]  

(2.8)
Now the system's response to the third pulse, $\tilde{F}(\omega) = \tilde{E}_3(\omega)$, can be written in the frequency domain as

$$\tilde{E}_{\text{out}}(\omega) \propto \tilde{E}_1^*(\omega)\tilde{E}_2(\omega)\tilde{E}_3(\omega). \quad (2.9)$$

If the two programming pulse envelopes are Guassian in nature, the Fourier transform of each individual pulse is itself a Guassian envelope in the frequency-domain. There is a time shift between these two Guassian input pulses, $\tau_{21}$. Because of this time delay, the Fourier transform of the second pulse includes a linear phase shift with respect to the Fourier transform of the first pulse, that is dependent upon the frequency and time delay, $e^{-i\omega \tau_{21}}$. This linear phase shift gives rise to a modulation in the power spectrum and thus a spectral grating in the medium with a period of $1/\tau_{21}$.

Much like spatial gratings, which can act to diffract pulses in the spatial domain, a spectral grating can diffract a pulse in the time domain. If the period of a spatial grating is tuned, the angular output for an incident wavelength is changed. If the period of a spectral grating is tuned, the time delay of the output is changed.

In figure 4 (a) the electric field of two programming pulses are shown with $\tau_{21} = 10$ ns. In order to simulate the output from the above linear analysis, the electric field must be oscillating much faster than the slowly varying envelope of the optical pulse. This essentially means that the pulse's bandwidth is much less than the carrier frequency. Here the total power spectrum is simulated so effects due to both spectral interference terms in eqn. (2.3) are included.
Figure 4. (a) Shows the two programming pulses, FWHM ≈ 1 ns, \( \tau_{21} = 10 \) ns. (b) Shows the resulting power spectrum. (c) The expected output as calculated using non-causal linear filter theory. One can see the SPE and the non-causal virtual echo, or VE.
Here the frequency of the electric field is set to 10 GHz and makes the pulse appear dark in the figure. The medium stores the power spectrum of these two pulses, which is shown in (b). The medium's excited state population would be directly proportional to this plot. Notice that there is a periodic spectral structure, that gives rise to the medium's stored spectral grating. The time period of this periodic structure is \(1/\tau_{21}\) and the overall spectral width of the grating is related to the temporal width of the individual pulses as \(\approx 1/\tau_p\). Finally, (c) shows the output electric field from a probe pulse at \(t = 30\) ns, which was diffracted off of the grating. Notice the echo output at \(\tau_{21}\) after the probe pulse. One will also notice the non-causal virtual echo, labeled VE, that happens \(\tau_{21}\) before the probe pulse. The occurrence of this echo is an artifact of the simulation. By causality, only signals which occur after the probe pulse are real.

By using the above framework, the systems response to various types of programming and probing can be predicted. One can use eqn. (2.9), to predict the temporal output from the medium. For example, if the first two input pulses are bandwidth limited temporal delta functions and have temporal widths, \(\tau_1\) and \(\tau_2\), which are much less than the temporal width, \(\delta \tau_3\), of a third input Guassian pulse (pulse widths at FWHM \(\tau_1 = \tau_2 \ll \delta \tau_3\), the respective power spectrums of the first two pulses can be considered constant over the bandwidth of the third pulse. Throughout this thesis, if a pulse's total temporal duration is short compared to other pulses incident upon the media, it will be referred to as a brief pulse. Here for example, pulses 1 and 2 are brief pulses. The first two pulses (with \(\tau_{21} < T_2\)) have a cross term in the
power spectrum with respect to each other giving rise to a periodic spectral feature for $\mathcal{H}(\omega)$. The incident third pulse (with $T_2 < \tau_3 < T_1$) gives the output from the system (the photon echo) whose respective power spectrum is exactly the same as the third Guassian pulse except now with the periodic spectral feature. If the third pulse was centered at a time, $t_3$ the echo output will occur at $t_3 + \tau_2$ due to the periodic spectral feature. This is the description of the SPE process.

Next, assume that the third pulse is not a Guassian pulse, but instead has some temporal structure. Here the fastest temporal structure of the pulse has a width $\delta t_3$ and the same condition from above holds ($\tau_1 = \tau_2 \ll \delta t_3$). Then, using (2.9),

$$\tilde{E}_{\text{out}}(\omega) \propto \tilde{E}_3(\omega)e^{-i\omega \tau_2}.$$  

This means that the SPE will mimic the probe pulse exactly, but delayed by $\tau_2$. This is just the TTD process discussed earlier. But echo processes are not limited to just TTD. One may notice that if the first pulse is some arbitrary input starting at $t_1$ (here the fastest temporal structure is $\delta t_1$, the second pulse is a brief reference pulse at $t_2$ ($\tau_2 \ll \delta t_1$), and the third pulse is some other arbitrary input pulse starting at $t_3$ ($\tau_2 \ll \delta t_3$), the echo output is found to be

$$\tilde{E}_{\text{out}}(\omega) \approx \tilde{E}_1^*(\omega)\tilde{E}_3(\omega)e^{-i\omega \tau_2}.$$  

(2.10)

When Fourier transformed to find the time-domain response, this is just the cross-correlation of $E_1$ and $E_3$ with a time delay of $\tau_2$, which has applications in correlating arbitrary signals with a desired signal [22].
Possible Methods of Solution For The Photon Echo

In the above linear filter theory approach to a solution for the output from the OCT system, several assumptions were made. These were that the input optical pulses were energetically weak, and that the medium was optically thin. If the pulses are energetically weak (i.e. the system is kept from saturating), the system records the power spectrum of the pulse as weak spectral features in the populations. Had the input optical pulses been strong enough to induce saturation, the spectral features stored in the populations are distorted and do not exactly resemble the power spectrum of the pulse that created them. Thus, the predicted output from the system using equation (2.9), would not represent the actual output. Also, the linear filter theory approach (given in eqn. (2.9)) to a solution breaks down as the absorption length increases. There are other approaches to predicting echo phenomena, ranging from easy analytic solutions to computationally complex numerical solutions. This section presents an overview of some possible solution approaches for various situations and their range of validity.

It is important to understand the limitations of the approach being used to predict echo efficiencies in TTD configurations. Figure 5 shows the effect of a thick medium on the power efficiency, $\eta$ of the 2PE and the SPE for two types of solutions. Here the echo power efficiency, $\eta$ is defined as the ratio of the echo peak power to the probe pulse peak input power. The probe pulse for the 2PE is the second pulse and the probe pulse for the SPE is the third pulse. In the figure, $\eta$ is plotted versus the absorption
length ($\alpha L$) of the medium. In this plot, the pulses were not energetically weak (pulse areas $\Theta_1 = \Theta_2 = \Theta_3 = \pi/2$) and all had the same temporal duration. In the linear regime, $\eta$ is expected to have a quadratic dependence on $\alpha L$, as will be discussed later. Quadratic fits predicted from the linear filter theory are shown in this plot with solid lines and labeled with "Q". This quadratic dependence continues to grow as the thickness of the medium increases, predicting extremely high echo efficiencies for large $\alpha L$. The numerically integrated solutions, shown with dotted lines and labeled with "MB", were solved with a full Maxwell-Bloch treatment of the medium (discussed below). As can be seen, the Maxwell-Bloch predicted echo efficiency initially follows the quadratic solutions of the linear filter theory. It then begins to fall off, peaking at values of $\alpha L \approx 2$ for the 2PE and $\alpha L \approx 3$ for the SPE. This fall off can be partly attributed to the fact that energy from the incident programming pulses is absorbed by the medium. Thus, further into the medium, the medium sees weaker and weaker programming pulses. At some point, the polarization which produces the echo is no longer creating enough optical energy to compete with the absorption of the medium. Subsequently the echo begins to decay away, as it continues to propagate through the medium. As this plot clearly shows, thick crystals can be utilized to increase echo efficiencies as compared to the optically thin case. However, there is an optimal thickness that depends on the application and programming strengths available.
Figure 5. Quadratic fits predicted from linear filter theory (solid and labeled with "Q") and numerical solutions (dashed and labeled with "MB") for the 2PE and the SPE power efficiencies. Here the quadratic solution was valid only for $aL \ll 1$. The numerical solutions were found using a full Maxwell-Bloch treatment of the medium and give a more realistic picture of the true behavior of the system. Here $\Theta_1 = \Theta_2 = \Theta_3 = \pi/2$.

The two solution methods shown in figure 5, represent vastly different amounts of calculational complexity. The linear filter theory, as shown above, results in simple analytic solutions for the output echo. These equations are often very easy to solve but only work under limited conditions. Whereas, the full Maxwell-Bloch treatment for a solution requires direct numerical integration of both the Bloch and Maxwell equations in order to determine the solution but can be used in almost any situation. These computationally intense algorithms can run for hours and require vast amounts
of memory. Solution methods also exist between these two extremes. Over the years many possible methods to predict photon echo processes have been developed [23, 24, 5, 13]. Each of these possible approaches to predicting the outputs are valid under certain conditions. Due to the significant amount of methods, and also the significant amount of conditions for their validity, a “roadmap” is provided here, to help direct the lonely OCT traveler towards a proper predictive tool. In figure 6, this OCT “roadmap” is shown. Here the traveler should start with an input, in this case a specific pulse in a sequence, and proceed to answer the questions on the “road” about that pulse. As a traveler answers questions, he/she will obtain indications about formulas to use, conditions for validity, and ultimately by combining the two, a predictive tool for that specific pulse. This process is repeated for all pulses in a sequence and the resulting formulas and conditions marked down for all the pulses. After all pulses have been processed with the “roadmap”, some reflection and thought is required on the part of the traveler. Certain methods can be effectively combined, resulting in quick solutions, which require little computational effort. But sometimes it doesn’t pay to mix and match solutions: For instance, if one pulse needs to be completely determined using the full computational complexity of a Maxwell-Bloch simulator, it will make sense to use this simulator for all pulses in the sequence and not worry about trying to combine solutions.
Figure 6. A diagram showing different approaches to a predictive tool for photon echoes. Each pulse in a sequence should be treated with this “roadmap”. One asks the specific questions about the pulse, slowly traveling through the “roadmap”, marking down the specific conditions, and equations along the way. Once an output is reached an equation for the polarization, \( P \), should have been determined as well as an equation for the output field in terms of that \( P \). Solution types can be mixed, see text for discussion.
The Maxwell wave equation for the OCT medium can be written as

\[ \nabla^2 E(z, t) - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} E(z, t) = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2}{\partial t^2} P(z, t). \] (2.11)

Here \( P(z, t) \) represents the macroscopic polarization of the medium. The field is written as

\[ E(z, t) = E_0(z, t) \cos(\omega t - k_z z + \phi(z, t)). \] (2.12)

Here again, \( E_0(z, t) \) is the slowly varying envelope, \( \omega t \) is the optical angular frequency, \( k_z \) is the wavevector in the direction of propagation and \( \phi(z, t) \) is some slowly varying arbitrary phase function. Note that in order to solve for the field, the polarization must be known. However, this polarization is also dependent upon the input field. Thus, there are two major parts for a predictive solution of a photon echo process. The first is to determine the macroscopic polarization within the crystal. This polarization determines the absorption on an input pulse, or subsequently, the emission of light in the form of a photon echo or other free induction decay phenomena. As will be shown later in this chapter, the atomic system for a specific location can be adequately modeled by a set of equations of motion called the Bloch equations. Through the use of these equations, the macroscopic polarization at a specific location in the crystal can be determined for any sequence of pulses. By utilizing certain assumptions, it has been found that these equations can be reduced in complexity. The second part to a solution is to describe the propagation of the field through the medium. When the medium is optically thin, the output electric field from the system is directly
proportional to the macroscopic polarization of the medium. Whereas, in optically thick cases, the optical pulses have to propagate through the medium, and interactions with the medium must be considered at every point along its path. The "roadmap" shown in figure 6 has both of these major components. As a pulse is taken through this "roadmap", equations or approaches to determine the macroscopic polarization, \( P(z,t) \) are given as well the proper equations or approaches to predict the propagation effects on the pulses and outputs. Next the approaches to determining the polarization and the propagation are outlined.

**Exact Solutions.**

Analytic expressions have been derived for a square pulse incident upon a two level system such as the famous solution by I. I. Rabi [25]. This solution, while originally intended for a spin 1/2 system, is applicable to optical pulses incident upon OCT media. If the pulse was a brief pulse, the solution only had a dependence on detuning in the phase factors. A transformation matrix gives a solution for the off diagonal elements of the density matrix as [24]

\[
\rho_{12}(t_f) = \rho_{21}^*(t_f) = \cos^2\left(\Theta_o/2\right) \rho_{12}(t_i) + \sin^2\left(\Theta_o/2\right) \phi_o^2 \rho_{21}^*(t_i) \\
- \frac{i\phi_o}{2} \sin\left(\Theta_o\right) \left(\rho_{22}(t_i) - \rho_{11}(t_i)\right) 
\]

(2.13)

The factor \( \phi_o \) has dependence upon the electric field strength, the dipole moment, and a phase dependence upon the detuning. Here, \( \Theta_o \) is the pulse area of the incident brief pulse, \( t_i \) is the time before the pulse and \( t_f \) is the time after application of the pulse.
The macroscopic polarization, for atoms in the inhomogeneously broadened crystal between frequencies $\omega$ and $\omega + d\omega$, is dependent upon these off diagonal elements as

$$P(z, t, \omega) = \mu_{12} \rho_{12}(z, t, \omega) \exp(i\omega t)Ng(\omega)d\omega + c.c. \quad (2.14)$$

Here $\mu_{12}$ is the dipole moment of the atoms, $N$ is the number density of the atoms, and $g(\omega)$ is a weighting function determined by the inhomogeneous broadening. Through the integration of this equation over $\omega$, the macroscopic polarization can be determined exactly with brief pulses and eqn. (2.13).

**Fourier Transform Approximation.**

The Fourier transform approximation, originally proposed by T. W. Mossberg [23], utilizes the fact that each frequency component of the optical field acts separately on the resonant atoms of that frequency in the weak field regime. The pulse area, previously defined as $\Theta$ is now defined for the frequencies other than the carrier frequency as

$$\Theta(\omega) = \frac{\mu_{12}}{\hbar} \int E_0(t) e^{-i\omega t} dt \quad (2.15)$$

If the pulse area for all frequencies is small enough, $\Theta(\omega) \ll 1$ (all $\omega$), the ground state spectral population distribution can be written as

$$\rho_{11}(\omega) \approx 1 - \Theta^2(\omega)/4 \quad (2.16)$$

The off diagonal elements are then solved for, using the transformation matrix approach described above. This allows for solution of the macroscopic polarization. The
contribution of this polarization to the SPE for a three pulse sequence can then be solved analytically, giving
\[ P(t) = \frac{i|\mu_{12}|^2}{\hbar} \int E_1^*(\omega)E_2^*(\omega)E_3(\omega)e^{-i\omega(t-\tau_3)} d\omega. \] (2.17)

This fundamental result is of tremendous value to the prediction of photon echoes, as was shown above in the linear filter theory. For further discussion of this approach see \[24, 5\].

**Bloch Equations Of Motion.**

If the pulses are too strong to treat using the Fourier transform approximation, the full Bloch equations of motion must be utilized. A generalized version of these equations will be derived below, and an expression for the macroscopic polarization, in terms of the Bloch vector components will be given. A numerical solution for this polarization is achievable by following the optical Bloch vector for the set of inhomogeneously broadened atoms. Essentially, the equations of motion for each atom is considered as it is resonant with the frequency components in the optical field.

**Thin Crystal \( \alpha L \ll 1. \)**

Along with the approaches to find the polarization, equations to determine the propagation must be considered. Now, the attention is turned to how the polarization, in conjunction with the Maxwell wave equation, can be used to get an output. The easiest approach to solving for the output field from an OCT medium is to assume the
medium is optically thin. As discussed above, this assumption allows the electric field to be directly proportional to the macroscopic polarization, thus, there is no need to take into account the propagation of the output fields. As the medium is optically thin, the absorption of the electric field is negligible and the approach is also known as the undepleted pump approximation. This method allows quick calculations, as no spatial integration is needed, however the regime in which it is valid yields poor echo efficiencies. For a thin $\alpha L$ then, the Fourier transform of the output field is written as [24]

$$E_o(L, \omega) = \frac{-i \hbar \omega}{2 \pi |\mu_{12}|^2} P(z = 0, \omega).$$ (2.18)

Here the Fourier transform of the polarization at the front of the medium $P(z = 0, \omega)$ is representative of $P(z, \omega)$ throughout the medium. For the SPE process if $\Theta_1, \Theta_2, \Theta_3, \ll 1$, $P(z = 0, \omega)$ is given as

$$P(z = 0, \omega) = \frac{i |\mu_{12}|^2}{\hbar} E_1^*(\omega) E_2(\omega) E_3(\omega) e^{-i \omega z_1}.$$ (2.19)

**Thick Crystal $\alpha L \geq 1$, Weak Pulses $\Theta \ll 1$.**

The propagation effects due to the Maxwell equation, can be utilized in the case for a thick crystal. If $\Theta \ll 1$, the effects of propagation can be adequately handled by Beer’s law (2.1), which can be utilized to get an accurate output field. This method allows a simple $z$ dependence of both the output field and the polarization. Essentially, as a field travels into the medium, the field strength will decay exponentially. Thus, the polarization as a function of propagation distance, will decay due to the decay
of the input field. The output field generated also decays due to absorption. The propagation of the field through the medium can then be written [24]

\[ \frac{dE_o(x, \omega)}{dz} = -\frac{\alpha(\omega)}{2} E_o(x, \omega) - \frac{iL\hbar \alpha(\omega)}{2\pi|\mu_{12}|^2} P(z = 0, \omega) \exp \left( -\frac{3\alpha(\omega)z}{2} \right). \]  

The exponential dependence on z in the term for the polarization is valid for either the 2PE or the SPE process. This equation has been solved exactly as

\[ E_o(L, \omega) = -\frac{i\hbar}{2\pi|\mu_{12}|^2} P(z = 0, \omega) (1 - \exp(-\alpha(\omega)L)) \exp \left( -\frac{\alpha(\omega)L}{2} \right). \]  

The polarization is given for the SPE process in eqn. (2.19).

**Thick Crystal \( \alpha L \geq 1 \), Area Theorem Approach.**

The approach outlined in eqn. (2.20) requires that the incident optical pulses all be weak. There is another approach to solving for the output in thick media analytically. This approach was outlined by Cornish [13]. In this case, both analytic solutions to the system as well as the Fourier transform approximation were utilized in thick media assuming that \( \alpha(\omega) \) is flat over the bandwidth of all incident pulses \( \alpha(\omega) = \alpha \). Here, the system responding to a brief pulse can be solved for analytically by utilizing the area theorem. This theorem states that the pulse area must obey [26]

\[ \frac{d\Theta(z)}{dz} = -\frac{\alpha}{2 \sin(\Theta(z))}. \]  

Through the use of this equation, the overall effect of the pulse on the polarization can be determined at any point in the medium. However, the output pulse shape during the brief pulse can not be determined, only its area. In this case, it is assumed that the
brief pulse need not be energetically weak, but that data pulses are energetically weak. Thus, these data pulses can be solved with the Fourier transform approximation. The systems response to the different pulses is examined after each pulse and then the system is examined where the echo output is expected. In each temporal window, the solution for the pulses are treated with Maxwell's wave equation giving the proper effects due to propagation. Calculations performed in this manner gave analytic echo efficiencies within 1% for brief pulse areas $\Theta_{\text{brief}} < 0.6\pi$ of numerically simulated results, as well as adequately predicting the output data shape. Readers are also referred to this method, as it gives a good explanation of how two solution approaches can be used in combination, allowing a combined predictive tool for the output from the medium. As discussed above, brief pulses of any pulse area can be used with this approach. However, due to possible pulse shaping problems [26], the output bandwidth, $B_{\text{out}}$, of the input brief pulse can change. If this output bandwidth is still significantly larger than any other j'th pulse bandwidth, $B_j$, that is $B_{\text{out}} \gg B_j$, it may still be possible to use this approach. However, due to possible pulse reshaping events, such as pulse breakup [19], the signal fidelity predicted using this approach, may be better than that observed in experiments. It is therefore suggested that pulse areas less than $\pi$ only be used with this approach.

Finally, the system may not be optically thin and the input pulses are all strong, requiring that the Maxwell wave equation be used in conjunction with the optical
Bloch vector. In this situation, the coupled Maxwell-Bloch equations must be directly integrated. These Maxwell-Bloch simulators are very powerful tools as they can handle any arbitrarily shaped pulse, any pulse area, and any optical thickness. However, there are drawbacks. These simulators are computationally demanding and require large amounts of memory. For problems such as accumulated programming sequences, the simulator may need to run for days. Despite these problems, the Maxwell-Bloch simulator has been useful in predicting greater than unit echo efficiencies \[12, 10\] as well as describing the energy source of the photon echo \[13\].

**Maxwell-Bloch With Arbitrary Phase**

Because of the complexity of the equations involved in treating the OCT medium quantum mechanically and including propagation, there are few analytic solutions for an optically thick medium interacting with strong pulses.\(^1\) In order to successfully predict the material's behavior to pulses of arbitrary shape and phase when these pulses are strong and the medium is optically thick one must resort to numerical integration of the Maxwell-Bloch equations.

Bloch simulators, which are based upon numerically integrating the equations of motion for the optical Bloch vector, assume \(\alpha L \ll 1\) as discussed above. This was so that the undepleted pump approximation could be assumed and thus the output

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See for example the famous self-induced transparency of McCall and Hahn [26]. Another analytic solution utilizing transformation matrices is given in [27] generalizing the McCall and Hahn solution.
electric field would be directly proportional to the macroscopic polarization. In this approximation, all the atoms see the same incident pulses, thus there is no depletion of these pulses as they travel through the medium. These simulators have been successful in predicting the dynamics of the system for many different input pulse sequences including chirped pulses [5]. Issues relating to the timing of echoes, saturation of the media and accumulation of pulse sequences have all been simulated using this approach [5, 28, 29]. However, since the Bloch simulator is limited to optically thin media, the results are not accurate when the output power efficiencies of the echo approach ($\sim 10^{-2}$). As advancements were made in research, it became apparent that the optimal optical thickness is $\alpha L \gtrsim 1$, though the actual optimal value to obtain the highest efficiency varies with application. However, the highest efficiency is not always the best [30] due to the effects of saturation. Along with efficiency, the fidelity of the echoes must be examined and ultimately, a balance between efficiency and good fidelity sought. In order to predict and study these phenomena, simulators based upon the coupled Maxwell-Bloch equations were developed. These simulators were used to predict highly efficient photon echo generation [12, 10] before their detection [11] for an optical storage application. The utility of such simulators, allows the dynamics of the population gratings and coherences to be followed through the thick medium. With these tools, the energy source for the photon echo was also determined [13].

The goal of this thesis is to demonstrate TTD of a broad bandwidth signal, with high efficiencies and good signal fidelity. It is known that chirped programming
provides a better overall method to achieve OCT TTD. To date, however, Maxwell-Bloch simulators have been based upon a field whose arbitrary phase function is zero. If the electric field is written

\[ E(z, t) = E_0(z, t) \cos(\omega t - k_0 z + \phi(z, t)), \]  

(2.23)

the previous simulators kept \( \phi = 0 \). Note that \( E_0(z, t) \) is the time dependent, slowly varying envelope function, which allows arbitrarily shaped pulses. The requirement that \( \phi = 0 \) does not allow simulation of pulses that have frequency shifts with respect to each other or for frequency chirped pulses. Thus, to properly study echo efficiencies, signal fidelity, and echo timing, the full Maxwell-Bloch equations, including arbitrarily ramping phase, had to be used. A simulator based upon these equations can then be utilized to study the power efficiencies, signal fidelities, and timing of gratings programmed with frequency offset linear frequency chirps. Ultimately, this simulator can be used to predict the optimal absorption length for efficient, high fidelity echoes. It is through this optimization and subsequent improvement in echo strength that will make OCT TTD applications attractive.

Two Level Systems and the Bloch Equations

In order to begin a discussion on the Maxwell-Bloch equations, one must have a bit of background in the quantum mechanical treatment of two level systems and their interaction with electric fields. The two levels in the system are described as \( |1\rangle \) and \( |2\rangle \) for the ground state and excited state, respectively. If the energy difference
between the two levels is \( \hbar \omega_a \), the Hamiltonian, \( \hat{H}_0 \), for the time independent two
level system is then

\[
\hat{H}_0 = \begin{pmatrix} 0 & 0 \\ 0 & \hbar \omega_a \end{pmatrix}.
\] (2.24)

If an electric field, such as in eqn. (2.23) is introduced the Hamiltonian must be
modified. The generalized Hamiltonian, \( \hat{H} \), can be found by adding a term to the
time independent Hamiltonian that appropriately takes into account the interaction
of the system with the electric field. Thus \( \hat{H} \) can be written

\[
\hat{H} = \hat{H}_0 + \hat{V}(z, t).
\] (2.25)

This interaction, \( \hat{V}(z, t) \) is described by the dipole interaction

\[
\hat{V}(z, t) = -\hat{\mu} E(z, t) = -\begin{pmatrix} 0 & \mu_{12} \\ \mu_{21} & 0 \end{pmatrix} E(z, t).
\] (2.26)

It is assumed that the field, \( E(z, t) \) and the dipole moment are aligned and that dipole
moment operator acts between the levels of interest. In this case, we do not consider
transitions between levels of different spin, and therefore do not account for changes
in polarizations of the fields. With these assumptions, \( \mu_{12} = \mu_{21} = \mu \) and can be
assumed real for simplicity. Finally, the total Hamiltonian can be expressed as

\[
\hat{H} = \begin{pmatrix} 0 & -\mu E(z, t) \\ -\mu E(z, t) & \hbar \omega_a \end{pmatrix}.
\] (2.27)

In order to obtain the Bloch equations, the density matrix formalism will be used.

For the two level system, the density matrix elements are

\[
\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}.
\] (2.28)
The equation of motion, or Schroedinger's equation, using a density matrix is given by

\[ \dot{\rho} = \frac{i}{\hbar} [\rho, H] = \frac{i}{\hbar} (\rho H - H \rho) \tag{2.29} \]

The differential equations for the individual density matrix elements are then

\[ \rho_{11} = \frac{i \mu E(z, t)}{\hbar} (\rho_{21} - \rho_{12}) \]
\[ \rho_{12} = \frac{i}{\hbar} (\mu E(z, t) (\rho_{22} - \rho_{11}) + \hbar \omega_a \rho_{12}) \]
\[ \rho_{21} = (\rho_{12})^* \]
\[ \rho_{22} = -\dot{\rho}_{11}. \tag{2.30} \]

To simplify the equations, it is common to choose a rotating frame. This rotating frame allows terms that are rapidly varying (frequencies of order \(2\omega_a\)) compared to the timescale of interest to be dropped. To do this the frame is rotated at a constant angular frequency, \(\omega_a\), which is close to the transition frequency. Now the density matrix elements, \(\varrho_{ij}\) in the rotating frame are defined as

\[ \varrho_{11} = \rho_{11} \]
\[ \varrho_{22} = \rho_{22} \]
\[ \varrho_{12} = \rho_{12} e^{-i\omega_a t} \]
\[ \varrho_{21} = \rho_{21} e^{i\omega_a t}. \tag{2.31} \]
Taking the eqns. (2.30) and substituting in the rotating frame elements gives the on-axis elements

\[ \dot{e}_{11} = \frac{i \mu E(z, t)}{\hbar} (\dot{e}_{22} e^{-i\omega_0 t} - \dot{e}_{12} e^{i\omega_0 t}) \]
\[ \dot{e}_{22} = -\dot{e}_{11} \] (2.32)

Using the off-axis description in eqn. (2.30) and taking the time derivative of eqn. (2.31) gives the off-axis equation

\[ \dot{e}_{12} = \rho_{12} e^{-i\omega_0 t} - i\omega_0 \rho_{12} e^{-i\omega_0 t} \] (2.33)

Substituting in from eqn. (2.30) gives

\[ \dot{e}_{12} = \left( \frac{i}{\hbar} \left( \mu E(z, t) (\dot{e}_{22} - \dot{e}_{11}) + \hbar \omega_0 \dot{e}_{12} e^{i\omega_0 t} \right) \right) e^{-i\omega_0 t} - i\omega_0 \dot{e}_{12} \]
\[ = \frac{i}{\hbar} \mu E(z, t) (\dot{e}_{22} - \dot{e}_{11}) e^{-i\omega_0 t} + i (\omega_a - \omega_o) \dot{e}_{12} \]
\[ \dot{e}_{21} = (\dot{e}_{12})^* \] (2.34)

Now the electric field can be entered and the formulas simplified. Here the electric field is rewritten as follows

\[ E(z, t) = \frac{\hbar}{2\mu} \Omega(z, t) \left( e^{i(\omega_1 t - k_z z + \phi(z,t))} + e^{-i(\omega_1 t - k_z z + \phi(z,t))} \right) \] (2.35)

Notice that the envelope function has been changed to have frequency units, \( \Omega(z, t) = \frac{\mu}{\hbar} E_0(z, t) \), where an explicit \( z \) and \( t \) dependence of the field is assumed. Entering the
electric field into the eqn. (2.32) the on-axis elements become

\[\dot{\varrho}_{11} = \frac{i\mu}{\hbar} \left( \varrho_{21} e^{-i\omega_0 t} - \varrho_{12} e^{i\omega_0 t} \right) \frac{\hbar}{2\mu} \Omega \left( e^{i(\omega_1 t - k_z z + \phi(z,t))} + e^{-i(\omega_1 t - k_z z + \phi(z,t))} \right)\]

\[= \frac{i\Omega}{2} \left( \varrho_{21} e^{-i\omega_0 t} e^{i(\omega_1 t - k_z z + \phi(z,t))} + \varrho_{21} e^{-i\omega_0 t} e^{-i(\omega_1 t - k_z z + \phi(z,t))} \right)\]

\[-\varrho_{12} e^{i\omega_0 t} e^{i(\omega_1 t - k_z z + \phi(z,t))} - \varrho_{12} e^{i\omega_0 t} e^{-i(\omega_1 t - k_z z + \phi(z,t))}\]

Notice that there are two terms in this last equation that oscillate extremely rapidly, one with \(e^{i(\omega_0 + \omega_1)t}\) and its complex conjugate. This oscillation essentially averages to zero on the time scale of the change in the density matrix elements and the two terms can be dropped or neglected. This is known as the rotating wave approximation. In this case then, the expression becomes

\[\dot{\varrho}_{11} = \frac{i\Omega}{2} \left( \varrho_{21} e^{i((\omega_1 - \omega_0)t - k_z z + \phi(z,t))} - \varrho_{12} e^{-i((\omega_1 - \omega_0)t - k_z z + \phi(z,t))} \right) . \quad (2.36)\]

By defining \(\gamma = (\omega_1 - \omega_0)t - k_z z + \phi(z,t)\) the equations further simplify to

\[\dot{\varrho}_{11} = \frac{i\Omega}{2} \left( \varrho_{21} e^{i\gamma} - \varrho_{12} e^{-i\gamma} \right)\]

\[= \frac{\Omega}{2} \left( i(\varrho_{21} - \varrho_{12}) \cos \gamma - (\varrho_{21} + \varrho_{12}) \sin \gamma \right) . \quad (2.37)\]

And the off axis elements become

\[\varrho_{12} = i(\omega_a - \omega_0) \varrho_{12} + \frac{i}{2} \Omega e^{i\gamma} \left( \varrho_{22} - \varrho_{11} \right) , \quad (2.38)\]

after the rotating wave approximation is made. Finally, the elements of the Bloch
vector in the rotating frame, \( \mathbf{r} \), are defined as

\[
\mathbf{r}_1 = \mathbf{q}_{21} + \mathbf{q}_{12} \\
\mathbf{r}_2 = i(\mathbf{q}_{21} - \mathbf{q}_{12}) \\
\mathbf{r}_3 = \mathbf{q}_{22} - \mathbf{q}_{11} .
\]

The time derivative for \( \mathbf{r}_1 \) is

\[
\dot{\mathbf{r}}_1 = \mathbf{q}_{21} + \mathbf{q}_{12} \\
= -i(\omega_a - \omega_o) \mathbf{q}_{21} - i \frac{i}{2} \Omega e^{-i\gamma} (\mathbf{q}_{22} - \mathbf{q}_{11}) + i(\omega_a - \omega_o) \mathbf{q}_{12} + i \frac{i}{2} \Omega e^{i\gamma} (\mathbf{q}_{22} - \mathbf{q}_{11}) \\
= -i(\mathbf{q}_{21} - \mathbf{q}_{12}) (\omega_a - \omega_o) + i \frac{i}{2} \Omega (\mathbf{q}_{22} - \mathbf{q}_{11}) (e^{i\gamma} - e^{-i\gamma}) \\
= - (\omega_a - \omega_o) \mathbf{r}_2 - \Omega \mathbf{r}_3 \sin \gamma .
\]

Similarly \( \mathbf{r}_2 \) is found to be

\[
\dot{\mathbf{r}}_2 = i(\mathbf{q}_{21} - \mathbf{q}_{12}) \\
= i \left( -i(\omega_a - \omega_o) \mathbf{q}_{21} - i \frac{i}{2} \Omega e^{-i\gamma} (\mathbf{q}_{22} - \mathbf{q}_{11}) - i(\omega_a - \omega_o) \mathbf{q}_{12} - i \frac{i}{2} \Omega e^{i\gamma} (\mathbf{q}_{22} - \mathbf{q}_{11}) \right) \\
= (\omega_a - \omega_o) (\mathbf{q}_{21} + \mathbf{q}_{12}) + i \frac{i}{2} \Omega (\mathbf{q}_{22} - \mathbf{q}_{11}) (e^{-i\gamma} + e^{i\gamma}) \\
= (\omega_a - \omega_o) \mathbf{r}_1 + \Omega \mathbf{r}_3 \cos \gamma .
\]

And finally for \( \mathbf{r}_3 \),

\[
\dot{\mathbf{r}}_3 = \dot{\mathbf{q}}_{22} - \dot{\mathbf{q}}_{11} = -2\dot{\mathbf{q}}_{11} \\
= -2 \left( \frac{\Omega}{2} (i(\mathbf{q}_{21} - \mathbf{q}_{12}) \cos \gamma - (\mathbf{q}_{21} + \mathbf{q}_{12}) \sin \gamma) \right) \\
= \Omega \mathbf{r}_1 \sin \gamma - \Omega \mathbf{r}_2 \cos \gamma .
\]
These equations give the time evolution of the Bloch vector, otherwise known as the Bloch equations and can be used to effectively determine the localized effect of the field on the medium.

\[\begin{align*}
  \dot{r}_1 &= -(\omega_a - \omega_b) r_2 - \Omega r_3 \sin \gamma \\
  \dot{r}_2 &= (\omega_a - \omega_b) r_1 + \Omega r_3 \cos \gamma \\
  \dot{r}_3 &= \Omega r_1 \sin \gamma - \Omega r_2 \cos \gamma
\end{align*}\] (2.43)

**Maxwell-Bloch Derivation**

If the field, defined in eqn. (2.23), acts upon the medium and in turn the medium acts back on the field, then the classical Maxwell equations must be coupled with the Bloch equations in order to predict the output from the medium. Note that through eqn. (2.35) solutions are restricted to forward propagating fields. Here, the Maxwell equation can be written in the rest frame as

\[\nabla^2 E(z, t) - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} E(z, t) = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2}{\partial t^2} P(z, t)\] (2.44)

Here the envelope of the field, the phase (and thus the Bloch vectors) are assumed to be slowly varying with respect to the optical frequency. This assumption, known as the slowly varying envelope approximation, will be used below to drop small terms [31]. Using the field as defined in eqn. 2.23, the slowly varying envelope approximation assumes

\[\frac{\partial E_o}{\partial z} \ll k_z E_o, \quad \frac{\partial E_o}{\partial t} \ll \omega_t E_o, \quad \frac{\partial \phi}{\partial z} \ll k_z, \quad \frac{\partial \phi}{\partial t} \ll \omega_t, \quad \frac{\partial r_i}{\partial t} \ll \omega_t r_i.\] (2.45)
To determine the effect of the medium on the field, the macroscopic polarization must be determined. The macroscopic polarization is given by a weighted integral over all the atoms in the medium. The inhomogeneous broadening is assumed to be a Gaussian function given by

$$g(\omega) = g_0 \exp \left(-\frac{(\omega_c - \omega)^2}{\sigma_i^2}\right).$$

(2.46)

Here $\omega_c$ is the center frequency of the inhomogeneous line, $\sigma_i$ is the half width at the $1/e$ point of the inhomogeneous line, and $g_0$ is the normalization, which has units of time. Here the normalization can be calculated as

$$\frac{1}{2\pi} \int_0^\infty g(\omega) d\omega = 1.$$

(2.47)

The macroscopic polarization, $P(z, t)$, written in the rest frame is defined as

$$P(z, t) = \frac{N \mu}{2\pi} \int_0^\infty \left[r_1 \cos(\omega_0 t - k_z z) - r_2 \sin(\omega_0 t - k_z z)\right] g(\omega) d\omega.$$

(2.48)

By taking the second derivative with respect to time for (2.48) and dropping terms that are small given by (2.45) one finds

$$\frac{\partial^2}{\partial t^2} P(z, t) \approx \frac{N \mu \omega_0^2}{2\pi} \int \left[r_2 \sin(\omega_0 t - k_z z) - r_1 \cos(\omega_0 t - k_z z)\right] g(\omega) d\omega.$$

(2.49)

Now, the attention is turned to the left hand side of eqn. (2.44). Again using the approximations in (2.45), the second derivative of the field with respect to $z$ can be expressed as

$$\nabla^2 E(z, t) \approx E_0 \left(-k_z^2 + 2k_z \frac{\partial \phi}{\partial z}\right) \cos(\omega_0 t - k_z z + \phi) + 2k_z \frac{\partial E_0}{\partial z} \sin(\omega_0 t - k_z z + \phi).$$

(2.50)
Likewise, the second derivative of the field with respect to time can be found as

\[
\frac{\partial^2}{\partial t^2} E(z, t) = E_o \left( -\omega_l^2 - 2\omega_l \frac{\partial \phi}{\partial t} \right) \cos (\omega_l t - k_z z + \phi) - 2\omega_l \frac{\partial E_o}{\partial t} \sin (\omega_l t - k_z z + \phi) .
\]

(2.51)

By combining the last two equations with the left hand side of eqn. (2.44) and by using the relation \( k_z = n\omega_l/c \) gives

\[
\nabla^2 E(z, t) - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} E(z, t) = 2E_o \left( k_z \frac{\partial \phi}{\partial z} + \frac{n^2\omega_l}{c^2} \frac{\partial \phi}{\partial t} \right) \cos (\omega_l t - k_z z + \phi) + 2 \left( k_z \frac{\partial E_o}{\partial z} + \frac{n^2\omega_l}{c^2} \frac{\partial E_o}{\partial t} \right) \sin (\omega_l t - k_z z + \phi) .
\]

(2.52)

Now the two sides of eqn. (2.44) can be equated. This gives

\[
2E_o \left( k_z \frac{\partial \phi}{\partial z} + \frac{n^2\omega_l}{c^2} \frac{\partial \phi}{\partial t} \right) \cos (\omega_l t - k_z z + \phi) + 2 \left( k_z \frac{\partial E_o}{\partial z} + \frac{n^2\omega_l}{c^2} \frac{\partial E_o}{\partial t} \right) \sin (\omega_l t - k_z z + \phi) = \frac{N\mu\omega_l^2}{2\pi\varepsilon_0 c^2} \int [r_2 \sin (\omega_o t - k_z z) - r_1 \cos (\omega_o t - k_z z)] g(\omega) d\omega .
\]

(2.53)

Up to here, the rotating frame has been generalized to rotate at a frequency, \( \omega_o \), that is close to the atomic frequency, \( \omega_a \). This was done so that any of the common rotating frames could be chosen, such as the laser frame or atomic frame. However, here it becomes necessary that the laser frame be chosen (\( \omega_o = \omega_l \)). The goal at this point is to try to equate like cosines and sines in order to get two equations, one for \( \phi(z, t) \) and one for \( E_o(z, t) \). But it is noticed that the terms on the right hand side of eqn. (2.53) are missing terms for the \( \phi(z, t) \). This requires that some trigonometric
equations be used namely,

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b.$$  \hspace{1cm} (2.54)

Here $a = \omega - k z$ and $b = \phi(z, t)$. With this relation, one finds the expression

$$A \{\cos a \cos b - \sin a \sin b\} + B \{\sin a \cos b + \cos a \sin b\}$$

$$= C \int [r_2 \sin a - r_1 \cos a] g(\omega) d\omega,$$  \hspace{1cm} (2.55)

with

$$A = 2E_o \left(k_z \frac{\partial \phi}{\partial z} + \frac{n^2 \omega \partial \phi}{c^2 \partial t}\right)$$

$$B = 2 \left(k_z \frac{\partial E_o}{\partial z} + \frac{n^2 \omega \partial E_o}{c^2 \partial t}\right)$$

$$C = \frac{N \mu \omega^2}{2\pi \epsilon_0 c^2}.$$  \hspace{1cm} (2.56)

Now the like sines and cosines can be equated on both sides giving two equations

$$B \cos b - A \sin b = C \int r_2 g(\omega) d\omega$$  \hspace{1cm} (2.57)

$$A \cos b + B \sin b = -C \int r_1 g(\omega) d\omega.$$  \hspace{1cm} (2.58)

These expressions can be rewritten after a little trigonometry and algebra as

$$B = C \int \{r_2 \cos b - r_1 \sin b\} g(\omega) d\omega$$  \hspace{1cm} (2.59)

$$A = -C \int \{r_1 \cos b + r_2 \sin b\} g(\omega) d\omega.$$  \hspace{1cm} (2.60)
Inserting the coefficients given by (2.56) one finds two equations, one for the slowly varying field amplitude and one for the slowly varying phase

\[
\left( \frac{\partial \phi}{\partial z} + \frac{n \omega I \partial \phi}{c \partial t} \right) = -\frac{N \mu \omega l}{4\pi \varepsilon_0 \varepsilon n c} \int \{ r_1 \cos \phi + r_2 \sin \phi \} g(\omega) d\omega \quad (2.61)
\]

\[
\left( \frac{\partial E_o}{\partial z} + \frac{n \omega I \partial E_o}{c \partial t} \right) = \frac{N \mu \omega l}{4\pi \varepsilon_0 \varepsilon n c} \int \{ r_2 \cos \phi - r_1 \sin \phi \} g(\omega) d\omega . \quad (2.62)
\]

Finally, by switching to a frame that moves at the speed of light in the medium [32] in the +z direction, one can essentially eliminate derivatives with respect to time. This is done as follows. Allow a new coordinate, \( t' \), in a new frame such that

\[
t' = t - \frac{n}{c} z, \quad \frac{\partial t}{\partial t'} = 1, \quad \frac{\partial t'}{\partial z} = -\frac{n}{c} + \frac{\partial t}{\partial z} . \quad (2.63)
\]

With some math the phase and field can now be written in the new frame as

\[
\frac{\partial \phi'(z,t')}{\partial z} = -\frac{1}{E_o'} \frac{N \mu \omega l}{4\pi \varepsilon_0 \varepsilon n c} \int [ r_1 \cos (\phi') + r_2 \sin (\phi') ] g(\omega) d\omega \quad (2.64)
\]

\[
\frac{\partial E_o'(z,t')}{\partial z} = \frac{N \mu \omega l}{4\pi \varepsilon_0 \varepsilon n c} \int [ r_2 \cos (\phi') - r_1 \sin (\phi') ] g(\omega) d\omega \quad (2.65)
\]

Note that the phase and the amplitude functions have been redefined in the new frame as \( E_o' \) and \( \phi' \), and are equivalent to the old functions when \( z = 0 \). Notice that the equations have been reduced to two coupled first order differential equations with respect to the propagation coordinate, \( z \). This allows numerical integration assuming there is only propagation in the forward direction (positive \( z \)). Finally, the absorption coefficient, \( \alpha \), written in terms of the material parameters is

\[
\alpha = \frac{N \mu^2 \omega c g_o}{2\varepsilon_0 h n c} . \quad (2.66)
\]
Next it is assumed that the laser is operating at the center of the inhomogeneous transition ($\omega_l = \omega_c$) given by eqn. (2.46). Also, a substitution from $\omega$ to detuning, $\Delta = \omega_a - \omega_b$ has been performed. Thus, the limits of the integration are now switched from 0 to $\infty$ to $-\infty$ to $\infty$, assuming that the weighting function is small for large $\Delta$.

The eqns. (2.64) and (2.65) are then rewritten to include the absorption coefficient, detuning, and Rabi frequency as

$$\frac{\partial \phi'(z, t')}{\partial z} = -\frac{1}{\Omega' \cdot 2\pi} \int (r_1 \cos(\phi') + r_2 \sin(\phi')) \exp\left(-\frac{(\Delta/\sigma_1)^2}{2}\right) d\Delta \quad (2.67)$$

$$\frac{\partial \Omega'(z, t')}{\partial z} = \frac{\alpha}{2\pi} \int (r_2 \cos(\phi') - r_1 \sin(\phi')) \exp\left(-\frac{(\Delta/\sigma_1)^2}{2}\right) d\Delta \quad (2.68)$$

Finally, two equations relating the slowly varying phase and amplitude to the first two elements of the Bloch vector have been obtained. With the above definitions for detuning and because of the moving frame, $\gamma = \phi'(z, t')$ and the Bloch equations can be written

$$r_1 = -\Delta r_2 - \Omega r_3 \sin \phi \quad (2.69)$$

$$r_2 = \Delta r_1 + \Omega r_3 \cos \phi \quad (2.70)$$

$$r_3 = \Omega r_1 \sin \phi - \Omega r_2 \cos \phi \quad (2.71)$$

As these equations were to be numerically integrated, the above equation for $\phi'$ (2.67) has the difficulty of being inversely proportional to $\Omega'$. This can introduce numerical complexities that can be easily avoided by keeping track of the in-phase
and in-quadrature parts of the field. By defining

\[ \Omega_c(z, t') = \Omega'(z, t') \cos(\phi') \]

\[ \Omega_s(z, t') = \Omega'(z, t') \sin(\phi') , \] (2.72)

the Bloch equations simplify to

\[ \dot{r}_1 = -\Delta r_2 - \Omega_s r_3 \]

\[ \dot{r}_2 = \Delta r_1 + \Omega_c r_2 \]

\[ \dot{r}_3 = \Omega_s r_1 - \Omega_c r_2 \] (2.73)

and the propagation equations then become

\[ \frac{\partial \Omega_c}{\partial z} = \frac{\alpha}{2\pi} \int r_2 \exp\left(-\left(\Delta/\sigma_\Delta\right)^2\right) d\Delta \] (2.74)

\[ \frac{\partial \Omega_s}{\partial z} = -\frac{\alpha}{2\pi} \int r_1 \exp\left(-\left(\Delta/\sigma_\Delta\right)^2\right) d\Delta . \] (2.75)

These equations are numerically integrated to give the Bloch vector \( \vec{r}(z, t', \Delta) \) for all \( z \), time and detunings. The field components \( \Omega_c(z, t') \) and \( \Omega_s(z, t') \) are also found for all \( z \) and time. Finally, the square of the field, \( |\Omega'(z, t')|^2 \), is proportional to the observed intensity, and can be found from

\[ |\Omega'(z, t')|^2 = \Omega_c(z, t')^2 + \Omega_s(z, t')^2 . \] (2.76)

The phase can be calculated from

\[ \phi'(z, t') = \arctan \left( \frac{\Omega_s(z, t')}{\Omega_c(z, t')} \right) . \] (2.77)
Maxwell-Bloch Simulations

Throughout this thesis, the above Maxwell-Bloch equations are numerically integrated in a specially developed Maxwell-Bloch simulator. The simulator has a graphical front end, which allows the user easy access to the simulator's parameters and a pictorial view of the input sequence of pulses. The graphical user interface was created using Visual C++ and is designed to run on a 32-bit Windows platform. A screen shot of the graphical front end is shown in figure 7. The output from this front end is a start file that contains all the information needed to run a simulation. The actual numerical integrator, the heart of the simulator, is itself a separate program and takes the start file as input. The separation was done for practical purposes. In this fashion, the numerical integrator can be fully optimized and easily compiled without having to deal with the graphical front end. The numerical integration code is also platform independent C++ code that can be compiled and run on many operating systems, UNIX®, LYNIX, or Windows®. This allows time consuming, parallel computations to be run on the campus supercomputer, Bigdog. Also, many versions of numerical integrators can be used for the same start file. Along with these benefits, the graphical front end can control the starting of the numerical integrator on separate computers. It transfers the start file and begins the numerical integration. It also handles the output from the numerical integrator and can display various outputs using the graphical package contained within a popular program called MATLAB®. All of these features allow easy, quick simulations to be performed.
As an example and a check of the simulators abilities, some different sequences are shown here but many are also included in the subsequent chapters. First, a simulation of the experimental echo sequence shown in figure 3 is given. Here the pulses are of Gaussian envelope with FWHM's of 100 ns, $\tau_{21} = 1100$ ns, each pulse area at zero detuning was $\pi/2$ and $\alpha L = 1.4$. This sequence was simulated and the output is
shown in figure (8). The input sequence (dotted line) is shown as well as the outputs, including the expected echoes (solid line). The echoes are observed at the proper time delay with efficiencies of \( \sim 5\% \) for the 2PE and \( \sim 10\% \) for the SPE. The pulse area for the 2PE has been calculated using the proper analytic solutions [13] and agrees to within \( \sim 3\% \) of the simulated 5\% efficiency result.

![Graph showing intensity over time](image)

Figure 8. A simulated echo sequence showing the 2PE and the SPE for an \( \alpha L = 1.4 \).

This suggests that the simulator is predicting the output appropriately. Further checks are shown later in this chapter. One of the nice aspects of numerical simulators is that they allow easy analysis of the medium’s properties at any time. In other words, the individual components of the Bloch vector can be analyzed at any time and for different locations within the medium. This can be very helpful in analyzing the response of the medium’s populations and coherences to a complicated series of pulses. For example, the spectral grating can be seen in the parameter, \( r_3 \). The
grating, shown in the top of figure 9, is a snapshot at time $5 \mu s$ at an $\alpha L \approx 0.3$. The value of $r_3$ is shown for all frequency detunings that were simulated. While this dependence could have been analyzed with a simple Bloch simulator, the Maxwell-Bloch simulator allows the detuning to be examined at any $\alpha L$, whereas the Bloch simulator is limited to $\alpha L \ll 1$. An example grating from the same sequence is also shown in the bottom of figure 9, this time for an $\alpha L = 1.4$.

Figure 9. The population grating, stored in the $r_3$ component of the Bloch vector versus detuning at time $5 \mu s$ from the above echo sequence. The top plot shows the grating for an $\alpha L \approx .3$, and the bottom plot shows the grating for an $\alpha L \approx .3$. 
Along with being able to analyze the components of the Bloch vector for any point within the crystal and at various times, the output field and phase are also known. Thus, the propagation of the input pulses and their corresponding absorption can be analyzed. The echo can also be analyzed and a picture of how the echo's amplitude grows as a function of propagation is also investigated. These effects are shown in figure 10 for the 2PE and in figure 11 for the SPE.

![3D plot](image)

Figure 10. The field amplitude for various times and absorption length of the first two pulses and the 2PE.

The medium starts in the ground state, and picks up energy from the initial pulses traveling into the medium. This can be seen as an overall absorption on the input
pulses as they propagate through the medium. As can be seen from figures 10 and 11, the echo typically starts with a very small amplitude for small $\alpha L$, and as it propagates through the medium, picks up energy and grows in amplitude. It was verified that this energy is essentially transferred from the populations, $r_3$, at the time of the echo leading to the mechanism for the energy source of the photon echo [13].

![Figure 11. The field amplitude for various times and absorption length of the probe pulse and the SPE.](image)

A further check to see if the simulator was behaving properly was to analyze sequences in the optically thin regime. In this regime both the intensities of the 2PE and the SPE should show quadratic dependence on the absorption length. The echo
power efficiencies, \( \eta \), are expected to follow

\[
\eta_{2\text{PE}} = \sin^4(\Theta_2/2)(\alpha L)^2
\]
\[
\eta_{\text{SPE}} = \frac{1}{4} \sin^2(\Theta_2) \sin^2(\Theta_3)(\alpha L)^2
\]

for a weak first pulse and \( \alpha \ll 1 \) [24]. Here the efficiency is with respect to the first pulse, which is considered the data pulse as described above (\( \tau_1 \gg \tau_2 = \tau_3 \)). In the simulations above, the pulse area on resonance of each pulse was \( \pi/2 \) and they all had the same bandwidths, thus violating these conditions for power efficiency. Thus eqns. (2.78) and (2.79) do not apply for this case, except as an upper bound on the efficiency. The actual efficiency is worse than when the conditions do apply. This is essentially because the second and third pulse do not act as \( \pi/2 \) pulses over the whole bandwidth of the first pulse. The power efficiency was confirmed to obey eqns. (2.78) and (2.79) by running another simulation for thin media. In this simulation, \( \tau_1 \gg \tau_2 = \tau_3 \), and the pulse areas were \( \Theta_1 = 0.01\pi, \Theta_2 = 0.5\pi, \) and \( \Theta_3 = 0.5\pi, \) thus the conditions for the validity of the power efficiency held. In this situation, eqn. (2.78) and (2.79) give \( \eta = 1/4(\alpha L)^2 \) for both the 2PE and the SPE. This was observed as shown in figure 12. Here the simulated power efficiencies for 2PE (o's) and SPE (x's) are shown versus absorption length. The solid line is (2.78) and (2.79). The exact match in the linear regime shows that the simulator is behaving appropriately.
Figure 12. The simulated power efficiencies for 2PE (o’s) and SPE (x’s) are shown versus absorption length. The solid line is the quadratic analytic solution with a coefficient of \(1/4\). Here \(\tau_1 \gg \tau_2 = \tau_3\), and the pulse areas were \(\Theta_1 = 0.01\pi\), \(\Theta_2 = 0.5\pi\), and \(\Theta_3 = 0.5\pi\).
The above echo sequences used $\phi' = 0$. However, the newly developed simulator has the necessary equations to properly handle arbitrary phase functions. One very important type of pulse, the linear frequency chirp, can be simulated by using the phase envelope function

$$\phi' = \pi \gamma (t - t_s)^2. \quad (2.80)$$

Here $\gamma = B/\tau_C$ is defined as the chirp rate of the pulse with $B$ as the bandwidth and $\tau_C$ as the chirp duration. It is typical to allow this chirp to be centered around $\omega_1$, however the simulator allows one to choose the offset. Note that the phase is synchronized with the front of the chirped pulse at time, $t_s$. In figure (13) two 1 $\mu$s chirped pulses are incident upon the medium ($\alpha L = 1.4$) separated by $\tau_{21} = 1.1$ $\mu$s. These two pulses, behave similarly to two brief pulses and program a time delay grating over the bandwidth that they chirp (40 MHz in this case). Here the $\Omega$'s of each chirp were set to 1.7 MHz. The probe pulse, $P_3$, is then incident upon this grating and creates an SPE that comes with the appropriate delay of 1.1 $\mu$s.

The spectral grating for linear frequency chirped programming has a flatter profile than its brief pulse counterpart as seen in figure 14. The spectral grating has a period that is the inverse of the programmed delay and can be used to create a stimulated photon echo.

Linear frequency chirps have several advantages for programming TTD gratings. One advantage is that, as can be seen in figure 13, the echo is more efficient with this type of programming method. Here the echo efficiency is $\sim 20\%$, which is twice as
large as the equivalent brief pulse programming scheme shown in figure 8. Chapter
3 includes a discussion of this effect, including simulations of power efficiencies for
optically thick crystals. Then later, in chapter 5, other advantages of linear frequency
chirped programming will be discussed.

Any phase function, assuming it is slowly varying compared to the optical fre­
quency, can be numerically integrated using this Maxwell-Bloch simulator. This
allows pulses with frequency shifts, CW phase modulated pulses, binary phase shift
keyed (BPSK) pulses, amplitude and phase modulated pulses, along with many oth­
ers to be simulated and studied. In each of these cases, one must remember that in
order for the approximations given by (2.45) to hold, slowly varying functions of the
phase must be represented. The phase can not be discontinuous but must be ramped
continuously for each of these types of arbitrary types of phase modulated pulses. All
of these types of pulses have been tested in the simulator, and some are presented in
the appropriate sections with experimental data.
Figure 13. The input and output intensities for a chirped pulse sequence from a crystal with $\alpha L = 1.4$. Here the echo efficiency is $\sim 20\%$.

Figure 14. The $r_3$ component of the Bloch vector as a function of the detuning. This shows the spectral grating created by two linear frequency chirped pulses. Here $\alpha L = 1.4$ and $t = 5 \mu$s.
In order to achieve high bandwidth OCT TTD, several practical issues must be sorted out. There are considerations with the material, OCT programming and probing, as well as high bandwidth modulation and detection. For example, the materials TBP's must be considered as well as lifetimes, as they play a crucial role in determining on what time scale and over what bandwidths TTD can be achieved. Along with this, the material parameters play a role in efficient programming and probing of OCT's. Since efficient spectral gratings are desired, the issues of how to program them into the OCT medium must be discussed. Other programming schemes have resulted in better than unit echo efficiencies [11, 12, 10]. These efforts were interested in data storage, and not in TTD. Thus the power efficiencies for TTD must be considered for desired programming methods using optically thick media. Another important issue, is that even if an efficient high bandwidth spectral grating is programmed, one must understand the limitations or nonlinear effects that can be expected when probing this spectral grating. These limitations have been studied before [30], and there relevance to the probing schemes for TTD should be examined. Finally, if these probes are to be high bandwidth signals, such as BPSK signals, the implications of how to create and detect these probes must be understood. For
large bandwidth probe signals the effects due to the inhomogeneous broadening may become important. Also, can the spectral grating be utilized to filter broadband signals in an efficient way creating better methods for detection of these signals?

This chapter discusses all of these important practical considerations to achieving high bandwidth OCT TTD. In the first section, the material parameters for Tm$^{3+}$:YAG will be presented. In the second section, a discussion of the implications for programming efficient TTD gratings is examined. This is followed by a brief overview of possible problems in probing these gratings. Finally, how to create and detect phase modulated signals is the topic of the last section. This last section concludes with a treatment on how the bandwidth of the crystal may influence these phase modulated signals as well as how spectral gratings can act as highly efficient filters for detecting phase modulated signals.

Material Considerations

There are many possible materials that could be used for spatial-spectral holography (SSH). Some possibilities were presented in chapter 2, and include rare-earth ion doped crystals, gasses of heated atoms, and systems of large organic molecules. Each of these materials has benefits and possible applications in various situations. It turns out, however, that for practical uses such as OCT TTD or other processes such as correlation or memory storage, rare-earth ion doped crystals have several strong advantages. These advantages include compactness, large TBP’s, transitions
at popular laser lines (≈790 nm, ≈1530 nm), long $T_2$'s and $T_1$'s, and low cost. [33]

Because of these advantages, the experiments performed in this thesis use the $^3H_6 - ^3H_4$ transition in 0.1% atomic Tm$^{3+}$:YAG. A representation of the atomic levels is shown in figure 15. This transition's resonant frequency is 793 nm. The overall defect and spectral properties of this crystal were studied by Wang and can be found in reference [17]. Spectral hole burning on the trivalent thulium ion was first observed by Macfarlane [34] as well as the first photon-echo measurements [14]. This crystal has a bottleneck state, the $^3F_4$ level. The electrons in the $^3H_4$ level decay either to the $^3H_6$ or the $^3F_4$ with a branching ratio, $\beta$, of approximately 0.56 and a decay time, $T_1$, of $\sim 600 \mu$s.\(^1\) The electrons on the $^3F_4$ level also decay to the $^3H_6$ level with a lifetime, $T_3$, of $\sim 12$ ms [36]. Dephasing times have been reported as $\sim 20 \mu$s and can range depending upon experimental conditions from a few $\mu$s to more than 30 $\mu$s as will be discussed in the following chapters. The inhomogeneous broadening's FWHM is approximately 20 GHz. For the longer $T_2$'s, the TBP is roughly $10^6$ for this crystal. These material parameters are highly suited for broadband TTD. Expected delays of several microseconds on broadband (>20 GHz) signals can be expected. Thus Tm$^{3+}$:YAG is a good candidate material for a TTD system.

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\(^1\) The $^3H_5$ level has an energy between the $^3H_4$ and the $^3F_4$ levels but the lifetime of this level, estimated to be 0.1-1.0 $\mu$s [35], is insignificant compared to the bottleneck lifetime and can thus be ignored.
Figure 15. The atomic levels of interest for this thesis for the rare-earth ion doped crystal Tm$^{3+}$:YAG.

**OCT Programming Efficiencies and Considerations**

As was stated in the introduction to this chapter, there are several important considerations when programming or probing a spectral grating in an OCT medium. As programming spectral gratings is pushed to higher bandwidths, perhaps the most important consideration is the efficiency of the programmed grating. The efficiency of a grating, $\eta$, is defined as the ratio of the peak echo intensity, $I_e$, to the peak intensity of the probe, $I_p$. That is

$$\eta = \frac{I_e}{I_p}. \quad (3.1)$$

Echo efficiencies for single shot, low bandwidth experiments are usually 1-3% for $\alpha L \approx 1$. However, as higher bandwidths are reached, the optical pulse has to excite more and more atoms, spread out over a larger bandwidth. This means that in order
to keep the efficiency the same, the probe must transfer more optical energy into the medium. For brief pulses, the only way to do this is to increase the power of the brief pulse, since the brief pulses temporal duration must get shorter to program a wider bandwidth. In order to reach multi-gigahertz bandwidths, picosecond pulses must be used.

In our lab, these pulses can be created from a mode-locked Ti:Sapphire laser, and along with a regenerative amplifier, can produce very powerful pulses, \( \sim 10^9 \) Watts. However, even with these large powers, it is still difficult to program an efficient spectral grating. The reason for this is that the pulses can not be focused too tightly into the crystal, or crystal surface damage will result. At a conservative spot size within the crystal of \( \sim 200 \mu m \), grating efficiencies are extremely low. For a single shot experiment, efficiencies of \( \sim 10^{-5} \) are expected for brief pulse programming with a pulse area of \( 0.05 \pi \). Compared with low bandwidth single shot experiments, this efficiency is clearly very low.

There are three main programming approaches that can help to improve grating efficiencies. The first is to use programming pulses that can spread their bandwidths over time, such as linear frequency chirps. This alleviates the problems associated with brief pulse programming because the temporal duration of the pulse can now be longer, while still programming large bandwidths. The second approach is to use optically thicker crystals. The third approach is to repetitively apply the programming pulses. Ideally, this should enhance the spectral grating and allow build up of
a stronger, more efficient grating. Of course all of these techniques could be used in combination, further enhancing the grating efficiency.

**Grating Efficiency vs. Bandwidth**

As was discussed in chapter 2, LFC pulses are a more efficient solution for programming compared to using brief pulses. However, achieving maximum efficiency requires optimizing the chirp parameters. The efficiency of a grating, $\eta$, was defined above in eqn. (3.1). This efficiency is equivalent to what is also termed the power efficiency, used by other authors [13]. First, the efficiency of chirps is compared to the brief programming pulse method. Then the power efficiency of these two methods is simulated and compared for optically thick crystals.

It can be shown that if a spectral grating was programmed by a series of brief programming pulses, the echo efficiency would fall as approximately $1/B^4$ if the peak intensity of the pulse is fixed. $^2$ In the linear regime, the echo signal in the frequency domain, $E_e(\omega)$ can be found from the relation [37]

$$E_e(\omega) \propto E_1^*(\omega) E_2(\omega) E_p(\omega).$$  

Here $E_1(\omega)$ and $E_2(\omega)$ are the Fourier transforms of the first and second pulses of

$$\eta = \left| \frac{E_e(\omega)}{E_p(\omega)} \right|^2 = |E_1^*(\omega)|^2 |E_2(\omega)|^2 \propto 1/B^4. \quad (3.2)$$

Thus the power efficiency for brief pulse programming very rapidly decreases as a function of bandwidth for power limited lasers.
the programming sequence respectively and \( E_p(\omega) \) is the Fourier transformed probe pulse. If the frequency chirps have a field amplitude in the time domain of \( E_c \) and \( E_1^*(\omega)E_2(\omega) \) is assumed flat over \( B \) and zero outside of \( B \), that is \( |E_1(\omega)| = |E_2(\omega)| = E_o \) on the interval \( 2\pi B \) of the LFC, then it can be shown, using Parseval’s theorem, that the following relation holds

\[
E_o = E_c \sqrt{\frac{T_C}{B}}. \tag{3.4}
\]

Using this equation, and \( I \propto \langle E^2 \rangle \) by definition, the intensity of the echo is found to be

\[
I_e \propto E_c^4 \left( \frac{T_C}{B} \right)^2 I_p. \tag{3.5}
\]

And using eqn. (3.1) the efficiency in the linear regime can be written as

\[
\eta \propto E_c^4 \left( \frac{T_C}{B} \right)^2. \tag{3.6}
\]

This equation shows that the echo efficiency with chirped pulse programming is proportional to \( 1/B^2 \), and thus chirps offer an immediate advantage over brief pulses. This is basically because as a chirp’s bandwidth is increased, its temporal duration can stay constant. Whereas for brief pulses, the duration must decrease in order to program a larger bandwidth. LFC programming has the added benefit of having the echo intensity proportional to \( T_C^2 \). Thus, the longer the chirp duration, the stronger the echo will be in the linear regime. So if the bandwidth is increased, the drop in efficiency can be compensated for by increasing the chirp time to keep the chirp rate
constant. This can be done provided that the chirps are completed within the bottleneck lifetime of the medium. It is these traits that make LFC pulse programming an attractive, practical solution for high bandwidth TTD.

The functional dependence of eqn. (3.6) was examined using the Maxwell-Bloch simulator for media that are not optically thin, in this case $\alpha L = 1.4$. This was done for two different situations. The first situation, kept $\tau_C$ constant while varying $B$. A series of 100 simulations were done with programming $B$'s from 20 MHz to 60 MHz in steps of 0.4 MHz. The probe pulse had a bandwidth of 10 MHz. The equivalent Rabi frequency, $\Omega_{eq}$, for a chirped pulse acting with a $\pi/2$ pulse area for a 4 $\mu$s, 20 MHz chirp is calculated as $\sim 0.6$ MHz using the formula [22]$^3$

$$\Omega_{eq} = 0.27 \sqrt{\frac{B}{\tau_C}}. \quad (3.7)$$

This simulation had a Rabi frequency of $\Omega = 0.125$ MHz, keeping it below the $\Omega_{eq} = 0.42$ MHz for the lowest bandwidth of 20 MHz. Here the time delay was controlled using frequency offsets, $\delta$'s, which will be discussed in chapter 5. These frequency offsets were varied to keep the time delay constant at 0.8 $\mu$s. In figure 16, the circles are the simulated echo efficiencies versus the programming bandwidth. As can be seen, the echo intensity decreases as a function of the bandwidth as expected. The $1/B^2$ analytic solution is also plotted as the solid line. The dashed line is representative of the brief pulse programming method dropping off as $1/B^4$. One can see

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$^3$ The author independently verified that this equivalent Rabi frequency gives an $r_3(\Delta = 0) = 0$ and an average $r_3(\omega)=0$ over the bandwidth of the chirp to within $\pm 3\%$ for various bandwidths and chirp durations.
a slight deviation in the simulation output as compared to the analytic solution in
the optically thin regime. Overall, the efficiency does not appear to be significantly
affected by the absorption length, $\alpha L = 1.4$.

Figure 16. Data points showing simulated peak echo height versus bandwidth for con­
stant programming powers. Here $\tau_C = 4 \mu s$ and $\delta$ is varied to keep $\tau_D$ constant. The
solid line is calculated using the analytic functional dependence of $1/B^2$ (normalized).
The dashed line is a plot of the analytic dependence of a brief pulse programming ver­
sus bandwidth and is normalized to the first point in the $1/B^2$ sequence for contrast.
The triangles are simulated sequences for a constant $\tau_C/B$ ratio with efficiencies given
by the right hand axis.
The second situation is when the ratio $\tau_C/B$ is kept constant. In this case, there should be roughly no change in echo efficiency. This situation is plotted as the triangles in figure 16. Here the righthand vertical axis gives the echo efficiencies for this plot. As can be seen, the simulated echo efficiencies for this situation remain relatively constant. This set of simulations was done with $B$'s from 20 MHz to 60 MHz in steps of 0.4 MHz. Here the $\tau_C$'s ran from 2 $\mu$s to 6 $\mu$s in steps of 0.04 $\mu$s, and $\delta = 8$ MHz giving a constant $\tau_D = 0.8$ $\mu$s. $\Omega$'s here were 0.5 MHz over the whole simulation. Note that the calculated ratio in efficiency (using eqn. (3.6)) between the constant sequence and the $B = 20$ MHz point of the $1/B^2$ sequence is a factor of 16. The simulated value using an average of $\eta = 0.03$ for the constant sequence gives a ratio of 13.7. The slight decrease from the expected value is most likely due to the fact that the constant sequence is further from the linear regime with an $\Omega = 0.5$ MHz. Here the calculated equivalent Rabi frequency is $\Omega_{eq} = 0.84$ MHz. The small fluctuations can be attributed to the fact that the bandwidth of the probe pulse and the time delays are of the same order as will be discussed in a later chapter. This situation shows the advantage of using LFC pulses for programming with a power limited laser. Here, as the bandwidth is increased, the efficiency of the grating can be kept constant if more optical energy is transferred to the medium by increasing the chirp duration. Ultimately, it is by balancing the $\tau_C/B$ ratio that high bandwidth gratings can be programmed efficiently into the OCT medium.
Highly Efficient TTD Using Chirped Programming

As was discussed briefly at the end of chapter 2, linear frequency chirped programming is a more efficient programming method. This could be seen in figure 13 where the echo was 2 times as efficient compared to the similar brief pulse programming case shown in figure 8. In this section, echo power efficiencies are examined as a function of the pulse areas (or Rabi frequencies) of the programming, as well as a function of $\alpha L$ for TTD programming.

Figure 17 shows the various input sequences studied in this section. In (a) a data storage programming scheme is shown. This programming scheme was extensively studied by C. Cornish [13] and was the first programming scheme where greater than unit power efficiencies were predicted [12] and observed [11]. This scheme is studied here as a check on the new Maxwell-Bloch simulator for large $\alpha L$.

Here, the first pulse in the sequence is a brief pulse with a significant pulse area, $\Theta_1$. The second pulse in the sequence is known as a data pulse, and has a bandwidth less than the first pulse and also has low pulse areas, $\Theta_2$. Finally, at a later time, the stored data can be recalled by a probe pulse identical to the first pulse with a significant pulse area, $\Theta_p$, stimulating an echo representative of the data pulse.

This sequence was simulated using the Maxwell-Bloch simulator, with the same parameters found in the work done by C. Cornish [13]. The power efficiency is plotted as a function of $\alpha L$ for various $\Theta_1 = \Theta_p = A$, ranging from 0.1 $\pi$ to 0.7 $\pi$ in figure 18.
Figure 17. Illustrations of each of the simulated input sequences studied for this thesis. In (a) a data storage configuration is shown. This configuration has shown better than unit efficiencies (see text) and is simulated here for comparison with new arbitrary phase Maxwell-Bloch simulator. (b) A brief programming pulse scheme for programming TTD echo sequences. (c) A linear frequency chirped programming scheme for producing TTD echo sequences.
Here the first and third pulses were Gaussian with FWHM's of 0.1 ns. The data pulse was also Gaussian and had a FWHM of 0.4 ns and $\Theta_2 = 0.01\pi$. As can be seen in the plot, efficiencies greater than unity are predicted around $\alpha L = 3$ for $A > 0.6\pi$.

This figure is in excellent agreement with the study shown in [13], where analytic and simulated results are shown, suggesting that this new simulator is working and producing the expected results for thick $\alpha L$.

While work had been done on the data storage sequence, there has been no discussion of the power efficiencies for TTD sequences. As was stated in chapter 2, TTD can be created using brief pulses or with chirped pulses. Figure 17 (b) shows an example of programming TTD using brief pulses. Now, two brief pulses, of significant pulse areas $\Theta_1 = \Theta_2$, write a spectral grating into the medium, then an arbitrary data pulse, with a low pulse area $\Theta_p$ and low bandwidth compared to the brief pulses, probes the grating, stimulating a TTD version of itself.

The power efficiencies for the brief pulse TTD programming sequence are shown in figure 19. In this simulation, $\Theta_1 = \Theta_2 = A$ and range from 0.1 to 0.9 $\pi$. The first and second pulses were Gaussian with FWHM’s of 0.1 ns. The data pulse was also Gaussian and had a FWHM of 0.2 ns and $\Theta_P = 0.05\pi$. It can be seen in this figure that the echo efficiency peaks around an $\alpha L = 2.3$, with a maximum efficiency of 33% for $A = 0.5\pi$. This is a significant efficiency, compared to thin $\alpha L$'s where efficiencies of 0.1-2% are common. Thus, as previously demonstrated for the programming
Figure 18. Power efficiencies for the data storage programming scheme shown in figure 17 (a). Here $\Theta_1 = \Theta_P = A$ and range from 0.1 to 0.7 $\pi$. The first and third pulses were Guassian pulses with FWHM's of 0.1 ns. Here, the data pulse was also Guassian and had a FWHM of 0.4 ns and $\Theta_2 = 0.01\pi$. 
Figure 19. Power efficiencies for the brief pulse TTD programming scheme shown in figure 17 (b). Here $\Theta_1 = \Theta_2 = A$ and range from 0.1 to 0.9 $\pi$. The first and second pulses were Gaussian with FWHM's of 0.1 ns. The data pulse was also Gaussian and had a FWHM of 0.2 ns and $\Theta_P = 0.05 \pi$. 
Figure 20. Power efficiencies for the linear frequency chirped pulse TTD programming scheme shown in figure 17 (c). Here $\Omega_1 = \Omega_2 = \Omega$ and ranges from 5 GRad/s to 19 GRad/s. The programming bandwidth was $B = 50$ GHz and $\tau_C = 1$ ns. The data pulse was Gaussian and had a FWHM of 0.04 ns and $\Theta_P = 0.05 \pi$. 
data sequences, highly efficient photon echoes can be produced for optically thick crystals. Unlike the data storage case, shown in figure 17 (a), efficiencies for TTD are still less than unity.

Now, the attention is turned to the linear frequency chirped programming sequence. In this case, as shown in figure 17 (c), the first two programming pulses are now linear frequency chirps. It is difficult to discuss pulse areas when talking about linear frequency chirps as their frequencies are chirping unlike the brief pulse case. Instead these pulses are characterized by their bandwidths \( B \), durations \( \tau_C \) and Rabi frequencies \( \Omega \). In this case, the power efficiencies are examined as a function of the absorption length and the Rabi frequency, keeping the bandwidth and duration constant. Like the brief pulse case, the two linear frequency chirps are energetically strong, with \( \Omega_1 = \Omega_2 \), and write a spectral grating. Then an energetically weak data probe stimulates the grating producing a TTD version of itself.

The power efficiencies for the chirped case are plotted in 20. Here \( \Omega_1 = \Omega_2 = \Omega \) and ranges from 5 GRad/s to 19 GRad/s. The programming bandwidth was \( B = 50 \) GHz and \( \tau_C = 1 \) ns. The data pulse was Gaussian and had a FWHM of 0.04 ns and \( \Theta_P = 0.05 \pi \). Here the power efficiency can be seen to peak around an \( \alpha L = 3.0 \) with a maximum power efficiency of 62% for \( \Omega = 12 \) GRad/s. It is interesting to note, that using eqn. (3.7) the \( \pi/2 \) equivalent Rabi frequency is calculated to be \( \Omega_{eq} = 11.8 \) GRad/s. This is in close agreement with the value for the peak efficiency Rabi of 12 GRad/s. This suggests that, as was the case for the brief pulses, that pulses with
areas close to $\pi/2$ program the most efficient gratings and give the highest power efficiencies for TTD. The power efficiency of 62% is approximately twice as large compared to the brief pulse programming sequence. This result suggests that linear frequency chirped pulse programming can be highly efficient, which is good news for applications such as TTD beam steering for phased array radar. This along with other advantages discussed later, make linear frequency chirped programming an attractive avenue for practical operation.

Finally, to ensure that these efficiencies would hold for arbitrary data pulses, a data sequence was used. This data sequence was a bi-phase, amplitude modulated sequence. Here, the data pulses had bit lengths (FWHM's) of 0.04 ns and individual pulse areas of 0.02 $\pi$. The seven bit input sequence was $+\pi/4, -\pi/4, +\pi/4, 0, \pi/4, 0, -\pi/4$, where 0 represents zero amplitude and the amplitude for the other bits is constant, whereas the phase of the bits can range from $\pm \pi/4$. The programming pulses had an $\Omega = 12$ GRad/s, $\tau_C = 1$ ns, $B = 50$ GHz, and a programmed time delay of $\tau_D = 1.5$ ns. Figure 21 shows the input probe pulse (dotted line) and the output at an $\alpha L = 3.06$ (solid line). An extremely high efficiency of 58% can be seen. The phase of the output bits was measured at the center of each bit and were found to be $0.90, -0.71, 0.82, 0, 0.85, 0, -0.75, 0$. The largest deviation from the expected value of $\pi/4$ is the first bit, which gives a percent error of $\sim 14\%$. Overall, good fidelity can be seen, suggesting that the linear frequency chirped programming method will be useful for practical applications that require high power efficiencies.
Figure 21. A simulation of bi-phase, amplitude modulated data undergoing highly efficient TTD. The input is the dotted line. The solid line is the output at $\alpha L = 3.06$ where the efficiency is close to 60%. This is more approximately twice the efficiency from the brief pulse programming scheme. Here the bits had FWHM's = 0.04 ns and $\Theta_{bit} = 0.02\pi$. The programming chirps had $\pi/2$ equivalent $\Omega = 12$ GRad/s, $B=50$ GHz, and $\tau_C = 1$ ns.

Accumulation of Spectral Gratings

Finally accumulation can be used to improve and maintain grating efficiencies. Accumulation is the process of repeatedly programming a spectral grating over many shots. This approach has been used to improve grating efficiencies from low pulse area programming pulses. The technique of accumulating simple spectral gratings is not new and has been studied in detail over the past years [38, 28, 39]. These
efforts have used the accumulation technique to build up and maintain efficient spectral gratings. Using this technique, it has been shown that lower power lasers can be used to create efficient gratings compared to using a single programming pulse pair to create the gratings. However, accumulation does put different frequency stability requirements on the programming pulses. Grating accumulation works because of the differences in the dephasing time, $T_2$, and the upper state lifetime or bottleneck lifetime, $T_3$. Essentially, the longest delay an OCT medium can recognize for a pair of programming pulses is $T_2$. However, this grating will persist for the bottleneck lifetime of the level. Thus, by waiting approximately $2T_2$ after the first pair of programming pulses, another set of identical programming pulses can be applied without worry that population gratings will be formed with these newly applied programming pulses. The subsequent application of new programming pulses after $2T_2$ leads to a strengthening of the periodic spectral grating that already existed within the medium.

Since $T_3$ for Tm$^{3+}$:YAG, is approximately 12 ms more than 400 programming shots can be accumulated. This allows significant enhancement of the grating efficiency. The process of accumulation is also known as continuous programming [40]. In the description above, the sequence of programming pulses was described as identical. But this need not be the case. In fact, if the programming pulses change on the order of the bottleneck lifetime, the stored spectral grating will also change accordingly, slowly adapting to the new programming sequence.\footnote{4}
The most stringent requirement for grating accumulation is that the phase difference between the pulses that make up each pulse pair vary much less than $\pi$ between any two coherently accumulated gratings. This requires that the frequency of the laser jitter by much less than the reciprocal of the programmed delay over the grating lifetime (typically milliseconds). Quantitatively, this requires that the frequency jitter of the laser, $\Delta f$, satisfy $\Delta f \ll \frac{1}{\tau_D}$ [28]. Here $\tau_D$ represents the delay programmed. If $\tau_D = 0.5$ $\mu$s the laser linewidth is required to be much less than 1 MHz. Thus, for most applications, especially ones requiring large $\tau_D$'s, a stabilized laser must be used in order to accumulate efficient gratings. This requires that the laser be locked to some frequency reference and continually compensated for its phase drifts. Frequency locked lasers are not new and have been studied in great detail [41, 42, 43, 44]. However, most schemes have used locking to high finesse cavities. Over time, the reference frequency of the cavity can drift causing unacceptable frequency drift of the laser off of the atomic transition. This problem has been overcome recently by using the process of locking to a spectral hole within the medium itself. This process, or locking to a regenerative spectral hole, has had great success and has led to several publications [36, 45, 46]. In fact, this process has been used to stabilize lasers that were used to coherently accumulate a spectral grating [28].

To learn more about the dynamics of the accumulation process using picosecond pulses see appendix 1.
Spatial Gratings and Spatially Isolated Echoes

Another aspect that makes OCT phenomena so attractive is that along with being able to store information in frequency gratings, these crystals can store spatial gratings. Spatial gratings are created by the complex index of refraction modulations within the crystal, ultimately due to the interference of spatially distinct beams. Once a spatial grating is formed, only certain directions are allowed to diffract from it. This results in spatial phase matching conditions for the echo. If the input pulses are spatially distinct, the direction that the echo will propagate through the crystal and appear as an output can be determined from the input pulses' wavevectors, $\vec{k}$.

In figure 22 the $k$-vectors or propagation directions for an SPE process are shown for the box geometry. Its effect is to spatially isolate the echo signal on the direction $\vec{k}_e$ allowing background free detection. If the i'th pulse has vector $\vec{k}_i$ then the direction of propagation for the SPE is defined as

$$\vec{k}_e = \vec{k}_3 + \vec{k}_2 - \vec{k}_1$$  \hspace{1cm} (3.8)

In the figure, the echo would thus propagate in the direction represented by e in the diagram. These types of geometry are of critical importance to most practical OCT applications and one OCT method, the continuous programmed continuous processor (CPCP) requires it [40]. The CPCP is a generalized description for an OCT processor, such as a continuous matched filter correlator, which is continually accepting programming and probe pulses asynchronously. The first proposal of a continuously
programmed continuous processor (CPCP) came in 1999 [47, 40]. Since that time, there have been several implementations, mostly done at low bandwidths [28] but higher bandwidth programming and processing has been explored with picosecond systems [39, 48].

Figure 22. The typical propagation directions for an SPE experiment in the boxcar configuration. Here 1, 2, and 3 represent the directions of pulses 1, 2 and 3, and e represents the direction for which the echo would propagate after the crystal.

Rare-earth ion doped crystals can also be programmed in several different spatial regions within the crystal. Typical experiments have spot sizes inside the crystal of about a hundred μm in diameter. Thus in a crystal with a surface area of 1 cm², over $10^4$ spots could be programmed. This is a significant amount of spatial-spectral gratings.

In summary, the ideal programming method would create multiple, highly efficient spatial-spectral gratings that would also act to spatially isolate the echo output. Due to the inherent advantages and a predicted 60% efficiency, most likely this method
would use some type of linear frequency chirp to program the grating. This grating could then be further enhanced or maintained through accumulation, ultimately resulting in a CPCP TTD system.

**Coherent and Incoherent Saturation Effects**

There are also considerations for probing in these materials. For certain CPCP applications, such as TTD for phased array radar, a continuous or quasi-continuous (temporal duration $\propto T_3$) optical processing signal may be desired. If a strong quasi-continuous optical signal is incident upon an unprogrammed OCT media coherent and incoherent saturation effects occur. This section details these effects and gives an upper limit on the incident photon flux allowed. Then, some experimental results, detailing coherent saturation and its effects are shown.

Coherent saturation, or coherently driving a collection of two level atoms, is a well known physical phenomena. This phenomena gives rise to what are called Rabi oscillations, and are related to the strength of the optical field and the absorption length of the crystal [19]. An optical pulse that has experienced Rabi oscillations is shown in figure 23.

Figure 23 shows that the medium is driven from its ground state (an absorbing medium) to its excited state (gain medium), producing oscillations around the input intensity level. The input pulse in this case was created by narrowband laser, thus all of the optical energy was concentrated around the laser frequency. Had this energy
been spread out in frequency, through broadband modulation, there would be a larger amount of atoms that would now have to be driven coherently. This would result in an overall increase in the period of the observed Rabi oscillations.

For TTD, it is desired to send in a quasi-continuous probe pulse. This pulse would have some broadband modulation encoded upon it, broadening its spectral profile. If the medium is to stay in a linear regime, which is not being saturated, the incident probe pulse must keep its incident photon flux below an acceptable limit. Essentially, the probe pulse should be weak enough so that it does not damage the already programmed spectral grating. But damage is not just limited through coherent saturation effects. After the dephasing time, the medium is no longer coherently
driven, and instead spectral hole burning occurs. Effects after the $T_2$ limit are known as incoherent saturation. The physical processes of coherent and incoherent saturation due to a broadband probe pulse incident upon the OCT medium were studied by Babbitt et. al [30]. What follows is a brief synopsis of their findings.

First, coherent saturation is examined. Over any time period $T_2$, if one examined the power spectrum of the probe pulse, one would find an average value as well as a maximum peak value in the power spectrum. The ratio of these two features is defined to be $\xi_c$. One can then define the maximum data pulse area, $\Theta_L$ over the bandwidth of the probe as

$$\Theta_L = \frac{4\pi\mu}{h} \sqrt{\frac{2\pi\xi_c T_2 I_d}{ncB_D}}.$$  \hspace{1cm} (3.9)

Here $I_d$ is the intensity of the third pulse, $n$ is the index of refraction, and $B_D$ is the bandwidth of the data pulse. It can be shown that the maximum allowable photon flux, $F_c$, for the material is

$$F_c = \frac{\lambda I_d}{hc} \leq \frac{\lambda h \Theta_L^2 n B_D}{32\pi^3 \mu^2 \xi_c T_2}.$$  \hspace{1cm} (3.10)

Finally, the degree of nonlinearity allowed determines $\Theta_L$. Here the acceptable nonlinearity, $\epsilon$, is defined through

$$\epsilon = \frac{\Theta_L}{\sin \Theta_L} - 1.$$  \hspace{1cm} (3.11)

For example, if $\epsilon = 10\%$ nonlinearity is an acceptable level, the maximum allowable pulse area is then $\Theta_L \approx \pi/4$. This and the other material parameters then limit the maximum allowable photon flux given in eqn. (3.10).
For incoherent saturation, the power spectrum of the probe pulse is now examined over the effective upper state lifetime of the material. When a bottleneck state is involved, the effective upper state lifetime, \( \tau' \) is defined as

\[
\tau' = \frac{\tau_{10}}{2} \left( \frac{\tau_{20} + 2\tau_{12}}{\tau_{12} + \tau_{10}} \right) .
\] (3.12)

Here \( \tau_{20}, \tau_{21}, \tau_{10} \) represent the lifetimes between the upper state and ground state, the upper state and the bottleneck state, and the bottleneck state and the ground state, respectively. The maximum of the power spectrum over this lifetime compared to the average of the power spectrum, giving a ratio defined to be \( \zeta \). Using \( \epsilon \), the maximum allowable photon flux for incoherent saturation, \( F_i \), can be written as

\[
F_i = \frac{3\lambda h n^2 B_D \epsilon}{16\pi^3 \mu^2 \zeta \tau'} .
\] (3.13)

Thus, once the amount of acceptable nonlinearity has been defined, the probe must have a photon flux that is lower than either \( F_c \) or \( F_i \) in order to avoid both coherent and incoherent saturation.

But what happens to the photon echoes if these conditions are not met? The author was recently involved in a study of how severe coherent saturation affects the stimulated photon echo. The research examined how a temporally long optical pulse, strong enough to induce Rabi oscillations, will diffract off of a spatial-spectral grating [49, 50]. Theoretical and experimental results have been obtained showing that indeed, there are also nutational effects upon the echo signal created from these long optical pulses. Figure 24 shows echoes for different strength probes. The echoes
clearly show nutational effects. It is interesting to note that for higher probe strengths, such as for 200 mW, the echo is smaller due to saturation effects from the probe pulse.\textsuperscript{5}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{echoes.png}
\caption{Echoes from various strength quasi-continuous optical probe pulses. The echo shows the nutational effects that are described with theory and experiment in appendix 2.}
\end{figure}

It can be seen that even though coherent saturation is occurring, a stimulated photon echo can still be produced. This is exciting news suggesting that perhaps probes of extremely low pulse areas may not be necessary. Through the research of this phenomena, there is now a better understanding of how quasi-continuous, strong probe pulses interact with the medium and how their echoes are effected. Future research should focus on how modulated probe pulses are effected by coherent saturation. The knowledge gained from such an experiment may help determine

\textsuperscript{5} For a detailed description of this process, involving theory, simulation with angled Maxwell-Bloch simulators, and experiment please see appendix 2.
the maximum allowable photon flux that is acceptable to produce high fidelity TTD outputs. In any case, the effects of coherent and incoherent saturation can be avoided by keeping the incident photon flux lower than either $F_i$ or $F_c$.

**Optical Modulation and Detection Considerations**

It is highly desired to use probe pulses that are phase or amplitude modulated at high bandwidths for the production of echoes. These subsequent echoes would also have phase or amplitude modulation as well as the appropriate TTD given by the programming. But high bandwidth processing can lead to problems. For example, the inhomogeneous broadening of Tm$^{3+}$:YAG is close to 20 GHz. Thus, signals that are CW modulated at this frequency, or signals that have bandwidths on this order, will have distorted outputs because of the absorption profile of the material. Or, perhaps the programming method involved leads to a spectral grating that is smaller than the full bandwidth of the probe pulse. How then, will the echo be effected? Ultimately, can these effects be used in some way to our benefit such as in efficient detection of high bandwidth phase modulated signals? These issues are adequately predicted by using linear filter theory and a spectral filter assuming that the input pulses are acting linearly on the system. The remaining part of this chapter is devoted to a discussion of how phase modulated signals are created, and how the linear filter theory can be used to predict the detected appearance of the spectrally filtered signals.
Phase Modulation And Spectral Filtering

Recently, electro-optic waveguide modulators, (EOM’s), have made it to the commercial stage and are being sold at affordable prices. These modulators are also known as integrated optics phase modulators. Unlike bulk phase modulators, these EOM’s have the ability to create broadband phase modulation on extremely high bandwidths (>40 GHz). This along with other advantages, such as low $V_{\pi}$’s, makes these EOM’s very popular for high bandwidth communications applications in long haul fiber optic lines. For a discussion on the basics of EOM’s and their practical uses see [51, 52].

In order to model the effects of spectral filtering, the spectral characteristics of CW phase modulated probe pulses are examined. The phase change due to the modulator can be written as

$$E(t) = E_0 \cos[\omega_l t + \beta \cos(\omega_m t)]$$

$$\beta = \frac{\pi V_o}{V_{\pi}}.$$  \hspace{1cm} (3.14)

Here, $\omega_l$ is the laser frequency, $\omega_m$ is the CW modulation frequency, $V_o$ is the amplitude of the applied voltage, and $V_{\pi}$ is the voltage needed to reach a phase shift of $\pi$. In order to model the spectral filtering induced by the medium, the SPE process is modeled as a linear system. It is assumed that the probe signal is incident upon an ideal TTD grating. The probe will then produce a photon echo, $E_e$, that has the programmed TTD, $\tau_D$. The echo output can be written as

$$E_e(t) = E_{eo} \cos[\omega_l (t - \tau_D) + \beta \cos(\omega_m (t - \tau_D))] \hspace{1cm} (3.16)$$
Here $E_{eo}$ is the echo amplitude and is given by the grating diffraction efficiency, $\eta$ as

$$E_{eo} = \sqrt{\eta} E_0 \quad (3.17)$$

If the delay is ignored on the optical carrier, the echo can be written as

$$E_e(t) = E_{eo} \cos [\omega_0 t + \beta \cos (\omega_m t - \phi_m)] \quad (3.18)$$

$$\phi_m = \omega_m \tau_D \quad (3.19)$$

Here, as discussed in chapter 1, the signal on the echo has a frequency dependent phase shift, $\phi$, with the proportionality constant being exactly the delay of the photon echo.

If the phase of the RF signal on the echo could be measured for different modulation frequencies one could show that the photon echo is truly obeying the properties of a TTD regenerator. Or, by keeping the frequency fixed, the delay, $\tau_D$, can be varied and the phase monitored. Thus (3.19) is an important equation for characterizing a TTD regenerator.$^6$

As mentioned above the medium or spectral grating may act as a bandwidth limiting spectral filter and thus it is important to examine what will happen to this echo signal under such effects. Equation (3.18) can be rewritten in terms of exponentials as

$$E_e(t) = \frac{E_{eo}}{2} \left\{ e^{i\omega_0 t} e^{i\beta \cos (\omega_m t - \phi_m)} + e^{-i\omega_0 t} e^{-i\beta \cos (\omega_m t - \phi_m)} \right\} \quad (3.20)$$

Using the relation

$$e^{i\alpha \cos \delta} = \sum_{n=-\infty}^{\infty} e^{ink/2} J_n(z) e^{in\delta} \quad (3.21)$$

In chapter 6, this phase relation is examined in more detail.
the echo signal can be written

\[ E_e(t) = \frac{E_{eo}}{2} \left\{ e^{i\omega t} \sum_{n=-\infty}^{\infty} J_n(\beta)e^{in(\pi/2+\omega_m t-\phi_m)} + c.c. \right\}. \hspace{1cm} (3.22) \]

By expanding the sum around \( n = 0 \), the signal can be rewritten as

\[ E_e(t) = E_{eo} \left\{ \cdots - J_1(\beta) \cos((\omega_l - \omega_m)t - \pi/2 + \phi_m) + J_0(\beta) \cos(\omega_l t) + J_1(\beta) \cos((\omega_l + \omega_m)t + \pi/2 - \phi_m) + \cdots \right\}. \hspace{1cm} (3.23) \]

Figure 25. The power spectrum of a simulated CW phase modulated signal on a carrier.

Here it is apparent that the phase modulation has put sidebands on the optical carrier at integer multiples of the modulation frequency. Figure 25 shows the power spectrum of a simulated CW phase modulated signal on a carrier. Here the modulation frequency is \( \omega_m = 10^2 \text{ Hz} \) on a \( 10^4 \text{ Hz} \) carrier. The \( n^{th} \) order sidebands are
marked, and their strength is related to $\beta$, which in this case is $\beta = 1.4$.

Now that the spectral characteristics of a CW phase modulated signal have been derived, the effects of a spectral filter can be examined. For example, the medium, as stated above, can act like a Gaussian filter in frequency space with a FWHM of $\sim 20$ GHz. Here dispersion is ignored. The effect of this filter would reduce the sum above to just a few components. The next step is to assume that the first order sidebands are the dominant spectral features besides the carrier ($\beta \ll 1$), and assume that the filter's FWHM is $\approx \omega_m$. With these assumptions, the filtering process can be approximated by eliminating all but the zero order and first order terms in eqn. (3.23) and then reducing the three terms left by separate constants, $A, B, C$. The constants $A, B, C$ simply represent the amount the Gaussian filter acts upon the individual sidebands. If the filter were actually centered on the carrier frequency then $A = C \approx 1/e$ and $B = 1$. However, this need not be the case and shifts on the order of $\omega_m$ of the center frequency can be allowed and still keep this approximation, however in this case the constants are not symmetric. So, after filtering (3.23) becomes

$$E_{\text{filtered}}(t) = E_0 \left\{ \begin{array}{c} -AJ_1(\beta) \cos((\omega_l - \omega_m)t - \pi/2 - \phi_m) + \\
BJ_0(\beta) \cos(\omega_l t) + CJ_1(\beta) \cos((\omega_l + \omega_m)t + \pi/2 + \phi_m) \end{array} \right\}.$$  

(3.24)

Finally in order to simulate what is observed on a detector, the intensity of the filtered echo signal must be squared and time averaged using

$$I_e(t) = \langle |E_{\text{filtered}}|^2 \rangle_{\nu}.$$  

(3.25)
Optical detectors have bandwidths that allow observation of the RF modulation but not the actual carrier oscillations. Thus $t' \ll t_o \ll t_m$ where $t_o$ and $t_m$ are the periods of the optical and RF signals respectively. One finds after some algebra that the intensity can be written

$$I_c(t) = \eta I_p \left\{ \frac{J_0 J_1 \cos(\omega_m t + \pi/2 + \phi_m) B [C - A] - }{AC J_2^2 \cos(2\omega_m t + \pi + 2\phi_m)} \right\} + I_{\text{const}} \quad (3.26)$$

Next assume the center frequency of the filter is detuned from the optical carrier, $A \neq C$. Equation (3.26) shows that there can be a doubling of the modulation frequency and also a doubling of the phase shift that is associated with the beating between the sidebands themselves.\(^7\) It can also be seen that through this spectral filtering process phase modulation can be turned into amplitude modulation with the same modulation frequency. It can be seen in the above equation that $(C - A)$ should be a maximum and $AC$ should be a minimum in order to produce an amplitude modulated signal at the modulation frequency without significant higher order harmonics. This condition can be accomplished by moving the center frequency of a Guassian filter towards either one of the sidebands.

Spectral Filtering and Efficient Detection

The discussion above has shown from an analytic perspective what is expected for filtered phase modulated signals. By relying on the linear filter theory discussed in chapter 2, any arbitrary filter can act upon a phase modulated signal. All that

\(^7\) A special spectral filter was created to observe this effect and is discussed in chapter 6.
is needed is the filter's frequency profile that determines $\tilde{G}(\omega)$ in eqn. (2.8). The phase modulated signal that will undergo filtering is then $\tilde{F}(\omega)$. The fact that phase modulation could be turned into intensity modulation with the same modulation frequency is an apparent result of the above theory. This suggests that perhaps spectral gratings or the medium could act to filter phase modulation into detectable amplitude modulation. This is not a new result. In the past, FM spectroscopy has been studied in detail for examining spectral holes or lineshapes [53]. Here the interest is in the use of spectral gratings as spectral filters on the phase modulated signals. A simple simulator was developed, which used common filter types such as Guassian or Lorentzian shapes, to look at spectral filtering of phase modulated signals. The spectral filtering simulator was used to analyze the strength of the created intensity modulation from CW phase modulated signals as a function of two parameters. The first parameter was the detuning, $\Delta_{sf}$. This detuning is defined as

$$\Delta_{sf} = \omega_l - \omega_c .$$

(3.27)

Here $\omega_l$ is the carrier frequency and $\omega_c$ is the filter's center frequency. The other parameter was the modulation frequency, $\omega_m$. Essentially, the intensity modulation strength at $(\omega_m)$ of a spectrally filtered CW phase modulation was analyzed as a function of $\omega_l$ and $\omega_m$. Here the desire was to simulate and experimentally demonstrate the physical effect of the inhomogeneously broadened transition on the phase modulated echo signal. It can be shown that in this case, the filter would act like a transmissive Guassian filter...
on the echo signal. A transmissive filter is one that has a large transmission at the filter’s center frequency, whereas an absorptive filter is one with a low transmission at the filter’s center frequency. If one looked at the probe pulse instead of the echo after the medium, the inhomogeneous broadening would act as an absorptive filter.

Figure 26. The reflection function of a fabry-perot etalon that is 3 mm thick measured using a frequency scan of a diode laser.

In order to experimentally examine the effect of absorptive and transmissive filters on phase modulated signals, spectral filtering was done using fabry-perot etalons. An etalon is essentially a cavity, composed of two reflecting surfaces [54]. Because only certain modes can exist within the cavity, the etalon acts as an airy filter. An airy filter is essentially just a repeating Lorentzian filter. In figure 26, a laser’s frequency is
scanned over the airy filter giving the reflection function of the etalon that is shown.

Notice that in this case, the reflection from the etalon represents an absorptive filter, whereas the transmission through the etalon represents a transmissive filter. The etalon was 3 mm thick and had a FWHM of ~ 700 MHz. A broadband EOM was used to CW phase modulate the carrier over a set of modulation frequencies from 0.5 to 13 GHz. The carrier frequency of the laser was also changed resulting in a scan of the detuning. The transmission through the filter was received on a broadband detector and analyzed with a network analyzer. The network analyzer returned the strength of the intensity modulation (at $\omega_m$) for a given detuning and modulation frequency. A plot of the output power (measured in dB) is shown in figure 27. Here the modulation strength, $\beta$ is approximately 1.2.

This experiment represents approximately what would be expected for a CW phase modulated echo filtered by the inhomogeneous broadening of the crystal. Notice that the largest intensity modulation happens when the filter is slightly detuned towards the sidebands away from the carrier. If the filter is not detuned at all, almost no intensity modulation is observed, which is as expected. To ensure that this shape was what was expected, the results were simulated, as shown in figure 28. Here the units on the axis are normalized to the FWHM of the Lorentzian filter. Good agreement between the experiment and simulation can be seen.
Figure 27. The strength of the intensity modulation at the modulation frequency for various detunings and modulation frequencies for experimental phase modulated signals.
Figure 28. Simulated strengths of the intensity modulation at the modulation frequency for various detunings and modulation frequencies using linear filter theory. Frequency axes units are in the FWHM of the filter.
The practical operating regime is near the top of the peaks in figures 27 and 28. Further away from the peaks is the region where the modulation bandwidth is significantly greater than the filters bandwidth, a region of impractical operation. As can be seen in the simulation, the peak occurs roughly when the modulation frequency, the filter center frequency, and the filter's width are all equal. This means that if one was to use this filter to change phase modulation into amplitude modulation, this is the most efficient point to do so. Since the Lorentzian filter is centered on one of the sidebands, the filter acts to reduce the power in the carrier, resulting in a loss of power for the signal. Ideal conversion efficiency would keep both the sideband and the carrier powers equal to their original values, thus maximizing the signal.

A square spectral filter, with a unit value would do this. If the spectral width of the square filter was larger than $\omega_m$, but less than $2\omega_m$, one sideband and the carrier could be filtered allowing optimal efficiency. When linear frequency chirps are used to program TTD gratings, the resulting gratings can act as square filters. These filters can then be utilized as spectral filters on phase modulated signals. For example, in the case above, assume the CW phase modulated signal is incident upon the TTD grating and the carrier and one sideband are within its bandwidth. Then the medium would produce a stimulated photon echo of the sideband and the carrier, producing a strong beat signal at the modulation frequency. As was discussed above, the efficiency of the grating, although not unity, can be fairly high (60%). Along with this, TTD has been accomplished.
In applications such as phased array radar, RF carriers are typically utilized [3]. These carriers are usually in the multi-GHz range and broadband data is encoded upon these RF carriers. In this situation, the sidebands, shown in figure 25, would have a spectral width containing the Fourier components of the encoded data. Several methods of detection could be utilized to detect this data after the signal was TTD. One such example would be self-heterodyne detection (discussed in chapter 4). In order to detect this signal appropriately using such a technique requires both sidebands and their spectral content. This would require programming a large bandwidth grating into the OCT medium to TTD both sidebands, which could reduce grating efficiency. Another more efficient approach, is to spectrally filter the sideband utilizing a lower bandwidth grating centered around the sideband. Here the grating would be large enough to preserve the encoded broadband data only. There is no need to keep the other sideband for detection purposes, and instead of self-heterodyne detection, the signal would be direct detected with the proper receiver, resulting in a more efficient detection scheme. Experimental demonstrations utilizing spectral gratings in the OCT medium to filter phase modulated signals are examined in chapter 6.

This chapter has shown the different considerations for broadband OCT TTD. These included material parameters such as the TBP, grating efficiency considerations due to programming methods, the effects of coherent and incoherent saturation, as well as spectral filtering. With these considerations in hand, the attention is turned to directly accomplishing broadband TTD in Tm$^{3+}$:YAG.
Programming high bandwidth spectral gratings into optical coherent transient media poses several challenges. One such challenge is the need to create an efficient spectral grating over these broad bandwidths. Essentially, the optical field needs to act on a large number of atoms each with different resonant frequencies in order to create the broadband grating. It must also make an efficient grating, as discussed in the previous chapter, so that probes will create high signal to noise ratio (SNR) echo pulses. High efficiency is extremely important at high bandwidths, since photodetectors have significantly higher noise levels at large bandwidths due to the fact that thermal noise levels are an increasing function of bandwidth. This means that for larger bandwidths, the minimum detectable optical power (MOP) increases, and is given as [55]

\[ MOP = NEP \sqrt{B} \]  

(4.1)

Here B is the bandwidth of detection and NEP is the noise equivalent power of the detector. For a typical high bandwidth detector, such as the New Focus 1554, NEP's are specified to be \( \sim 100 \text{ pW} / \sqrt{\text{Hz}} \). This means that for practical purposes (signal to noise ratios \( \sim 10 \)), echo signals must have powers of 50 \( \mu \text{W} \) in order to be observed with 3 GHz of bandwidth using these detectors. In the previous chapter, TTD echo
efficiencies were predicted to be in excess of 60% under certain conditions. But con­straints in practical systems such as limited programming power may lead to echo efficiencies which are significantly lower. Efficiencies of broadband accumulated TTD gratings have been measured to be around 0.1% [39], requiring that probe pulses have optical powers greater than 50 mW in order to create detectable echoes. It is desired to use probes that are encoded with high bandwidth phase modulated signals from integrated optics modulators. Unfortunately, current off-the-shelf integrated optic modulators, driven at high frequencies, are power limited at 793 nm (the $^3H_6-^3H_4$ transition in Tm$^{3+}$) due to photorefractive damage of the LiNbO$_3$ waveguides. Optical input powers for these devices at this wavelength are less than 10 mW with optical output powers typically less than a milliwatt. This power level is inadequate to demonstrate the desired high bandwidth applications of spatial-spectral holography, creating a need for reliable high bandwidth amplification devices at 793 nm.

There has been limited work on amplifiers at this wavelength range. One approach was to use a Tm doped fiber [56]. This approach has several drawbacks including a need for a high power single mode pump laser (~1W), pulsing of the pump laser due to population dynamics, and cooling of the fiber to 77 K. While amplification has been demonstrated from such a device, the drawbacks would limit the practical operation of a continuously programmed, continuously probed photon echo TTD device. In this chapter, an amplifier, based upon injection locking methods, is described and characterized. The benefits of such a system include ease of use, compactness,
integrability and gains of more than 20 dB over large bandwidths. However, injection locking with semiconductor diode lasers has several inherent challenges that must be understood in order to utilize the approach. Overall, the injection locking system described here was developed and used in several high bandwidth experiments of this thesis and enabled experimental results that could not have been obtained otherwise.

**Semiconductor Diode Lasers And Injection Locking Theory**

The operational premise of a laser injection locking system is shown in figure 29. Here a master laser creates the field that will be injected into another laser. Typically the master laser is just a simple laser operating in a single mode with very narrow linewidth, such as an external cavity diode laser (ECDL) [57]. The laser that receives the injected light is known as the slave laser. For the case of amplification, the slave will have a much larger optical power output than the master laser. Assuming the injected signal is close in frequency to the free running frequency of the slave laser, and the injected signal is strong enough, the slave laser will oscillate at the master laser’s frequency and is said to be injection locked. This output field will also have the same optical power as the free running slave, thus providing amplification to the injected field. These amplifications (23 dB) provide the needed optical power for SSH applications.
For an ideal amplifier, the output would be identical in every aspect to the input signal, except for the increased amplitude of the output field. Thus phase or amplitude modulated signals could be amplified. However, in practical situations, there is some bandwidth to the amplifier that will limit the largest modulation frequency that can be amplified. For an injection locking amplifier, the relaxation oscillations of the laser give approximately the largest modulation frequency that can be amplified by the system [58]. These relaxation oscillations thus define the bandwidth of the injection locking amplifier. Until the advent of the semiconductor laser, the relaxation oscillations of laser cavities were very low due to long cavity lengths and high cavity fineses [59]. This severely limited the injection locking amplifier's bandwidth. Thus there was not much interest in using injection locking as a signal amplifier until the semiconductor diode laser came along. Because of the unique characteristics of the diode laser, such as short round trip times, high gains, and poor cavity fineses, the diode laser possesses relaxation oscillations on the order of several gigahertz [60, 61]. These large relaxation oscillations allow the possibility of using injection locking of
semiconductor diode lasers as high bandwidth amplifiers [58]. But the diode laser's unique characteristics also lead to interesting effects for injection locking including dynamical instabilities and regions of chaos for certain operating conditions [62, 63, 64, 65, 66, 67, 68].

To begin a discussion on the dynamical aspects of injection locking semiconductor diode lasers, one must understand some of the physical characteristics of the diode laser itself. Specifically there are five parameters of the laser that are important in describing its operation. These parameters are the cavity decay rate, \( \gamma_c \), the spontaneous carrier relaxation rate, \( \gamma_s \), the differential carrier relaxation rate, \( \gamma_n \), the nonlinear carrier relaxation rate, \( \gamma_p \), and the linewidth enhancement factor \( b \) [62].

The cavity decay rate is related to the finesse of the laser cavity, becoming larger as the reflectance of the facets is decreased. The cavity photon lifetime, \( \tau_p \), is related to the cavity decay rate and can be expressed as \( \tau_p = 1/\gamma_c \). The spontaneous carrier relaxation rate describes the rate at which carriers recombine, and are removed from the gain region, \( g(\omega, n) \), of the laser. Here \( n \) is the carrier density in the active region. The differential carrier and nonlinear carrier relaxation rates are more complicated and describe the effects of the differential gain with respect to carriers, \( dg(\omega, n)/dn \), and the differential gain with respect to the photon density \( dg(\omega, P_0)/dP_0 \), respectively [69, 67]. Here \( P_0 \) represents the photon density inside the cavity. The differential gain, \( dg(\omega, n)/dn \), expresses how the gain of the active region changes with respect to the carriers. Because the index of refraction of the active region is very dependent
upon the carrier concentration, this leads to the rapid frequency control of the diode
through the injected current, $J$. But there is also an asymmetry in the differential
gain, giving rise to the linewidth enhancement factor, $b$. The linewidth enhancement
factor is defined as

$$b = \frac{d\chi_R}{dn} \left( \frac{d\chi_I}{dn} \right)^{-1}.$$  \hspace{1cm} (4.2)

Here, $\chi_R$ and $\chi_I$ are the real and imaginary parts of the complex susceptibility of the
laser, $\chi$. The linewidth enhancement factor is related to the linewidth, $\Delta f$, as

$$\Delta f = \Delta f_{ST}(1 + b^2). \hspace{1cm} (4.3)$$

Here $\Delta f_{ST}$ is the linewidth predicted by the modified Schawlow-Townes formula [70].
This linewidth enhancement factor gives rise to an asymmetry in the relaxation oscil-
lation sidebands of the laser, which has been utilized to give accurate measurements
of $b$ [61]. These relaxation oscillation sidebands can be observed in optical spectra
of the diode laser, giving an indication of the overall modulation bandwidth of the
diode laser. The frequency of the relaxation oscillations, $f_r$, can be written as [69]

$$f_r = \frac{1}{2\pi} \sqrt{\frac{n_r g'(\omega, n) P_o}{c \tau_p}}.$$ \hspace{1cm} (4.4)

Here $n_r$ is the average index of refraction in the medium and $P_o$ is the photon density
of the cavity, which is proportional to $J$. The modulation bandwidth of the laser
can then be increased by either increasing $P_o$ or decreasing $\tau_p$ for a given material.
Thus, AR coated diode lasers, which have extremely low reflectance on the front facet
($0.001\%$) and thus small $\tau_p$'s, are expected to have a larger modulation bandwidth.
than standard diode lasers. For example, if a diode laser had a front facet reflectance of 3% and \( f_r = 3 \text{ GHz} \), if the front facet reflection was instead 0.001% then \( f_r = 5.5 \text{ GHz} \), thus increasing the modulation bandwidth of the laser, by almost a factor of 2.

It has been theoretically and experimentally demonstrated that injection locking of semiconductor lasers leads to an increase in the relaxation oscillation frequency as well as a decrease in the noise characteristics of the slave laser [58, 71, 72, 73, 74]. These results are promising for using a diode laser as an injection locked amplifier for modulated signals. While diode lasers have been used as injection locked amplifiers in the past most of the modulation techniques involved directly modulating the current of the slave laser diode while light was injected from a master laser [75, 76, 77, 78, 79]. The largest data rate reached was 10 Gbit/s (~5 GHz bandwidth) shown in reference [77]. Unfortunately, since these were directly current modulated, the output is chirped in frequency. This is undesirable for use in optical coherent transients, and thus, another method of modulation must be found. Some other research efforts have injected light from a phase modulated master laser into the slave laser giving good amplification and faithful reproduction up to 2 GHz CW phase modulation frequencies [80]. Pseudorandom frequency or phase shift keying of the master laser has been demonstrated showing that data could be amplified at rates of 1 Gbit/s (~ 0.5 GHz bandwidth) [81]. These two papers saw no degradation in signal fidelity, most likely because the frequency modulations used were not large enough to reach the relaxation oscillations of the slave laser. Interest in amplifying frequency chirped
external cavity diode lasers also led to an investigation of using injection locking as a high power amplifier for ~2 GHz linear frequency chirps [82], in which good signal fidelity was achieved as well as high gains. These successes gave hope that injection locking of semiconductor lasers could be used as a possible amplification tool of phase modulated signals for high bandwidth spatial-spectral holographic applications.

A diode laser undergoing injection locking can be controlled by three operational parameters. These parameters are the injected current parameter, the amount of light injected into the slave, and the frequency detuning of the master laser frequency with respect to the slave's free running frequency. Of course the slave laser's intrinsic parameters, defined above, may shift as a result of adjusting the current. A mathematical model has been derived [68], which describes the functional dependence of the injection locked slave laser on the operational parameters as well as the inherent parameters of the slave laser. These equations are

\[
\begin{align*}
\frac{da}{dt} & = \frac{1}{2} (1 + a) \left[ \frac{\gamma_c \gamma_n}{\gamma_s} \tilde{n} - \gamma_p \left( 2a + a^2 \right) \right] + \xi \gamma_c \cos (\Omega_0 t + \phi) + F_a & (4.5) \\
\frac{d\phi}{dt} & = -\frac{b}{2} \left[ \frac{\gamma_c \gamma_n}{\gamma_s} \tilde{n} - \gamma_p \left( 2a + a^2 \right) \right] - \frac{\xi \gamma_c}{1 + a} \sin (\Omega_0 t + \phi) + \frac{F_\phi}{1 + a} & (4.6) \\
\frac{d\tilde{n}}{dt} & = -\gamma_s \tilde{n} - \gamma_n \left( 1 + a \right)^2 \tilde{n} - \gamma_s \tilde{f} \left( 2a + a^2 \right) + \frac{\gamma_e \gamma_p}{\gamma_c} \tilde{f} \left( 2a + a^2 \right) \left( 1 + a \right)^2 & (4.7)
\end{align*}
\]

Here $a$ is the normalized field amplitude, $\phi$ is the phase difference of the slave laser with respect to the injected field and $\tilde{n}$ is the normalized carrier density of the slave laser. The Langevin noise terms, $F_a$ and $F_\phi$, are also included in these equations and a discussion of their effect and calculation can be found in [83]. Here the operational
parameters are controlled through $\tilde{J}$, the injection current parameter, $\xi$ the injection parameter that describes the amount of light injected into the slave, and $\Omega_o$ the angular frequency detuning of the injection field from the slave. The injection parameter can be written as $\xi = \eta_c |A_i|/|A_0|$, where $\eta_c$ is a coupling factor into the cavity, $A_i$ is the injected field, and $A_0$ is the free running field produced by the slave.

Because the inherent laser parameters change as a function of the injected current, the operational parameter $\tilde{J}$ is usually set at a constant. This then fixes the intrinsic parameters of the laser and allows the dynamics of the system to be studied through tuning of the other operational parameters, $\xi$ and $\Omega_o$. Typically, maps of the dynamic regions of stability and instability for the slave laser are shown as a function of these two parameters both for theoretical and experimental investigations [62, 67]. It is important to map out the dynamical regions to find regions of stable locking so that the system can be used as an amplifier. This stable locking region is where faithful reproduction of modulated signals is expected, whereas other regions of dynamic instability can not be utilized for amplification. These dynamical regions of stability and instability are the result of a combination of the intrinsic properties of the laser and the values of the operational parameters. The linewidth enhancement factor, $b$, plays a very important role in the dynamics [67]. This parameter is important in determining the limits of the range of nonlinear dynamics, as well as whether these regions will contain chaos. Ultimately, our interest lies in the boundary of the stable region, as it is within this region in which amplification of signals will be performed.
A simulator has been developed that numerically integrates the rate equations defined above. Through the use of this simulator the dynamical regions found in the references cited above have been confirmed. These regions include regions of stable locking, regions of chaotic behavior, regions of undamped relaxation oscillations, regions of multi-mode mixing, period doubling regions of undamped relaxation oscillations, as well as four-wave mixing regions and unlocked regions. The standard period doubling route to chaos [68] was observed with this simulator and is shown in figure 30. This standard route to chaos is done by keeping the detuning $\Omega_0 = 0$ and increasing the optical injection parameter, $\xi$. When this is done, the stable region of locking is the first region, and is shown in (a) for a value of $\xi = 0.0013$. Note that the strong peak occurs at the free running oscillation frequency of the slave laser, which in this simulation was chosen to be 200 GHz. This frequency was chosen because the computing requirements to simulate THz carrier frequencies are too great. However, 200 GHz is still much greater than the fastest frequency oscillations of interest. The next region to be encountered is the undamped relaxation oscillations, which were measured from the simulated spectra to be $5.72 \pm 0.02$ GHz. This value for the undamped relaxation oscillations is a result of the chosen simulation parameters that are given below. This value matches the relaxation oscillations of the free running slave laser, which were measured from simulated spectra to be $5.7 \pm 0.3$ GHz.

1 A thorough description of the simulator as well as each of these regions is beyond the scope of this thesis and the reader is referred to references [62, 63, 64, 65, 66, 67, 68].
The difference in precision of these two measurements is related to the fact that the features are $10^5$ times stronger in the undamped relaxation oscillation region. This region, also known as a period one region, is shown in (b) for a $\xi = 0.0027$. In (c) a period doubling region is shown, where another set of oscillation sidebands can be seen half way between the period one oscillations. Here $\xi = 0.0077$. Finally, chaos is encountered where the slave laser emits in a random manner and spreads its energy over many frequencies. This is shown in (d) for $\xi = 0.0183$. In these simulations $\gamma_c = 625$, $\gamma_r = 4.55$, $\gamma_p = 2.0$, $\gamma_s = 0.4545$, $\gamma_n = 2.096$ given in GHz, $b = 4$, and $\bar{J} = 0.6$.

As was stated above, our purpose in studying injection locking was to use it as a high powered amplifier for high bandwidth phase modulated signals. Through the above theoretical discussions and other lower bandwidth experimental demonstrations [80, 81], it was expected that amplification of phase modulation could be supported by the injection locked slave laser in a stable locking region, at least out to the free running relaxation oscillation frequency [58]. In the following experiments, a high power, single mode, AR coated slave laser was used as the slave laser. An AR coated slave was chosen for two reasons. An AR coating leads to a reduced $\tau_p$, and subsequently an increase in the relaxation oscillations. Second, the modes for the cavity become less distinguishable when there is a low reflectance output coupler and the laser acts more like an amplifier. This allows, at least in theory, easier injection locking of the slave laser. In order to characterize the injection locking, both
CW phase modulated and binary phase shift keyed signals (BPSK), at multi-GHz modulation frequencies and bandwidths respectively, were used. As will be shown below, some unexpected results, which limit the bandwidth for phase modulation were observed.

![Simulated Optical Spectra](image)

**Figure 30.** The simulated optical spectra for an injection locked slave laser with a free running frequency of 200 GHz showing the standard period doubling route to chaos with $\Omega_o = 0$, and (a) $\xi = 0.0013$, (b) $\xi = 0.0027$, (c) $\xi = 0.0077$, (d) $\xi = 0.0183$.

**Experiments**

As discussed in chapter 3, high bandwidth integrated optics phase modulators
are available commercially as fiber pigtailed units. These units have bandwidths
from 1 to over 40 GHz. The modulator used in the following experiments had a 3 dB
bandwidth of 13 GHz. Note that the frequency response was not entirely flat over this
bandwidth and is discussed below. Input powers for this device at 793 nm were to
be kept lower than 10 mW. The measured insertion loss of 12 dB with the modulator
used in these experiments led to output powers for modulated signals of \( \sim 800 \ \mu W \).
This modulator was tested using both CW phase modulation and binary phase shift
keyed (BPSK) signals. For CW phase modulation, the modulator was driven by a
13.5 GHz HP 8719ES network analyzer. For BPSK signals, the modulator was driven
by a 12 GBit/s Advantest D3186 pulse pattern generator (PPG) and amplifier. This
PPG is capable of producing binary signals from 0.5 V to 2 V peak to peak at various
multi-gigabit data rates that were then amplified to 7 V peak to peak \( (V_x \sim 3.5 \ V
\) for this modulator).

The first step towards amplifying the phase modulated signals was to build a
reliable injection locking system and show that locking could be attained at our wave­
length of interest. Figure 31 details the experimental setup for the injection locking.
In this experiment, a homemade ECDL, operating in the Littman configuration, was
used.\(^2\) This laser had a measured linewidth of \( \sim 100 \) kHz and could be manually tuned
from 780 nm to 810 nm. A peizo electric transducer adjusts the cavity length and
allowed for continuous (mode hop free) tuning over approximately 65 GHz with a 150

\(^2\) This laser was designed by Greg Switzer from John Carlsten’s lab and machined by Zachary Cole.
volt supply. The output mode of the laser observed with an optical spectrum analyzer (OSA) shows typical sidemode suppression ratios of ~40 dB across the tuning range. The slave for this experiment, a free running diode laser (FRDL), uses an AR coated semiconductor diode laser operating at approximately 793 nm. This laser was originally a 100 mW single mode diode laser and after AR coating produced 90 mW of output power with a threshold current of approximately 40 mA. Above threshold a single mode output is observed with a sidemode suppression ratio of 25 dB.

Figure 31. Optical injection locking setup using a fiber coupled integrated optics phase modulator. See text for discussion.

For efficient injection locking, the injected field must be well coupled into the slave cavity. To do this and still allow for spatial isolation of the output field can be tricky.
In this case, a Faraday rotator and linear polarizers are used to spatially isolate the injected field from the output field. As shown in figure 31, the light from the master laser travels first through an anamorphic prism pair (APP) for beam shaping and then through an optical isolator giving greater than 40 dB of isolation. The beam is then passed through a half wave plate and polarizing beam splitter (PBS) to allow adjustment of the optical power sent to the slave laser without having to adjust the ECDL directly. The complementary output from the PBS is used to monitor the properties of the master laser. The output going to the slave laser is then fiber coupled to a single mode polarization maintaining fiber optimized for 800nm. The coupling efficiency was around 60%. This fiber can then be attached either directly to a fiber coupler on the slave side of the experiment or through the pigtailed phase modulator for the phase modulation experiments. The output from the fiber is then passed into the complimentary output of a polarizing beamsplitter Faraday isolator arrangement, so it passed freely and is injected into the slave laser.

The light from the slave laser passes through an anamorphic prism pair and effectively through two faraday isolators. For alignment, the waveplate before the first faraday rotator can be adjusted allowing the output of the slave laser to pass out the complementary port of the PBS and towards the single mode fiber coupler. The light is then coupled into a single mode fiber. The coupling efficiency at this location is typically 60%. This fiber is then attached (as discussed above) directly to the master side fiber or to the output of the pigtailed phase modulator. This
alignment step insures that the light from the master laser is optimally coupled with
the slave laser. This is because, the optical mode of the slave, which couples into the
single mode fiber (slave laser), will be very similar to the output mode from that fiber
(master laser). After alignment, the waveplate before the faraday rotator is adjusted
so that the slave laser passes through the PBS and on through a 40 dB isolator. A half
wave plate and polarizing beam splitter combination is used after the final isolator
allowing a small portion of the output to be analyzed simultaneously with the OSA
and a scanning fabry-perot cavity (SFPC), here a Coherent 240 with a 7.5 GHz free-
spectral range and a finesse of 150. The main portion of the beam passes through the
polarizer towards the chosen experiment. This optical configuration allows maximum
efficient use of the injected fields while maintaining isolation of the master laser from
the slave laser.

Initial Injection Locking and Locking
Regions

The modes of the master laser and the slave laser before and after injection locking
were examined with the OSA. Figures 32 (a), (b), and (c) show the master laser’s
optical spectrum, the slave laser’s spectrum before injection locking, and the slave
laser’s spectrum after injection locking in a stable region, respectively. Upon injection
of the master laser, the slave laser jumps to the carrier frequency of the master, and
also the sidemode suppression ratio of the slave laser increases from 25 dB to 35 dB.
There is still evidence of the original facet modes of the slave laser and these play an
important role in the carrier frequencies at which injection locking can take place. In order to injection lock, the frequency of the master laser did not have to be close to the free running frequency of the AR coated slave laser. As long as the master laser was positioned close to a facet mode of the slave laser, the slave would be injection locked.

Figure 32. Optical spectra of (a) master laser, (b) slave laser before injection locking, and (c) slave laser after injection locking.

As was discussed above, there is a standard route to observing chaos in an injection locked semiconductor laser [68]. This period doubling route to chaos was
observed by locking the slave laser with low injected powers at zero detuning and then increasing the injected power. To our knowledge this has not been observed before using an AR coated diode as a slave laser. There are some differences expected with AR coated lasers, which are expected and examined below. The four main regions, discussed in figure 30, were experimentally observed as the injected power was increased. The experimental optical spectra are shown in figure 33 for a slave output of $P_{\text{out}} = 88 \text{ mW}$. The four characteristic spectra shown in figure 33 represent regions of (a) stable locking shown with $P_{\text{in}} = 9 \mu\text{W}$, (b) undamped relaxation oscillations shown with $P_{\text{in}} = 70 \mu\text{W}$, (c) period doubled relaxation oscillations shown with $P_{\text{in}} = 132 \mu\text{W}$, and finally (d) the chaotic region shown with $P_{\text{in}} = 352 \mu\text{W}$.

The relaxation oscillations of the free running slave laser, measured using a heterodyne technique [61], increased as a function of injected current as expected. At the operational point of $P_{\text{out}} = 88 \text{ mW}$ the relaxation oscillations were measured to be $\nu_r = 5.6 \pm 0.3 \text{ GHz}$, in good agreement with period one oscillation sidebands, $\nu_{P_1} = 5.5 \pm 0.1 \text{ GHz}$, obtained from figure 33(b). As was discussed above, it was thought that the maximum modulation frequency for amplification of phase modulated signals would be close to these 5.5 GHz sidebands [58].
Figure 33. Optical spectrum of the different regions on the period doubling route to chaos showing (a) Stable locking, (b) undamped relaxation oscillations, (c) period doubled relaxation oscillations, and (d) the chaotic region.
Next it was desired to map the regions of dynamic instabilities and to identify the region of stable locking. To determine these different regions and their dependence on the injected power and detuning an automated experiment was set up. This experiment, controlled by a computer, recorded scans of the SFPC and the OSA for the master and the slave laser for various injected powers and detunings. The culminated data was then analyzed and the different optical spectra categorized by the dynamical regions described above. The resulting map of the dynamical instabilities and stable locking is shown in figure 34. This map shows good agreement with features found using non-AR coated diodes and distributed feedback lasers \[84, 85\].

As has been discussed above, one effect of the AR coating increases the relaxation oscillations compared to a similar non-coated laser. Thus, as might be expected, an overall scaling in the map along the detuning axis is expected, and observed, compared to the map with a similar non-AR coated laser \[67\]. In those experiments, the relaxation oscillations were measured to be \(f_r \approx 3\) GHz compared to the 5.5 GHz measured here. Here, the injected power level ranged from \(P_{in} = 0.8\) \(\mu\)W to \(P_{in} = 792\) \(\mu\)W and the master's frequency, \(f_M\), was stepped from -10 GHz to 10 GHz with respect to the nearest facet mode frequency, \(f_S\), of the free running slave laser. The detuning parameter has been defined as \(\Delta = \Omega_o / (2\pi) = f_S - f_M\). The injection ratio, \(\zeta\), is defined as the input power directly in front of the slave laser cavity divided by the output power from the slave without inclusion of cavity coupling factors \(\zeta = P_{in} / P_{out}\). The defined regions on the map are stable locking
undamped relaxation oscillations (P1), period doubled relaxation oscillations
(P2), chaotic regions (C), four wave mixing (4W), multi-longitudinal mixing (M),
period four relaxation oscillations (P4), a combination region of chaos and relaxation
oscillations (U) as well as regions that were unlocked. The different markers on the
map delineate observations of the approximate boundaries between regions. The lines
and shading are interpolated and intended to guide the eye.

As can be seen from the figure, a large region of stable locking, (S) is found. This
region is the important region in the map because it is where faithful reproduction of
the master laser is expected. At low injection parameters, the stable region is sym-
metric about the zero detuning point. But as the injection parameter is increased the
stable region quickly becomes asymmetrical and is found only for positive detunings
being bounded on one side by period one regions and regions of unlocked operation
or four wave mixing. The width of this region is fairly constant at approximately 2.5
GHz above $\zeta = 4 \times 10^{-4}$.
Figure 34. Observed regions of injection locking versus the detuning, $\Delta$ and the injection ratio, $\zeta$ for a $P_{out} = 88$ mW. The symbols represent observations of the boundaries between different regions while the lines and shading are there to guide the eye. The various regions are stable locking (S), undamped relaxation oscillations (P1), period doubled relaxation oscillations (P2), chaotic regions (C), four wave mixing (4W), multi-longitudinal mixing (M), period four relaxation oscillations (P4), an undefined region of both chaos and relaxation oscillations (U), as well as unlocked regions.
CW Phase Modulation

Once the region of stable locking had been adequately mapped, the slave laser was injection locked in that region with a phase modulated optical signal. First, the system’s response to CW phase modulation was examined. The phase modulator, operating with CW phase modulation, was tested without injection locking to ensure that the modulators output was as expected. This was done by recording optical spectra using the SFPC and a computer controlled digitizing oscilloscope. The left plot in figure 35 shows the master laser’s spectrum when phase modulated at (a) 1 GHz and (b) 3 GHz. The modulation frequency was then stepped from 0.8 GHz to 3.4 GHz in steps of 0.1 GHz. At each step, the optical powers of the master laser carrier and the lower and upper frequency first order sidebands were measured. These powers are plotted as a function of the modulation frequency in the right plot of figure 35. Due to nonlinearities in the modulator, a set of appropriate input RF powers had to be found for the series of modulation frequencies to give a carrier to first order sideband power ratio of ~2:1. The 2:1 ratio was picked to help determine signal fidelity in the injection locked signals and to reduce the 2nd order sidebands.
Figure 35. (Left) CW phase modulation of the master laser with modulation frequencies of (a) 1 GHz, and (b) 3 GHz. (Right) Plot of the peak powers vs. modulation frequency for the carrier and sidebands of the phase modulated master. At each modulation frequency the rf power was adjusted to achieve roughly a 2:1 ratio of carrier to sidebands.

After characterizing the phase modulated master, this phase modulated signal was injected into the slave laser with detuning $\Delta \approx 6$ GHz. The power of the CW phase modulated light just outside the slave cavity was $P_{in} = 400 \mu W$, giving a $\zeta = 4.5 \times 10^{-3}$. Using this combination of $P_{in}$ and $\Delta$ the injection locking was placed in a stable locking region with a detuning width of $\sim 2.7$ GHz. A set of measurements was made for several different output powers, $P_{out} = 9, 29, 50, 66, 88$ mW, of the slave laser giving gains of $A = 14, 18, 20, 22$ and 23 dB, respectively. The output of the injection locked slave laser was recorded using the SFPC for each
modulation frequency and for each different slave output power. Figure 36 plots the optical powers of the carrier and sidebands as a function of the modulation frequency.

Figure 36. Injection locked peak powers vs. modulation frequency for (a) lower frequency first order sideband, (b) carrier, and (c) upper frequency first order sideband. The different power levels correspond to gains of $A = 14, 18, 20, 22$ and $23$ dB.

Figure 36 shows (a) the lower frequency sideband, (b) the carrier, and (c) the upper frequency sideband. Each measurement set for a given slave output power was done within a two minute time frame eliminating long term laser or SFPC frequency drift. Figure 36 shows that locking was achieved for each output power. However, the
signal fidelity degrades as the modulation frequency increases. The ratio of carrier to sidebands deviates by roughly 20% at 3 GHz for $P_{\text{out}} = 88$ mW. Above this frequency, the fidelity of the signal rapidly degrades, with the lower frequency sideband increasing in power while the higher frequency sideband and carrier are depleted of power. In fact this trend is noticeable on all output power levels. This asymmetry in the sidebands is a significant problem and leads to unwanted amplitude modulation of the output signal. From the discussions above, one might expect that the maximum modulation frequency would be approximately the relaxation oscillation frequency. This frequency, as stated above was measured to be $f_r = 5.6$ GHz for a $P_{\text{out}} = 88$ mW. Here one notices that the maximum modulation frequency without serious asymmetry in the sidebands is roughly 3 GHz for all output powers. Thus, an upper limit for phase modulation has been reached and does not appear to be dependent upon the output power of the slave. Unfortunately, this limit is less than the expected value of $f_r = 5.6$ GHz. This upper limit is likely the result of a set of complex physical processes within the injection locked diode laser and will need to be simulated to identify the key physical parameters involved in its determination. Simulations are currently being pursued to further study this effect but are beyond the scope of this thesis.

To further study the asymmetry of the sidebands, the residual amplitude modulation (RAM) at the modulation frequency was measured with a fast detector and the network analyzer. The RAM was measured directly after the phase modulator to
characterize the phase modulators performance. It was also measured after injection locking at the output of the system. Figure 37 shows the RAM measured as a function of modulation frequency for both cases. By comparing the RAM before locking with that after locking one would expect an amplification due to the gain of the system. In figure 37 the RAMs are offset by approximately 20 dB, however, these measurements were not calibrated and thus do not reflect the absolute gain of the injection locking. Of more interest is whether the functional dependence of the RAM for both cases is preserved. As can be seen from the figure, the RAM after the injection locking system undergoes a significant increase beyond 3 GHz as compared to the RAM of just the phase modulator. This is as expected, because of the increasing asymmetry in the sidebands shown in Figure 36. This result reinforces the earlier conclusion that indeed the upper limit at this injection level for CW modulation is 3 GHz as that is where the shapes of the two RAM curves depart significantly from each other.

These CW phase modulation experiments demonstrate that the injection locking system can handle and reproduce reliably modulation frequencies out to 3 GHz. However, CW phase modulated signals, while useful in characterizing the system, are not used to communicate information. Instead, interest lies in signals with large bandwidths such as BPSK modulated signals. How the signal responds to such signals and whether the amplifier would work to the 3 GHz bandwidth found for CW phase modulation must be determined.
Figure 37. The residual amplitude modulation for both the phase modulated master and the injection locked signal. A deviation at around 3 GHz is apparent.

Broadband Operation (BPSK)

In order for BPSK signals to be observed, the phase shifted signals must be heterodyned with some kind of reference waveform. This can be done in many ways, including heterodyning with a stable carrier [54]. Phase variations due to thermal drifts or other effects create a need for a phase-locked loop that constantly adjusts for the phase difference. The approach utilized here for detection of the BPSK signals was to heterodyne the signal with itself. The approach has less phase variation, and can be used without a phase locked loop over several seconds without need for adjustment. Essentially, the BPSK signal is passed through an interferometer with
one path having an optical delay of close to the period of one bit. The original signal and the one-bit delayed version are then spatially combined onto a detector. By using this method, the detector becomes a differential phase detector. It detects the changes in the phase of the BPSK sequence. This technique is known as delayed-self-heterodyning. A system of beam splitters and a peizo adjustable mirror were used to observe BPSK signals of both the BPSK modulated master and the BPSK output field from the slave laser. The peizo was used to adjust the path length of the interferometer, and thus give fine control over the phase of the field along the delay path.

To explore the slave laser's ability to faithfully reproduce a digital signal, the pulse pattern generator was used to produce a set of test binary data sequences. Three test data rates were chosen, 2 Gbit/s, 3 Gbit/s and 6 Gbit/s. The upper limit on the data rate was 6 Gbit/s because of the 3 GHz bandwidth limit of the 10 GSample/s digitizing oscilloscope used to capture the outputs. The output data sequences were delayed-self-heterodyned in order to observe the phase shifts. To help adjust the phase of the delayed signal, the beginning of the data sequence was padded with 1's. The peizo loaded mirror was adjusted so that the padded region would destructively interfere at the output port.

The laser was set to operate within the stable locking region defined above ($\Delta \approx 6$ GHz). The BPSK phase modulated field was then injected into the slave laser.
The input power, $P_{in}$, just outside of the slave laser was $P_{in} = 400 \mu W$. The delayed-self-heterodyned output from the injection locked slave is shown in figure 38 for (a) 2 Gbit/s, (b) 3 Gbit/s, and (c) 6 Gbit/s. The output power for these sequences was $P_{out} = 88$ mW giving a gain of 23 dB. The expected delayed-self-heterodyned output is shown as a dotted line in each plot. It can be seen from these plots that the injection locking follows the expected output quite well for each bit rate. Over the 256 bit test sequences employed, no logic errors were observed. The eye diagrams of the 256 test bits for each data rate are shown next to each of the example signals in figure 38. As might be expected, the 6 GBit/s eye diagram is slightly less open than the other two. This could be the result of a combination of factors including the upper bandwidth limitations of the scope, a mismatch of the delay from the delay arm with the bit period, as well as the upper bandwidth limitations of the injection locking as seen from the CW phase modulation results above. This faithful reproduction of the binary signals up to 6 GBit/s is a six fold increase in the BPSK data rate of injection locked signals over previous demonstrations [81].

Unlike the experiments performed in [80], the modulation frequency for the CW phase modulated signals was large enough to encounter a fundamental limit that introduced a significant asymmetry in the sidebands. This suggests that at these injected powers, there is a fundamental limit to the phase modulation bandwidth of the diode laser that is less than the frequency of the relaxation oscillations. Future work on this subject should explore the physical mechanisms that limit the modulation
bandwidth to less than the relaxation oscillations of the slave. This is an important consideration as it would be desirable to phase modulate beyond 3 GHz bandwidths. Perhaps, as discussed in [58, 71, 72, 73, 74], with stronger injected fields the modulation bandwidth of the slave laser can be enhanced allowing these higher bandwidths to be amplified. However, the modulator’s power limit at this wavelength will ultimately limit the amount of injected field. In any case, it is apparent from the good reproduction of these BPSK signals that the injection locking system is fully capable of amplification to 6 Gbit/s with 23 dB. This is an exciting result because BPSK signals can now be used as probe signals for TTD applications. In chapter 6, this system is utilized to create high bandwidth BPSK signals that are subsequently TTD and then detected. Through the successful efforts shown in this chapter, a compact, easy to use, high bandwidth amplification device was created and characterized, and is utilized throughout the rest of this thesis.
Figure 38. Bits 1 through 15 of the delayed-self-heterodyne injection locked outputs of BPSK data at (a) 2 Gbit/s, (b) 3 Gbit/s, and (c) 6 Gbit/s. The expected output is shown as dotted lines. To the right of the data sequence is the eye diagram for the total 256 bit test sequence.
CHAPTER 5

TEMPORALLY OVERLAPPED LINEAR FREQUENCY
CHIRPED PULSES

As was discussed in the introduction, linear frequency chirped pulses can be used in place of temporally brief pulses to program a true-time-delay grating [7]. There are several key advantages to do so, such as being able to program efficient high bandwidth gratings with low power diode lasers. In this chapter, a discussion of several chirped programming methods is given. Along with this, the efficiency of these methods is discussed. Also, a new programming method, where the linear frequency chirps are temporally overlapped, is discussed. This method truly shows the flexibility of OCT programming, namely that the programming of gratings, which give time-domain responses, can occur through frequency shifting of temporally overlapped pulses. This raises questions as to how to determine phase matching conditions and which echoes will rephase. These issues as well as the several advantages of the new approach are presented in this chapter from both a theoretical and experimental vantage. Initial demonstrations of the technique are shown as well as high bandwidth TTD gratings that were created using a CECDL. The dynamics of accumulated gratings is also examined. The accumulation technique can allow for even lower power lasers to program broader bandwidth efficient spectral gratings or allow for larger spot areas in the crystal to be programmed giving greater throughput power.
Chirped Pulse Programming

As was shown in chapter 2, a set of two brief pulses separated by some time delay, $\tau_{21}$, programs an OCT crystal with a spectral grating. Such a grating, known as a TTD grating, will act upon a probe pulse producing a replica of the probe pulse with the time delay, $\tau_{21}$. It was also shown that this programming method can be replaced by two linear frequency chirped (LFC) pulses [7]. An LFC pulse is an optical pulse that has its instantaneous frequency vary linearly from the beginning to the end of the pulse. When a pair of temporally separated identical LFC pulses illuminates an IBA, the ions record the time separation between when the instantaneous frequencies of the first and second LFC pulses match a given ion's resonant frequency. The frequency dependent phase shift from this delay results in a periodic spectral population grating. As was shown at the end of chapter 2, these gratings have a spectral period equal to the reciprocal of the time separation of the chirped pulses, similar to brief pulse programming. However, by programming with LFC pulses one has the advantage of spreading the spectral energy of the pulse over a longer duration and thus producing efficient gratings with much lower peak intensities of the programming pulses. The square spectral features of linear frequency chirped pulses have advantages in spectral filtering, as will be discussed.
Programming TTD gratings with two temporally separated LFC pulses has the added advantage that the programmed delay can be tuned by simply adjusting the start frequencies of the LFC pulses. As previously demonstrated, a TTD grating can be set up with two LFC programming pulses, each with the same chirp rate, $\gamma$, as seen in figure 39(a) [6]. These LFC pulses, each of temporal length $\tau_c$ and bandwidth $B = \gamma \tau_c$, are separated by a delay, $\tau_{21}$. In the simple case that both LFC pulses start out with the same frequency, the delay programmed, $\tau_D$, is equal to $\tau_{21}$. However, if there is a difference in the start frequency between the two pulses then
the programmed delay can be shifted. The programmed delay is then given by [7]

\[ \tau_D = \tau_{21} + \frac{\delta}{\gamma} \]  

(5.1)

If \( \gamma > 0 \), \( \delta \) is defined as the start frequency of the first LFC pulse minus the start frequency of the second LFC pulse. The bandwidth of the probe pulse signal should not be greater than the overlap of the two LFC pulses' bandwidths. This technique has great promise for TTD applications since the time delay can be shifted simply by changing the offset between the start frequencies of the LFC pulses. Delay times of several microseconds have been programmed with errors in delay accuracies of less than 100 psec [5].

While there are significant advantages in using LFC pulses over brief pulses for programming TTD gratings, there are several drawbacks in the above approach. First, the time delay created in this method must be less than \( T_2 \) (on the order of ten microseconds) in order to produce efficient gratings. This requires that \( \tau_{21} \) be less than \( T_2 \), which in turn requires that the duration of the individual LFC pulses be less than \( \tau_{21} \). For high bandwidth, extremely high chirp rates (several GHz per microsecond) are required, putting unrealistic demands on the LFC source. Also, the power required to efficiently program an IBA is proportional to \( B/\tau_C \). Thus, limited LFC pulse duration translates to higher power requirements as the bandwidth increases. The next drawback comes from the fact that in practice \( \delta \ll B \) in order to utilize most of the LFC bandwidth [7]. When this condition is followed, the tunable range of delay times is limited to much less than the LFC pulse duration (much less
than $T_2$). Finally, in order to accumulate a spectral grating, a phase relationship between the two LFC pulses must be constant for all the programming sequences within the grating lifetime. The minimal requirement is that the frequency jitter of the laser, $\Delta f$, satisfy $\Delta f \ll \frac{1}{2\tau_D}$ [28]. Such stability, while achievable for single frequency laser sources, is extremely difficult to achieve with LFC laser sources. This is because LFC laser sources, such as CECDL's, change their effective cavity length. The frequency resolution on the actual chirp is typically worse than the required stability. When the laser returns to a steady state CW output after the chirp, its frequency may be different by more than the stability amount required by the above relation. Even if the chirped laser returned to the same frequency, the phase of the waveform may be different. In either case, the phase between sets of programming chirps cannot be controlled precisely enough to allow accumulation of LFC pulses created with CECDL's.

The above analysis assumed chirps that were separately created. However, this need not be the case. A beam splitter can be utilized to split a single LFC pulse. One of these pulses can then travel through a delay line creating a delay $\tau_{21}$ with respect to the other pulse. These pulses can then be focused into the crystal to create a spectral grating. The time delay can be tuned, as discussed above, by frequency offsetting one of the chirps, or tuned via the path length of the delay line. This method can also be used for accumulation, since the use of single LFC chirp reduces some of the laser requirements for accumulation. Here, the phase difference between the two
programming pulses is related to the change in the path length difference during \( \tau_{21} \). Thus, for stable optical setups, the phase differences between programming pulses is usually not a limiting factor. However, the start frequency of the next set of linear frequency chirps is important. Here it is required that the start frequency not change by more than \( 1/2 \) the period of the spectral grating between shots otherwise no grating will accumulate.

The approach proposed here is based on the realization that the two LFC pulses need not be temporally delayed, but overlapped, to the point when \( r_{2i} = 0 \), as shown in figure 39(b).\(^1\) If \( r_{2i} = 0 \), the two LFC pulses can be derived from the same LFC source eliminating the need for a delay line. This means that a single chirped pulse from one CECDL could be used to program the spectral grating. To do this, a frequency offset is imparted to the single LFC pulse, creating a pair of LFC pulses each with different start frequencies, but the same chirp rate. This frequency offset can be created by using low bandwidth acousto-optic modulators (AOM) driven at different RF frequencies. If a collinear geometry is desired, the original LFC pulse can be focused tightly into the AOM, keeping the two output pulses within the diffraction limit of the AOM and thus spatially indistinguishable. Or if two spatially separate pulses are required, two different approaches could be used. First, a beam splitter could be utilized, producing two versions of the original LFC pulse. Each

\(^1\) While LFC programming has been described in general [7, 5, 86], it has always been suggested or implied that \( r_{2i} \neq 0 \). The smallest value found in these previous approaches was \( r_{2i} = T_c/2 \). There has been no direct discussion, nor demonstration, of setting \( r_{2i} = 0 \) in the previous literature.
of these pulses then travels through separate AOM's each driven at slightly different frequencies giving an overall frequency difference between the two pulses. The second approach is to not focus tightly into a single AOM that is driven by two frequencies. The output of this AOM will be two diffracted beams at different angles that are frequency offset.

This technique has several advantages. The programmed TTD of the grating is now directly proportional to the frequency offset between the two programming LFC pulses, or

\[ \tau_D = \frac{\delta}{\gamma} \]  

(5.2)

A significant advantage of temporally overlapped LFC’s is that the pulse duration can now far exceed \( T_2 \), as long as the delay programmed is less than \( T_2 \). This is because it is the time delay between when an ion is effectively excited by the pulses that records the delay and not the overall time between two temporally separated pulses. Now, the duration, \( \tau_C \), of the programming pulses is limited by \( T_1 \) instead of \( T_2 \). Along with this, the full range of delays, from hundreds of picoseconds to \( T_2 \) can be programmed with the frequency offset by using large \( \tau_C \)’s and small \( \delta \)’s. Another advantage of using large \( \tau_C \)’s is that the power requirement on the LFC pulses is reduced by roughly a factor of \( \tau_C/T_2 \) compared to the temporally separate LFC programming method described above. Finally, the stringent phase requirements for accumulation, are reduced to a chirp linearity requirement on the LFC source.

This method of using a single LFC source and using a frequency offset is not
limited to the creation of single delays. In fact, if one uses multiple frequency
offsets, one can program multiple delays into the medium. If a probe pulse is incident
upon this multiple delay grating, multiple echoes will be produced- one for each delay
programmed. This process is known as pulse shaping or arbitrary waveform gen-
eration and has several applications. This technique, as well as a low bandwidth
demonstration, is discussed in appendix 3.

Phase Matching Conditions

Because the linear frequency chirped pulses are temporally overlapped, one may
wonder what happens to the phase matching conditions. For brief pulse programming,
the phase matching condition for a stimulated photon echo is

\[ \vec{k}_e = \vec{k}_3 + \vec{k}_2 - \vec{k}_1. \]  (5.3)

Here, the pulses are easy to distinguish because the arrival time of the pulses dictates
their ordering. However, in the situation of temporally overlapped linear frequency
chirped pulses, one is not afforded that luxury. Instead, a new method, besides pulse
arrival time, must now delineate the pulses. In this case, one must turn to what the
atoms see in order to figure out which "pulse" is \( \vec{k}_1 \) and \( \vec{k}_2 \).

Let us assume for example that the two temporally overlapped linear frequency
chirped pulses have separate wave vectors, \( \vec{k}_a \) and \( \vec{k}_b \). Let us also assume that the
start frequency of the LFC pulse with wave vector \( \vec{k}_a \) has a higher start frequency,
\( \omega_{\text{start}} \) and that the chirp rate is positive, \( \gamma > 0 \). Then, the ions at frequency \( \omega_{\text{start}} \)
will be illuminated by the pulse with wave vector $\vec{k}_a$ first. This sets those ions into a coherence and then at the appropriate $\tau_D$ this coherence is changed into a population grating by the illumination of the atoms by pulse with wave vector $\vec{k}_b$. In this case then, the wave vector $\vec{k}_a$ acts like the first pulse, $\vec{k}_1 = \vec{k}_a$, and $\vec{k}_b$ acts like the second pulse, or $\vec{k}_2 = \vec{k}_b$. The phase matching condition is then,

$$\vec{k}_e = \vec{k}_3 + \vec{k}_b - \vec{k}_a .$$  \hspace{1cm} (5.4)

Here $\vec{k}_3$ is the wavevector of the probe pulse and is sometimes represented by $\vec{k}_p$ since in some cases, like accumulation, the probe pulse is not the third pulse.

Figure 40. Diagram of a typical temporally overlapped linear frequency chirped pulse experiment.
Low Bandwidth Demonstrations

Experimental Setup

It was desired to show that the temporally overlapped frequency chirped programming method indeed could be used to create TTD gratings. Thus, a series of experiments that would detail the characteristics of this method were performed. These initial demonstrations were carried out at low bandwidth \((B = 40 \text{ MHz})\). An external cavity diode laser was used as the laser source amplified with the injection locking method described in chapter 4. Two AOM's were used effectively in series to create the temporally overlapped LFC pulses. The first AOM created an LFC pulse with \(B = 40 \text{ MHz}\). The second AOM was driven by the voltage

\[
V = A [\cos (2\pi (f_m - \frac{\delta}{2}) t) + \cos (2\pi (f_m + \frac{\delta}{2}) t)] ,
\]

resulting in two LFC pulses with different frequency shifts. Here \(f_m\) is the center frequency of the AOM, \(\delta\) is the offset frequency, and \(A\) is the amplitude. The light was focused tightly into the second AOM to keep the angle between the two frequencies close to the diffraction limit, ensuring spatial overlap of the two frequency offset pulses. This was because for the initial demonstrations a collinear pulse arrangement was desired. In the collinear scheme, \(\vec{k}_e = \vec{k}_a = \vec{k}_b = \vec{k}_p\). The same series of AOM's created the probe pulse whose optical frequency was centered within the chirped pulse's bandwidth with the first AOM and by keeping \(\delta = 0\) on the second AOM. The probe pulse was 50 ns long in all experiments unless noted otherwise. The power
of the collinear pulses before they were focused into the crystal was 35 mW and the beam waist diameter was \( \sim 35 \mu m \). In this experiment, the crystal was held at liquid helium temperatures \( \sim 4.1 \) K with an \( \alpha L = 1.4 \). \( T_2 \) was measured to be approximately 15 \( \mu s \) using temporally separated chirped pulse programming and the SPE. In these experiments, the output from the crystal was incident on a 1 GHz silicon photodiode and recorded on a digitizing oscilloscope with a bandwidth of 300 MHz. The time between single shot experiments was much greater than the grating lifetime to eliminate shot to shot interference. (i.e. There is no accumulation of gratings involved in these experiments.)

**Delay Versus Frequency Offset**

The first experiment performed measured the time delay of an echo versus the frequency offset of the LFC pulses. This was done to test the linear dependence found in eqn. (5.2). To do this, the temporally overlapped LFC programming pulses were followed 35 \( \mu s \) later by a 50 ns probe pulse, generating a delayed echo output. In figure 41 a probe pulse is incident upon the OCT medium that was programmed using \( B = 40 \) MHz temporally overlapped LFC pulses each with a \( \tau_C = 3 \) \( \mu s \). The frequency offset between the programming pulses was then changed from \( \delta = 3 \) MHz to 19.8 MHz in steps of 1.2 MHz. One can see that the echo’s delay does increase linearly as the frequency offset is increased. These experimental results were also simulated using the Maxwell-Bloch simulator discussed in chapter 2. The results are
shown in figure 42.

Next, the functional dependance of the time delay on the frequency offset is examined. According to eqn. (5.2) the time delay should be linear with frequency offset. The experimental time delay was determined by finding the time of the echo’s maximum power, and is plotted versus the frequency offset as circles in figure 43.

This plot includes echo delays created from many different programming durations, \( \tau_C = 0.5, 1, 3, 5, 15, 30, \text{ and } 100 \mu s \) to demonstrate the wide tunability of the programmed TTD. In each case, a linear fit to the data was made using eqn. (5.2) (solid lines).

Table 1 shows the expected inverse chirp rates as well as the slope, \( \kappa \), from the linear fits along with the percent difference and the root mean square (RMS) deviations for the data sets in figure 43 with more than 20 data points. The observation of echoes at \( \tau_C = 15, 30 \text{ and } 100 \mu s \) shows that indeed \( \tau_C \) can be considerably longer than \( T_2 \) allowing more optical energy to be transferred to the medium assuming that the programmed delay is much shorter than \( T_2 \).

<table>
<thead>
<tr>
<th>Chirp Time (( \mu s ))</th>
<th>0.5</th>
<th>1.0</th>
<th>3.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected ( \gamma^{-1} ) (( \mu s/\text{MHz} ))</td>
<td>0.0125</td>
<td>0.0250</td>
<td>0.0750</td>
<td>0.1250</td>
</tr>
<tr>
<td>( \kappa ) (( \mu s/\text{MHz} ))</td>
<td>0.0116</td>
<td>0.0244</td>
<td>0.0746</td>
<td>0.1248</td>
</tr>
<tr>
<td>Error (%)</td>
<td>7.4</td>
<td>1.8</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>RMS Deviation (ns)</td>
<td>4.0</td>
<td>2.0</td>
<td>2.2</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 1. Expected and measured inverse chirp rates for echoes from figure 43 along with RMS deviations.
Figure 41. Echoes from a probe pulse shown for various frequency offsets from $\delta = 3$ MHz to 19.8 MHz in steps of 1.2 MHz. Here $B = 40$ MHz and $\tau_C = 3 \mu s$.

Figure 42. Simulated programming and probe pulses and the echo output for various frequency offsets, $\delta = 0.66$ MHz to 19.785 MHz in steps of 2.125 MHz. Here $\tau_C = 3 \mu s$ and $B = 40$ MHz.
Figure 43. Measured echo delays vs. frequency offset for several different $\tau_C$'s with linear fits using (5.1). The points for $\tau_C = 30$ and 100 $\mu$s demonstrate the reduced chirp rate requirements.
The standard error of \( \kappa \) is approximately \( 10^{-4} \) for each sequence, except the 5 \( \mu s \) data, which is \( 2 \times 10^{-4} \). Table 1 does show some significant percent errors in the measured and expected slopes. The measurement method described above, used the delay found at the maximum echo height. This method is prone to error, especially for low signal to noise ratios. A better method was also used for the \( \tau_C = 3 \mu s \) data. This method used a least squares fit of a Gaussian peak to each echo peak. The subsequent time offset of the Gaussian peak relative to the first echo was then plotted against the frequency offset in the top portion of figure 44.

![Graph](image)

Figure 44. (Top) Data points and a best fit line for the time delay versus frequency offset of the 3\( \mu s \) data shown in the previous figure. Here the method to measure the time delay was a Gaussian fit to the echo peaks. (Bottom) The residuals of the above data points giving an RMS = 1.6 ns.
Also shown in this plot is the linear least squares fit to this data. The slope using this method is 0.0748 with a standard error of \(5 \times 10^{-5} \text{ } \mu s/\text{MHz}\). The residuals are shown in the lower plot and give an RMS deviation of 1.6 ns. This is close to the 2.2 ns that was achieved with the other method, indicating that the RMS deviation is coming from the experiment and not necessarily the method used to analyze the data. As will be discussed in the next section, these low bandwidth experiments are prone to echo oscillations. These are induced by a complex process that is mainly the result of low time-bandwidth products (TBP's) for the chirps and time delays on the order of the temporal duration of the probe pulse. The significant errors and RMS deviations are likely the result of these oscillations, which fortunately can be eliminated for large TBP's (and thus high bandwidths). \(^2\)

To show that the grating could reproduce any arbitrary sequence, the grating was probed using a data sequence. Figure 45 shows the results of TTD for this amplitude modulated probe pulse. The probe, a binary coding of 101011001 at a rate of 20 Mbit/s, was sent into the crystal 40 \(\mu s\) after the programming LFC pulses with \(\tau_c = 100 \mu s\) and \(\delta = 0.25 \text{ MHz}\). The echo signal is delayed by the expected \(\tau_D = 625 \text{ ns}\). Good signal fidelity (no bit errors) as well as strong echo efficiency (\(\sim 2.5\%\)) have been achieved.

These initial demonstrations show that this method is roughly producing the

\(^2\) RMS deviations for high bandwidth chirps utilizing the temporally overlapped programming method are shown in chapter 6 to be better than 60 ps.
expected output according to eqn. 5.2. However, some periodic intensity fluctuations of the echoes as a function of programmed frequency offsets were noticed. This was only noticed for chirped programming pulses with low time bandwidth products. The next section discusses these intensity fluctuations of the echoes and gives some possible explanation for their occurrence.

Figure 45. An example of a true time delayed data sequence (right x30). Here $\tau_c = 100 \mu s$ with a programmed time delay of 625 ns. The data sequence (left) is 101011001 at a data rate of 20 Mbit/s.
Operating Conditions

Under ideal conditions, for perfect TTD grating programming using LFC’s, the spectral amplitude of the chirp would be flat over the full frequency extent of the probe. However, in reality an LFC’s power spectrum includes oscillations. These oscillations in the spectral domain depend on the time bandwidth product (TBP), $B\tau_C$, of the chirp. Figure 46 shows these spectral oscillations and their effect on the population inversion, $r_3$, as a function of detuning within the crystal. Assuming two temporally separated chirps are incident on the material, with no frequency offset, a periodic spectral grating is written. However, that grating has a spectral envelope whose shape is similar to the chirp shown in 46. In other words, there is a fast
oscillation (with the period of the grating $1/\tau_D$) with an amplitude that has slower oscillations (the spectral envelope) as shown in figure 46. If the time delay is tuned by changing the temporal separation of the chirps, the grating period changes, but the spectral envelope does not. This leads to essentially constant echo intensities for tuning with temporal separations. This can be seen in figure 47. Here there are several different chirp sequences with different time delays ($\tau_D = 1.3, 1.5, 1.7, 1.9, 2.1 \mu s$). The rapid oscillations are all evident, however the spectral envelope always stays the same.

Figure 47. The simulated $r_3$ component of the Bloch vector versus detuning for several different temporally separate LFC pulses. Notice both the rapid oscillations (the spectral grating with period $1/\tau_D$ and the chirp envelope. Here $B = 40$ GHz, $\tau_C = 1$ ns and $\alpha L = 0.3$.
Figure 48. The simulated $r_3$ component of the Bloch vector versus detuning for temporally overlapped LFC pulses. The two plots show two different frequency offsets, and show that the spectral envelopes of the spectral grating changes leading to the possibility of intensity fluctuations in the echo. Here $B = 40 \text{ GHz}$, $\tau_C = 5 \text{ mus}$ and $\alpha L = 0.3$.

Now consider the case where the time delay is tuned via the frequency offset, $\delta$. In this case, the spectral envelope of the two chirps is shifted with respect to each other. As the frequency offset is changed to tune the delay, the period of the grating changes, as well as the overall spectral envelope of the grating. If this spectral envelope is changing so too can the overall power in the echo. These spectral oscillations can be seen in figure 48. The top plot shows the simulated $r_3$ vector versus detuning for a temporally overlapped frequency offset LFC whose individual spectral envelopes combine to create a fairly steady envelope for the grating. However, in the bottom plot, the spectral envelopes combine to create stronger oscillations, thus allowing intensity fluctuations in the echoes as the time delay is changed. Because of the complexity of this problem, an analytic description of this process is not easily found.
Finally, power fluctuations of the echo can also occur if the period of the spectral grating is of the same order as the bandwidth of the probe pulse. In this case, slight changes to the period of the spectral grating can lead to more or less absorption of the probe, and thus more or less diffraction of the probe. This results in a power fluctuation as the time delay is changed.

![Graph](image)

Figure 49. Experimental echo intensities plotted versus delay time for (a) TBP = 40, (b) TBP = 120, and (c) TBP = 200. For TBP < 120, a periodic intensity fluctuation is observed.

Through a combination of the above processes, echo intensity fluctuations were observed experimentally. These fluctuations were also noticed to diminish in amplitude as the TBP was increased. Figure 49 shows echo peak intensities versus delay time for (a) TBP = 40, (b) TBP = 120, and (c) TBP = 200. For each plot shown $B = 40$ MHz. The probe pulse had a duration of 50 ns ($B = 20$ MHz). The largest
spectral period (shortest time delay) was 12.8 MHz (0.8 μs). As can be seen from the figure, a strong periodic intensity fluctuation on the echo is seen for TBP = 40. As the TBP is increased this periodic event diminishes, as in (b) where TBP = 120, and then almost completely disappears, as in (c) where TBP = 200. As stated, this is the result of a combination of factors, including the increase in time delay, resulting in smaller spectral periods compared to the bandwidth of the probe, as well as a diminishing effect of the spectral envelope oscillations as the TBP increases. Thus, for practical operation, the TBP should be kept as high as possible, as well as keeping the time delay long compared to the bandwidth of the probe pulse \((τ_D \gg τ_p)\), in order to avoid such echo fluctuations. As higher chirp bandwidths are reached both of these conditions will be met. Thus, these fluctuations should be of little consequence as TBP’s on the order of \(10^4\) and higher will be used in conjunction with long delays compared to probe bandwidths.

Low Bandwidth Accumulation Experiments

As was discussed in chapter 1, accumulation of spectral gratings involves repeated application of programming pulses to build up and maintain efficient spectral gratings. This allows much lower power lasers to be used to create efficient gratings compared to using a single programming pulse pair to create the gratings, but does put different frequency stability requirements on the programming pulses [28, 39]. As was discussed in chapter 3, the most stringent requirement for brief pulse programming is that the
phase difference between the programming pulses vary much less than $\pi$ between two coherently accumulated gratings. If the grating shifts by a factor of $\pi$ between accumulation pairs, the grating will be wiped out. The next two sections show the dynamics of the accumulation process, using both experiments and simulations.

**Demonstrations Showing the Dynamics of Accumulation**

To demonstrate accumulation using temporally overlapped LFC pulses, two different experimental arrangements were used. The first utilized a frequency stabilized Ti:Sapphire laser that was locked to a regenerative spectral hole resulting in a laser jitter of approximately a few kHz [28]. This laser was used to help study the build-up and maintenance dynamics of the accumulated gratings. One may expect to see three different situations in accumulation for various programming strengths. The first is that the programming pulses are not strong enough to accumulate an efficient grating before population decay destroys the grating. The second case is that the programming pulses are too strong and through saturation phenomena lead to low grating efficiencies. In between these two cases, the ideal programming strength will build up and maintain a grating with maximum efficiency. This process will be described in greater detail below. It was the goal of the following experiments to identify the regions with different dynamics by varying the programming strength of the LFC pulses.

A computer controlled experiment was set up, where a series of $N$ temporally
overlapped LFC programming pulses would accumulate a spectral grating. After the N’th pulse, the computer would record the echo caused by a probe pulse 30 μs after the last programming sequence. Here the repetition time, τr, was 31 μs and the probe was a 50 ns brief pulse. The echo peak heights versus programming number could then be plotted to detail the dynamics of the accumulated gratings. The individual Rabi frequency of a programming pulse was used to characterize the strength of the programming pulse. To do this, the Rabi frequencies for pulses that were not chirped were derived from observations of the optical nutation [87]. In this paper, it is found that the first half period, τ, of the Rabi oscillation on a strong pulse is Θ = Ωτ = 1.66π for a thin medium. However, simulations in media of αL = 1.4 show that the first 1/2 oscillation gives

$$\Theta = \Omega \tau = 2.1\pi. \quad (5.6)$$

By comparing the optical powers of each individual chirp to that of a strong pulse that had evident Rabi oscillations, the individual chirp Rabi frequencies could be found.

Figure 50 details the echo power efficiency as a function of N for various programming pulse strengths defined at the center of their Guassian profile as \( \Omega = 0.5, 0.6, 0.9, 1.3, 1.7, 2.2 \) MRad/s. For this figure, \( \tau_C = 1 \) μs and \( \delta = 10 \) MHz, yielding a delay time for the echo of \( \tau_D = 250 \) ns. The crystal was again held at 4.1 K and had an \( \alpha L = 1.4 \). The bandwidth of the chirps was \( B = 40 \) MHz. For the weaker programming strengths, \( \Omega \leq 0.6 \) MRad/s, the repeated programming pulses can not accumulate
an efficient grating before population decay sets in. For the larger programming strengths, \( \Omega \geq 1.3 \) MRad/s, the programmed grating peaks quickly, then saturates, leading to inefficient gratings. But between these two regimes, the best steady-state accumulation can be found such as at the \( \Omega = 0.9 \) MRad/s programming strength.

These accumulation results were the first to show the dynamics for temporally overlapped LFC pulses. These results were the first to show all three different dynamical regions using the same optical setup. Similar experiments showing the dynamics for brief pulses have recently been done [88].

A similar experiment was conducted in the original setup using the ECDL and the injection locking system. Because there was no stabilization system for the laser, it was expected that the \( \tau_D \)'s would be limited by the inequality \( \delta f \ll 1/2\tau_D \). Figure 51 details the accumulation results. For these experiments, \( \tau_C = 1 \) \( \mu \)s and \( \delta = 5 \) MHz giving a delay time for the echo of \( \tau_D = 125 \) ns. Again the crystal was \( \alpha L = 1.4 \) and the temperature was held at 4.1 K. Here the Rabi frequencies were \( \Omega = 0.98, 1.28, 1.87, \) and 2.45 MHz. The results have more noise than the locked laser results, however, similar trends are observable indicating that indeed the injection locked laser could produce efficient gratings such as the \( \Omega = 1.28 \) MHz results. The frequency stability of this laser has been measured to be \( \sim 100 \) kHz. Thus the inequality \( \delta f \ll 1/2\tau_D \) is roughly satisfied allowing the creation of accumulated gratings.
Simulations Showing the Dynamics of Accumulation

Simulation of accumulation sequences requires that the population dynamics between shots be taken into account. Since Tm$^{3+}$:YAG is a three level system, the accumulation simulations must include the decay rates from each of the excited levels. Since there are no pulses between programming shots and the time between programming is longer than $T_2$, we can follow a simple analytic solution between programming shots [39].

In this system, the 793 nm transition is between the $^3H_6 - ^3H_4$ levels. The atoms on the $^3H_4$ level decay either to the $^3H_6$ or the $^3F_4$ with a branching ratio, $\beta$, of 0.56 and a decay time, $T_1$, of 800 µs [45]. The atoms on the $^3F_4$ level also decay to the $^3H_6$ level with a lifetime, $T_3$, of 10 ms [45]. By incorporating the population dynamics of all three levels we can successfully simulate the accumulation dynamics for the temporally overlapped LFC's and compare the results to the experiment.

For the accumulation simulations the programming pulses had $B = 40$ MHz, $\tau_C = 1$ µs, $\tau_D = 250$ ns, a repetition rate of $\tau_r = 31$ µs, and the crystal had $\alpha L = 1.4$. These values are identical to the accumulation experiments done with the frequency stabilized Ti:Sapphire laser. A series of programming Rabi frequencies was also chosen to simulate the dynamics. In the simulator, the Guassian beam profile of the optical pulses is not taken into account. Thus the Rabi frequency, defined here is considered to be the Rabi frequency for all locations in the planar wavefront incident upon the crystal.
Figure 50. Echo power efficiencies vs. programming number (lower axis) or time (upper axis) for a frequency stabilized Ti:Sapphire laser system locked to a spectral hole. The different plots are various programming strengths (Rabi frequency, Ω) as shown in the legend. Here τ_c = 1 µs, B = 40 MHz, τ_D = 250 ns, and τ_r = 31 µs.
Figure 51. Echo intensities vs. programming number (lower axis) or time (upper axis) using the injection locked ECDL laser system. The different plots are various programming strengths (Rabi frequency, $\Omega$) as shown in the legend. Here $\tau_c = 1\,\mu s$, $\tau_d = 125\,ns$, and $\tau_r = 46\,\mu s$. 
Figure 52. Simulated accumulation sequences with $\tau_C = 1 \mu s$, $\tau_D = 250$ ns, and $\tau_r = 31 \mu s$. The echo power efficiencies are plotted versus programming number (lower axis) or time (upper axis). The different lines represent different programming strengths (Rabi frequency, $\Omega$) with (a) showing programming Rabi frequencies from $\Omega = 0.3$ MRad/s to 1.2 MRad/s in steps of 0.1 MRad/s, and (b) showing programming Rabi frequencies from $\Omega = 1.3$ MRad/s to 1.8 MRad/s in steps of 0.1 MRad/s, then to 2.8 MRad/s in steps of 0.2 MRad/s.
In figure 52, the echo power efficiencies from the accumulation simulations are shown for (a) programming Rabi frequencies from $\Omega = 0.3 \text{ MRad/s}$ to $1.2 \text{ MRad/s}$ in steps of $0.1 \text{ MRad/s}$, and (b) programming Rabi frequencies from $\Omega = 1.3 \text{ MRad/s}$ to $1.8 \text{ MRad/s}$ in steps of $0.1 \text{ MRad/s}$, then from $1.8 \text{ MRad/s}$ to $2.8 \text{ MRad/s}$ in steps of $0.2 \text{ MRad/s}$. The overall dynamics of the accumulation match that from the experiments. As discussed in the experimental section, the larger programming strengths, $\Omega > 1.3 \text{ MRad/s}$, are too strong and saturate the medium leading to inefficient gratings. The weaker programming strengths, $\Omega < 0.9 \text{ MRad/s}$, are not strong enough and can not accumulate an efficient grating before the population decay sets in. For the simulations, the best accumulation can be found with $\Omega$ between $0.9$ and $1.3 \text{ MRad/s}$.

Another way to view these three regions is to examine the $r_3$ component of the Bloch vector. This component details the population difference between the excited and ground state of the atom and will essentially show the TTD grating that has been stored. By examining the grating versus the number of shots, the dynamics of the accumulation can also be shown. For example, the accumulation dynamics for three different Rabi frequencies are shown. First, $\Omega = 0.5 \text{ MRad/s}$ is shown in figure 53. Here $r_3$ is shown versus detuning and programming number. The grating can be seen to start out extremely weak and slowly build up. Unfortunately, the grating does not build up strongly and at the end of 900 sequences, the maximum grating height is only $-0.5$. The $\Omega = 0.5 \text{ MRad/s}$ Rabi frequency leads to inefficient gratings as was shown above by the echoes produced from such gratings. The extra grating
structure is due to the way in which the spectral envelopes of the chirps are offset with respect to each other as discussed in the section on operating conditions.

A different story is encountered with $\Omega = 1.2$ MRad/s as shown in figure 54. Here the grating builds up fairly quickly and reaches a steady state. The maximum of $r_3$ at the end of the 900 programming sequences is close to -0.1 with very deep and open modulations. This Rabi frequency is one that leads to a good steady state value for the echoes and thus has the most efficient gratings.

Of course, increasing the Rabi frequency beyond 1.2 MRad/s in this case does not help. This is exemplified by the grating build up of the $\Omega = 2.8$ MRad/s LFC pulses as shown in figure 55. Here the maximum grating height rapidly reaches the saturation height of 0.0. Do not be fooled into thinking that because the grating height is the largest of the these three gratings that this will be an efficient grating. If one looks at the start of the grating dynamics, one will notice that the low parts of the grating are increasing as $N$ is is increased. This is due to the saturation effects. At those locations, the grating modulation begins to fill in a non-linear fashion. This leads to a poor grating that does not produce strong echoes.

A comparison of the gratings at the end of 900 programming sequences is shown in figure 56. The figure shows the overall grating produced from the (a) $\Omega = 0.5$ MRad/s programming, (b) $\Omega = 1.2$ MRad/s programming, and (c) $\Omega = 2.8$ MRad/s programming.
Figure 53. Grating accumulation sequences for $\Omega = 0.5$ MRad/s with $\tau_C = 1$ $\mu$s, $\tau_D = 250$ ns, and $\tau_p = 31$ $\mu$s. The $r_3$ components are plotted versus detuning and programming number.

Figure 54. Grating accumulation sequences for $\Omega = 1.2$ MRad/s with $\tau_C = 1$ $\mu$s, $\tau_D = 250$ ns, and $\tau_p = 31$ $\mu$s. The $r_3$ components are plotted versus detuning and programming number.
Figure 55. Grating accumulation sequences for $\Omega = 2.8$ MRad/s with $\tau_C = 1 \mu s$, $\tau_D = 250$ ns, and $\tau_r = 31 \mu s$. The $r_3$ components are plotted versus detuning and programming number.

One can see that the grating in (a) is not as strong as that in (b), thus probe pulses diffracting from these gratings would lead to stronger echoes in the (b) case. Note also that although the grating is strong in (c) the modulation is very nonlinear in nature and does not resemble a proper cosine modulated TTD grating. Whereas, the grating in (b) much more closely represents a cosine modulation. The deviation from this cosine structure leads to weaker 1st order echoes from the grating shown in (c). Analysis of the accumulation process has shown that higher order gratings occur with this programming method [40]. This deviation from a true cosine grating is the result of these higher order terms and occurs for any accumulated grating. Note that the (b) grating has less nonlinearities and thus more closely represents a true cosine
function giving stronger diffracted echoes than for (c).

These characteristics are typical of the three regions of accumulated programming dynamics. Here gratings such as (a) are created by programming pulses that are too weak, (c) is the result of programming pulses that are too strong and (b) is the result of just the right amount of programming strength. This optimal Rabi frequency is not unique in that if the chirp parameters changed or if $\alpha L$ changed so would the optimal Rabi frequency. For example, if the chirp duration was shortened the optimal Rabi frequency would increase. Similarly, if $B$ was increased, the optimal Rabi frequency would increase. Finally, if $\alpha L$ increased, so too would the optimal Rabi frequency, as more optical energy would be needed to reach the efficient steady state gratings.

Overall, these accumulation results for LFC programming pulses are a significant step for future high bandwidth accumulation studies. For example, if one needed to program efficient extremely high bandwidth gratings, but was power limited in the high bandwidth regime, this technique could be utilized to effectively transfer more optical energy into the medium, producing more efficient gratings. By utilizing this technique, in combination with temporally long, high bandwidth chirps the next generation of ultra-high bandwidth OCT phenomena will become accessible with low power affordable lasers. For more information on the dynamics of accumulation see appendix 1.
High Bandwidth Experiments

In recent years there has been much development of chirped external cavity diode lasers (CECDL's). These lasers typically have an electro-optic crystal within the cavity for rapid tuning of the optical path length of the cavity [82]. Thus the optical frequency of the cavity can be changed by adjusting the voltage of the electro-optic crystal. With the appropriate amplifiers, frequency chirp rates of GHz/μs have been
achieved with bandwidths of 3 GHz [82]. By putting an angular cut on the electro-optic crystal and matching the refraction and grating angles, bandwidths in excess of 40 GHz have been demonstrated [8]. By utilizing these types of lasers with the temporally overlapped LFC pulse programming method, high bandwidth TTD gratings can be programmed.

One such CECDL, with a bandwidth of ~3 GHz, was used in an initial demonstration of high bandwidth TTD gratings. The output from this laser was rather low, typically less than a couple of milliwatts. Thus, the output LFC's from the CECDL were amplified using the injection locking system described in chapter 3. For some experimental details on injection locking frequency chirped sources see [82] and chapter 4. The output powers from the injection locking system for the high bandwidth experiments were ~ 70 mW. The amplified LFC pulses were then used to create the TTD gratings using the temporally overlapped LFC programming method.

Two main experiments were performed and their experimental setup is shown in figure 57. In both, a frequency offset, created by AOM1, was introduced onto the amplified LFC pulse, creating two spatially overlapping LFC pulses. These LFC pulses were then focused into the Tm$^{3+}$:YAG crystal giving a beam waist diameter of ~ 35 μm. This created large bandwidth TTD gratings within the crystal. The crux of the experiment became how to observe the high bandwidth grating that had been programmed into the medium. There are many ways this can be done. One

---

3 This laser was built by Robert Peters with a design from Kevin Repasky.
can imagine simply diffracting a high bandwidth probe pulse, such as a picosecond pulse off of the grating. Unfortunately, the picosecond pulses at our disposal had to be fiber coupled to reach the table with the chirped laser.

Figure 57. Diagram of the high bandwidth CECDL experiment utilizing temporally overlapped linear frequency chirped pulses.

Thus they were power limited and the echoes from such pulses would be below detectable limits. Another way to probe a spectral grating is to use a low power LFC pulse to examine the absorption spectrum of the gratings. In order to probe an absorption spectrum in this way, the probe pulse must not significantly alter the medium, and thus the optical powers of the probe are kept very low. The probe must also be temporally long so as to give high frequency resolution. After the LFC probe has traveled through the medium, the absorption features of the transmitted probe
pulse are indicative of the spectral grating it was incident upon. A simulation of this
effect is shown in figure 58. Here a low intensity LFC optical probe pulse is incident
upon a TTD grating that was previously programmed with a periodicity of 1 GHz and
a bandwidth of 10 GHz. The LFC probe pulse had a $\tau_c = 15$ ns and a bandwidth of
15 GHz, giving a chirp rate, $\gamma = 1$ GHz/ns. The dashed line is the original LFC pulse
before entering the medium and the solid line is the LFC pulse after transmission
through the medium with an $\alpha L = 1.4$. One can see the absorption spectrum of the
grating imparted upon the transmitted probe pulse. In this case the spectral grating
had portions that were inverted and thus produced amplification of the probe pulse.
Overall, the absorption features of the grating are shown giving verification that the
spectral grating exists within the medium. This approach has the advantage that
it is directly measuring the grating. There is no need for expensive and noisy high
bandwidth detection systems such as those needed in an echo experiment. However,
the slow probe pulse does induce a latency time in order to scan over the spectral
grating. If one wants to measure the TTD from the grating period, post processing,
specifically a fast Fourier transform, is needed. In order to ensure that echoes could
be created from this high bandwidth TTD grating, several low bandwidth probes can
be used to diffract off of the grating. By changing the optical frequency of the probe
pulse over that of the programmed grating, one can observe the echo characteristics,
giving information on how well the TTD grating was programmed over its bandwidth.
This approach has the advantage that it does not have any latency time, and no post
processing is needed to observe the time delay. However, since the probes are low
bandwidth, the whole grating is not probed and the optical frequency needs to be
changed in order to examine the whole grating.

![Figure 58](image_url)

Figure 58. A low intensity LFC optical probe pulse before (dashed line) and after
(solid line) transmission through the medium with an \( \alpha L = 1.4 \). Here \( \tau_c = 15 \) ns and
\( B = 15 \) GHz, giving a chirp rate, \( \gamma = 1 \) GHz/ns.

For the direct absorption measurements a \( \tau_C = 100 \) \( \mu \)s LFC pulse with a band-
width, \( B = 2.4 \) GHz was used to probe a TTD grating. The programming LFC pulse
had bandwidth \( B = 2.0 \) GHz, and \( \tau_C = 30 \) \( \mu \)s giving \( \gamma = 68 \) MHz/\( \mu \)s. It was passed
through an AOM that created two LFC pulses with an offset frequency of \( \delta = 15 \)
MHz. Figure 59(a) shows the transmitted \( B = 2.4 \) GHz probe pulse as a function of
the instantaneous frequency of the probe. The 2.0 GHz TTD grating is evident as a
series of oscillations, as expected, in the intensity of the probe pulse. Figure 59(b)
shows an enlarged portion of the absorption features seen on the probe pulse. Here
the periodicity in frequency corresponds to \( \sim 4.5 \) MHz.
Figure 59. (a) This figure shows the transmission of a low intensity 2.4 GHz LFC probing a 2.0 GHz TTD grating programmed with the TOLFC method. The large oscillation on the transmission is a frequency oscillation corresponding to a grating period $1/\tau_D$ where $\tau_D = 0.22 \, \mu s$. (b) Shows these oscillations in greater detail.
Figure 60. This figure shows echo intensity of a 50 ns probe pulse versus the optical frequency of the probe pulse (data points). The solid line shows the expected position of the programmed TTD grating.

When this plot is Fourier transformed, a frequency component is found at $0.22 \mu s$, which matches the expected TTD of $\tau_D = \delta / \gamma = 0.22 \mu s$. This direct observation of the TTD grating ensures that it possesses the appropriate characteristics.

The grating was also probed using a low bandwidth ($\sim 20$ MHz) Fourier transform limited pulse. The optical frequency of this pulse was then varied over the bandwidth of the programming chirps. Figure 60 shows the peak echo intensity as a function of the carrier detuning. Echo pulses can be seen when the probe overlaps the grating, and no echo when the probe does not. This approach allowed a more direct indication as to how well the TTD grating was programmed and whether there were any nonlinearities associated with it. This will be discussed below.
By changing the bandwidth of the programmed grating and probing with the Fourier transform limited pulse, one can verify equation (3.6), repeated here

\[ \eta \propto E_C^4 \left( \frac{\tau_C}{B} \right)^2. \]  

This equation shows that the power efficiency of the photon echo should drop as $1/B^2$ as the bandwidth is increased. To do this one simply needs to track the echo intensity as a function of the programmed bandwidth. Figure 61 shows the peak intensity of the echo pulse from a 50 ns probe as a function of the LFC's bandwidth. Here the programming pulses had $\tau_C = 4 \mu s$ and a frequency offset of $\delta = 20$ MHz. The echo delay times are all shorter than 0.6 $\mu s$ and thus $T_2$ effects are negligible. As can be seen, the echo intensity decreases, but this is expected. This is because the bandwidth has been increased but the overall programming time has been kept the same. This means that the same amount of energy is being spread over more atoms, and since the probe is transform limited, its peak height must drop. The solid line in the figure is a fit using a functional dependence of $1/B^2$. As can be seen, the echo intensity roughly decreases as predicted with a $1/B^2$ dependence.

Figure 62 plots the echo delay of the low bandwidth probes versus the frequency of the optical carrier (circles). In an ideal case, this plot would be a constant value. But as can be seen from the plot there is a slope to the plot with a decrease in time delay for the higher optical frequencies. A linear fit is made to the data showing the overall decrease as a function of bandwidth and the equation is shown in the plot. This indicates that the chirped programming pulse from the CECDL deviated slightly
from true linearity. In fact, the chirp rate increases towards the end of these high bandwidth chirps, translating into a shorter delay for the higher optical frequency echoes. These non-linearities are definitely a problem and would lead to distortions of high bandwidth probe pulses. Another problem with CECDL's are the predicted difficulties in frequency stabilizing these lasers. Because the driving voltages of these crystals is high, small fluctuations in voltage are expected. These fluctuations lead to chirp non-linearities that can not be repeated from shot to shot in an accumulation programming scheme. Because of the strict tolerances for accumulation of gratings, this method is expected to not perform well for accumulation of gratings. These problems led us to explore other methods for programming high bandwidth TTD gratings and the results are presented in the next chapter.
Figure 61. This figure examines the experimental echo peak heights as a function of grating bandwidth created with the CECDL laser system. Here the programming power and $\tau_C$ are kept constant, $\tau_C = 4 \, \mu$s and the offset frequency is kept at $\delta = 20$ MHz. The solid line is a fit using a functional dependence of $1/B^2$ according to eqn. (3.6).

Figure 62. This figure examines the experimental echo delay times versus the optical frequency of the 50 ns probe pulse. The echo delay time can be seen to decrease indicating non-linearities in the LFC pulse.
CHAPTER 6

LINEAR SIDEBAND CHIRPS AND THEIR APPLICATION

In the last chapter, linear frequency chirps were created directly either by using acousto-optic modulators or linear frequency chirped lasers. There is another method that can create a type of linear frequency chirp. The method, called here linear sideband chirping or linear sideband chirps (LSC's), is a technique that utilizes a broadband electro-optic phase modulator such as that described in chapter 3. Essentially, the sidebands, created by driving the modulator with an RF voltage, are chirped by allowing a linear change in the instantaneous frequency of the driving RF voltage. This technique can chirp over extremely large bandwidths (>40 GHz) and has several advantages over other high bandwidth chirping methods. Among the advantages, the LSC's can be produced with stabilized lasers and in general can have better chirp linearities than other chirp sources. LSC's, in combination with the temporally overlapped programming method and injection locking, can be used to program efficient spatial-spectral gratings. This novel programming technique as well as demonstrations of high bandwidth spatial-spectral gratings programmed with LSC's is the topic of this chapter.
Linear Sideband Chirps

As described in chapter 3, a broadband electro-optic modulator can be driven by an RF voltage to produce sidebands. These sidebands are spaced around the carrier by the modulation frequency, \( \omega_m \). If one wants to shift the position of these sidebands, they can do so by simply increasing or decreasing \( \omega_m \). It is the process of changing these sidebands linearly over time that is at the heart of the linear sideband chirping technique.

Here the field after traveling through the modulator is written as

\[
E = E_0 \cos \left[ \omega t + \beta \cos(\phi) \right].
\]  
(6.1)

Notice that if \( \phi = \omega_m t \), one would get the typical result of evenly spaced sidebands around the optical carrier, discussed previously. But if the instantaneous phase of the cosine inside eqn. (6.1) is allowed to vary as

\[
\phi = \pi \gamma t^2 + \omega_s t,
\]  
(6.2)

then the sidebands can be allowed to change linearly with time, with a chirp rate of \( \gamma \). Here \( \omega_s = 2\pi f_s \), where \( f_s \) is the start frequency for the beginning of the chirp. Using this approach, only the sidebands chirp, allowing the carrier frequency to remain stationary. Because this technique uses an external modulator to do the sideband chirping, a stabilized laser can be used as its source. As was discussed in previous chapters, accumulation of spectral gratings requires that the laser source be
extremely stable. Thus, this technique offers an immediate advantage over CECDL's, which chirp the optical frequency directly. As was discussed in the previous chapter, in order to stabilize CECDL's new, complicated stabilization circuits will need to be developed in order to allow for accumulation of LFC's. However, accumulation of linear sideband chirps should be possible by simply utilizing current stabilized laser sources.

In order to examine these linear sideband chirps in greater detail, the Maxwell-Bloch simulator described in chapter 2 is utilized. Here the phase function shown in eqn. (6.2) was used to produce chirped sidebands on an optical carrier. The effect of the linear sideband chirp after passing through the medium ($\alpha L = 1.4$) can be seen in the $r_3$ component of the Bloch vector, $\vec{r}$, which is shown in figure 63. Here the chirp bandwidth, $B = 20 \text{ GHz}$, $f_s = 30 \text{ GHz}$, $\tau_c = 8 \text{ ns}$, and $\beta = 1.2$. In this figure, it is apparent that LSC's have some fundamental differences to regular LFC's. First, one notices that there are two chirps, one due to the up-shifted first order sideband, and one due to the down-shifted first order sideband. Because the chirp rate, $\gamma = B/\tau_c$, is positive, both chirps start close to the carrier and chirp away from the carrier. Because there are two separate chirps, the optical power is spread out between the two, reducing the overall power in a single chirp as compared to a single LFC. There are also higher order chirps due to the higher order sidebands. The beginnings of the second order chirps are just visible in figure 63 starting at approximately 60 GHz.
Figure 63. An example of $r_3$ of a linear sideband chirped pulse after passing through the medium. Here the bandwidth of the LSC is 20 GHz and the start frequency is 25 GHz. Note the start of the second order chirps at approximately 60 GHz.

These second order LSC's can interfere spectrally with the first order LSC's within the crystal. Thus it is important either to avoid chirping more than an octave, or to minimize the effect of the second order sidebands in relation to the first. In order to minimize the first chirp with respect to the second, one can examine the functional dependence of the sideband power with respect to $\beta$. The sideband powers are related through the ordinary Bessel functions as $J_i(\beta)^2$. Here $i$ represents the order of the sideband. Typically one wants to maximize the ratio in power between the first and second order sidebands. The functional dependence of $J_1(\beta)^2/J_2(\beta)^2$ is examined in figure 64 with respect to the right hand axis. This plot also shows the square of each individual Bessel function on the left hand axis. $J_1(\beta)^2/J_2(\beta)^2$ is a maximum at 0. So, to minimize the effects of the second order chirps if chirping more than an octave, one would want $\beta \ll \pi$. However, if $\beta$ is small, there will be a very small amount of
power in the sidebands. Thus, \( \beta \) must be set such that the ratio is at an acceptable level for a given application. Since, small \( \beta \) is an inefficient use of the optical power, the non-octave method is a better approach for distortion free TTD. However, as will be shown, digital creation of this phase modulation can help to alleviate this problem.

Figure 64. A plot of the functional dependence of \( J_1(\beta)^2/J_2(\beta)^2 \) (right hand axis) as well as the square of \( J_1(\beta) \) and \( J_2(\beta) \). The maximum of \( J_1(\beta)^2/J_2(\beta)^2 \) occurs at 0 and an acceptable level must be chosen.

As discussed in chapter 3, electro-optic modulators are reaching bandwidths in excess of 40 GHz and could potentially be used to produce LSC’s over these bandwidths. However, one must remember that these LSC’s must be driven by a chirping
RF voltage. The choice of how to drive these modulators in order to chirp the side-bands is of fundamental importance for chirp linearity. Currently, high bandwidth chirping RF voltages can be created with YIG oscillators or pulse-pattern generators.

A YIG oscillator is a specially designed voltage oscillator, whose oscillation frequency depends upon an externally applied voltage. Thus, if one linearly ramps the voltage, the oscillation frequency of the device will chirp. YIG oscillators have been produced to operate at high frequencies (0.5 to >20 GHz) with their operating bandwidths spanning multi-octaves [89]. Unfortunately, these devices are not highly accurate, with resolutions of one part in a thousand. This would lead to frequency nonlinearities of 1 MHz over a 1 GHz bandwidth chirp. Along with this, these devices typically have 4-6 MHz of hysteresis. Although good for single-shot type chirping, these devices, due to their resolution and hysteresis, could not be used for accumulation experiments. However, for extremely high bandwidth ~ 40 GHz chirps, YIG oscillators may be the only choice [90].

Pulse-pattern generators are also capable of producing chirping signals. This is done by using the pulse-pattern generator as if it were an arbitrary waveform generator. If one wishes to create the digital equivalent of a sine wave, one would use a digital “on” when the sine wave is above zero and a digital “off” when the sine wave is below zero. The same approach can be followed for a chirping waveform. A chirping sine wave is shown in 65 (a) and the digital approximation to this waveform is shown in 65 (b). By using this approach, a chirping signal can be approximated.
using a digital pulse-pattern generator. It is important to realize that some amount of oversampling may be needed in order to properly replicate the chirp. This means that the clock rate of the pulse pattern generator must exceed the maximum instantaneous frequency of the chirp. The amount of oversampling varies depending upon how many harmonics are needed to make up the signal. Four times oversampling is expected to perform well for simple chirping functions [91]. In the following experiments, the maximum instantaneous frequencies were usually kept below 3 GHz. The pulse-pattern generator had a sampling rate of 12 GBit/s, thus maintaining a factor of four oversampling.

![Figure 65](image.png)

Figure 65. (a) A chirping sine wave used as a drive voltage. (b) A digital approximation to the chirping sine wave.
It can be shown that the effect of having chirping square waves instead of smooth chirping sine waves modulated on an optical carrier is to transfer more energy of the carrier into the first order chirp, giving stronger chirping sidebands. If the peak to peak of the square wave chirps is exactly $\pi$, the carrier is completely extinguished as well as even order sideband chirps, leaving only the odd order sideband chirps. This is shown in figure 66.

![Figure 66](image)

Figure 66. (top) The power spectrum of a square wave phase chirp on a carrier. (bottom) The power spectrum of a regular phase chirp on a carrier. In each case the chirps have $\tau_C = 1 \, \mu$s, $B = 0.2 \, \text{GHz}$, $\beta = \pi/2$, and a 10 GHz carrier.

The top plot shows the power spectrum of the square wave phase chirps and the bottom plot shows the power spectrum of regular phase chirps, both on carriers. In each case the chirps have $\tau_C = 1 \, \mu$s, $B = 0.2 \, \text{GHz}$, $\beta = \pi/2$, and a 10 GHz carrier.
In an experimental arrangement, due to bandwidth limitations of the modulator the sharp edges of the square waves will be smoothed out allowing some carrier and second order sidebands to remain in the spectrum. Even when this is the case, this type of square wave phase modulation still has the added benefit of removing more of the power in the second order sidebands, which reduces the spectral interference between the first and second order chirps for chirps spanning more than an octave.

**Spatial-Spectral Grating Programming**

The question now becomes, can these LSC’s be used to program a spatial-spectral grating? These LSC’s have very similar characteristics to standard LFC’s so one might expect that programming techniques that worked for LFC’s might also work for LSC’s. By utilizing the already well described chirped pulse programming methods used in earlier chapters spectral gratings can be produced. But there are a few exciting new developments due to the fact that the first order sidebands chirp in opposite directions. These will be examined in the phase matching section below.

By utilizing the temporally overlapped linear frequency chirped programming method described in chapter 5, a spectral grating can be produced with LSC’s. It is assumed that a phase modulator driven with a chirping waveform (less than an octave) is used to create LSC’s on an optical carrier. The optical pulse is split into two beams with a beamsplitter, and passed through AOM’s driven at different frequencies. As was described in the previous chapter, this process results in a frequency offset
between the two optical pulses. After focusing these pulses into an OCT crystal, gratings should be programmed. Remember that there are two first order chirps on the carrier, an up-shifted chirp and a down-shifted chirp. So, LSC’s act just like normal LFC’s except there are now two of them chirping in opposite directions. Unlike LFC’s, this temporally overlapped programming method using LSC’s gives rise to two spectral gratings: one for the up-shifted first order chirp and one for the down-shifted first order chirp. Another simulation was performed to show how a spatial-spectral grating is programmed using temporally overlapped LSC pulses. In figure 67, the \( r_3 \) component of the Bloch vector is shown for the two frequency offset LSC pulses after traveling through the medium \((\alpha L = 1.4)\). As expected two frequency gratings are created, one for the upper frequency sideband chirps and one for the down-shifted first order sideband chirps. Here \( B = 20 \text{ MHz}, \tau_C = 5 \text{ } \mu\text{s}, \quad f_s = 30 \text{ MHz}, \quad \delta = 2 \text{ MHz}, \quad \text{and} \quad \beta = 1.2 \). These low bandwidth parameters were chosen to match a low bandwidth initial demonstration. Higher bandwidth demonstrations and simulations are shown later. The time delay is still given by eqn. (5.2), which is

\[
\tau_D = \frac{\delta}{\gamma}.
\]

This creates an expected time delay of 0.5 \( \mu\text{s} \) or a grating period of 2 MHz. This grating period is evident from the figure. Notice also that the frequency offset of 2 MHz shows up between the two holes burned from the carriers of the LSC pulses.
Figure 67. The effect of two temporally overlapped frequency offset linear sideband chirped pulses incident upon the medium. Here the $\tau_3$ component of the Bloch vector is plotted as a function of frequency.

So from simulations, it is apparent that two spectral gratings will be produced from two frequency offset LSC’s in much the same way as those produced from two frequency offset LFC’s. The question now becomes, how can these gratings be probed? Somehow, a new carrier must illuminate this grating at the center frequency of either sideband grating. This can be accomplished in a variety of configurations using either the same laser as the LSC’s or a separate laser. The simplest configuration conceptually is to use two lasers. One laser is used to create the LSC pulses and the other is detuned from the first and centered on the spectral grating. A diagram of this method is shown in figure 68. Here the vertical axis is both amplitude (solid lines) and instantaneous frequency (dashed lines). The first pulses that are incident upon the medium are the frequency offset LSC’s. One can see both of the sideband...
chirps and the two carrier frequencies of the programming LSC pulses separated by \( \delta \). The probe pulse is created from a different laser source at frequency \( f_{L2} \). This frequency is centered within the upper frequency grating and will thus produce an echo pulse with a time delay given by eqn. (6.3). This probe pulse can be any kind of amplitude or phase modulated data waveform, assuming its bandwidth does not exceed the bandwidth of the spectral grating for perfect high bandwidth TTD.

![Diagram](image)

Figure 68. Programming and probe pulses created by separate laser sources. Here the programming pulses are LSC pulses and the probe pulse frequency \( f_{L2} \) is centered on the up-shifted first order grating.

Another possibility for probing gratings created from LSC pulses utilizes a single laser and a collinear geometry. The method is to use a CW phase modulated probe
pulse whose carrier frequency remains the same as that of the original LSC probe pulse. Since a single laser is used to create the programming and probe pulses, and since the geometry is collinear, the overall setup and detection of the echoes is simplified.

This method relies upon the CW phase modulation of the probe pulse to put a first order sideband onto the center of the up-shifted and down-shifted first order gratings. These sidebands will then be diffracted from the gratings creating two echo pulses at each sideband frequency. If a collinear geometry is utilized, the two echo outputs are also spatially overlapped. This results in a frequency beat between the two pulses due to the fact that they are spatially overlapped on a detector and each have different optical frequencies. In fact, if the modulation frequency of the CW phase modulation is $\omega_m$, the beat frequency between the two sidebands should be $2\omega_m$ since that is the frequency separation between the two first order sidebands.

An input pulse diagram is shown in figure 69 for this single laser, collinear probing method. Here the programming pulses are created as before, however, now the probe pulse is phase modulated as described above. The vertical grey dashed lines represent the time that the CW phase modulation is turned on. This is done by turning on the RF drive voltage to the modulator only during the grey dashed lines. During this time, then, the CW phase modulation creates sidebands that are centered in the spectral gratings producing echoes. Notice that the echoes’ temporal length is limited to when the CW phase modulation is on, thus the echoes are temporally shorter in
time then the whole optical pulse diagrammed in this figure. Notice also that there are two frequencies present during the echo pulse, which should create a $2\omega_m$ beat at the detector.

![Diagram of input pulse and phase matching](image)

Figure 69. An input pulse diagram for a single laser experiment. This experiment utilizes the same laser to create the programming and probe pulses. Here the probe pulses have a CW phase modulation that pushes first order sidebands to the center frequency of the grating allowing echoes to be produced.

**Phase Matching**

At this point, the phase matching conditions must be reexamined because of the fact that there are now two spectral gratings. One must know for an angled beam experiment how the phase matching is determined from either of these two sidebands.
Let us assume that the two temporally overlapped LSC pulses have been spatially separated from each other. One of these pulses, the one with the lower carrier frequency, has a wave vector of $k_a$, and the other, the one with the higher carrier frequency, has a wave vector of $k_0$. Remember from chapter 5, the phase matching conditions for a chirp rely upon the start frequency, $f_{\text{start}}$, of the chirps and whether the chirp rate, $\gamma$, is positive or negative. Here it is assumed that $\gamma > 0$. Essentially to determine what the phase matching conditions are for either the down-shifted or up-shifted first order grating, the ions with frequency $f_0 \pm (f_{\text{start}} + \delta/2)$ must be examined, where $f_0$ is the center frequency between the two frequency offset optical carriers. The chirp that excites these ions first is then treated as $k_1$. In the case of the down-shifted first order sideband grating, the first chirp that excites the ions with frequency $f_0 - (f_{\text{start}} + \delta/2)$ is the chirp with wave vector $k_0$. Whereas for the up-shifted first order sideband grating, it is the chirp with wave vector $k_a$ that excites the ions with frequency $f_0 + (f_{\text{start}} + \delta/2)$ first.

In a typical stimulated photon echo experiment for TTD, where a probe pulse of wave vector $k_p$ is incident upon a spectral-spatial grating, the phase matching conditions are

$$k_e = k_p + k_2 - k_1. \quad (6.4)$$

Here $k_1$ represents the chirped pulse that excites the ions with frequency $f \pm (f_{\text{start}} + \delta/2)$ first and $k_2$ the chirped pulse that excites these ions second. Notice that depending upon which sideband grating you probe, the phase matching condition changes. In
other words, if the probe pulse is incident upon the upper frequency grating, \( \vec{k}_1 = \vec{k}_a \) and \( \vec{k}_2 = \vec{k}_b \). And if the probe pulse is incident upon the down-shifted first order grating, \( \vec{k}_1 = \vec{k}_b \) and \( \vec{k}_2 = \vec{k}_a \). This leads to the phase matching conditions

\[
\begin{align*}
\vec{k}_{\text{upper}} &= \vec{k}_p + \vec{k}_b - \vec{k}_a \\
\vec{k}_{\text{lower}} &= \vec{k}_p + \vec{k}_a - \vec{k}_b.
\end{align*}
\] (6.5)

Here \( \vec{k}_{\text{upper}} \) represents the wave vector of the echo created from the up-shifted first order grating and \( \vec{k}_{\text{lower}} \) the echo created from the down-shifted first order grating. Thus, depending upon which grating is probed, the phase matching conditions will change in a predictable manner.

This is a fairly unique situation that holds promise for use in spatially separating frequency components. One can imagine a situation, say for networking, where optical pulses with a certain frequency are overlapped spatially and need to be routed in separate spatial directions. Two temporally overlapping, frequency offset LSC’s can create two spatial-spectral gratings that will spatially isolate these two signals. A special direction can be picked as well, with

\[
\vec{k}_p = \frac{\vec{k}_a + \vec{k}_b}{2}
\] (6.6)

In this situation, the frequency components of the probe pulse that are incident on the up-shifted grating will create an echo with wavevector \( \vec{k}_{\text{upper}} = \vec{k}_b \). Likewise, the frequency components of the probe pulse that are incident upon the down-shifted grating create an echo with wavevector \( \vec{k}_{\text{lower}} = \vec{k}_a \). This allows the echo direction to
be found quickly and easily by lining up detectors with the direction of the programming pulses, although the echoes are then not spatially distinct from the programming pulses. Due to the conditions for diffraction off of a spatial grating, this situation requires that the angle between $\vec{k}_a$ and $\vec{k}_b$ be much less than 1.

**Initial Demonstrations**

A Maxwell-Bloch simulation of the collinear pulse arrangement shown in figure 68, was done with $f_{L2}$ overlapping first the down-shifted grating, then $f_{L2}$ situated at the original LSC carrier, and then with $f_{L2}$ at the upper frequency grating. The expected output for such a sequence of probe pulses would be an echo with time delay $\tau_D$ when the probe pulse overlapped the down-shifted and up-shifted first order gratings and no output for when it overlaps the original LSC carrier. The echo outputs from the simulation are shown in figure 70 for an $\alpha L = 1.4$. Here the $\tau_C = 8 \mu s$, $B = 15$ MHz, $\omega_s = 25$, $\delta = 1.4$ MHz, and $\beta = 1.2$. This gives a programmed time delay of $\tau_D = 750$ ns. This delay is evident from the echo outputs on the first and third pulses whose frequencies overlapped the spectral gratings. The probe pulses had temporal widths of 200 ns. Here the detuning of the probe pulses is shown in the figure and was -32 MHz for the first pulse, 0 MHz for the second pulse, and +32 MHz for the third pulse measured with respect to the initial LSC carrier. This simulation shows that echoes can be stimulated from either of the LSC programmed gratings as expected.
Figure 70. A simulation showing the output of three separately detuned probe pulses. The detunings are -32 MHz, 0 MHz and +32 MHz for the first, second and third pulse respectively.

For an initial low bandwidth experimental demonstration of the linear sideband chirped pulse programming method acousto-optic modulators were used to create the temporally overlapped LSC pulses. An experimental setup, similar to that shown in figure 40 was used. In this case, AOM 1 was used to create the LSC pulse that was then amplified by the injection locking. The output from the amplifier was then passed through AOM 2, which created the frequency offset LSC pulses for the programming. The probe pulse was also created by AOM 2, which was detuned in frequency to overlap the down-shifted first order grating (first pulse), the center frequency between the two gratings (second pulse), or the up-shifted first order grating.
(third pulse). For this experiment, the AOM's bandwidth was 40 MHz, which limited the programming bandwidth to \( B = 15 \text{ MHz} \) and \( f_s = 5 \text{ MHz} \). Here \( \delta = 1.4 \text{ MHz} \) and \( \tau_C = 8 \mu s \) resulting in \( \tau_D = 750 \text{ ns} \). As in the simulation above, the probe pulses for this experiment had temporal widths of 200 ns. The power of the collinear pulses before they were focused into the crystal was 35 mW and the beam waist diameter was \( \sim 35 \mu \text{m} \). In this experiment, the output from the Tm\(^{3+}\):YAG crystal was captured with a 1 GHz silicon photodiode and recorded on a digitizing oscilloscope with a bandwidth of 300 MHz.

The output from the three incident pulses are shown in figure 71. Here the detuning is labeled in the figure and is \(-12 \text{ MHz}\) for the first pulse, \(0 \text{ MHz}\) for the second pulse, and \(+12 \text{ MHz}\) for the last pulse. As can be seen in the figure, echoes are observed with the appropriate delay times of \(\sim 750 \text{ ns}\). This was the predicted delay time from the above analysis, showing that LSC's can be utilized to program spectral gratings. Note that there is some spectral structure at zero detuning and this results in some minor transient outputs on the middle pulse. These transients are also evident in the simulation.

This initial demonstration showed that the LSC programming method could be utilized to create spectral gratings and that these gratings could be probed to successfully create echo signals with the programmed time delay. However, these demonstrations had limited bandwidths due to the utilization of AOM's. It was next desired to see if this method could be used with the broadband EOM's as discussed in the
introduction to this chapter to create high bandwidth spectral gratings.

Figure 71. An experimental demonstration of LSC programming, showing the output of three separately detuned probe pulses. The detunings are -12 MHz, 0 MHz and +12 MHz for the first, second and third pulse respectively.

High Bandwidth Single Laser Experiments

Due to the collinear nature and easy experimental setup, the programming and probing scheme shown in figure 69 was used for an initial broad bandwidth experimental demonstration. Figure 72 shows the experimental setup for the single laser method. As can be seen, the injection locking setup, as described in chapter 3 was utilized in conjunction with the fiber pigtailed integrated optics phase modulator to create amplified versions of the LSC pulse and the CW phase modulated probe pulse.
The EOM was driven by using the digital chirping method that was described above, utilizing the 12 GBit/s pulse-pattern generator. In all of the following experiments, $\beta$ was determined from CW phase modulation experiments, and set accordingly. This was done by monitoring the power of the phase modulation sidebands while the drive voltage from the CW phase modulation source was adjusted. For the non-octave chirps shown, $\beta$ was kept close to $\pi$ by driving with approximately a peak to peak voltage of 7 V. This drive voltage is essentially a constant for a given modulator and once set, does not need to be monitored.

After passing through the injection locked amplifier, the LSC pulse traveled through AOM 1, which created the temporally overlapped, frequency offset LSC programming pulses. The CW phase modulated probe pulse was also passed through this AOM driven at the center frequency of the AOM. Here the optical pulse was longer than the CW phase modulated portion, allowing a similar timing effect as that shown in figure 69. This meant that the echoes would be shorter in duration than the total optical probe length. The power of the collinear pulses before they were focused into the crystal was 35 mW and the beam waist diameter was $\sim 35 \mu m$. 
Figure 72. The experimental setup for the single laser, collinear programming and probing method.
In these experiments, the output from the crystal could be analyzed with either a 1 GHz silicon photodiode and recorded on a digitizing oscilloscope with a bandwidth of 300 MHz, or with a high bandwidth 12 GHz optical detector and recorded with a digitizing oscilloscope with a bandwidth of 3 GHz. Here the slower detector was used to observe the envelope of the spatially overlapped echo pulses and the faster detector was used to examine the frequency beat on the echoes. The repetition rate of the experiment was slow enough to allow the gratings to decay away, making this a single shot experimental situation.

Echoes were observed using this single laser technique. In figure 73 several echoes are shown for various frequency offsets. This shows that the time delay programmed can be tuned according to eqn. (6.3) in the same way that it was in chapter 5. The detected signal was further bandwidth limited to 20 MHz to enhance the signal to noise ratio in this figure. It is also apparent that the observed echoes are shorter in duration than the incident probe pulse, due to the limited duration of the RF signal. Dashed lines have been added to show the approximate time that the CW phase modulation is applied to the probe pulse. Notice that the time duration of the echoes roughly matches that of the CW phase modulation as expected. In this figure, $B = 300$ MHz, $\delta$'s range from 0.8 MHz to 3.6 MHz in steps of 0.2 MHz, $f_s = 1.275$ GHz and $\tau_C = 50$ $\mu$s. The CW phase modulation, $f_m = 1.425$ GHz, was turned on for 50 ns in the middle of the 150 ns probe pulse leading to echoes that were 50 ns in temporal duration.
Figure 73. Example echoes from the single laser LSC programming technique. Here the echoes are shown for various frequency offsets with a 20 MHz bandwidth detection limit. See text for discussion.

Each echo shown in figure 73 is really the result of two spatially overlapped echoes from each spectral grating. As was discussed above, a beat frequency should be apparent on this echo if indeed two echoes were created from both gratings. This frequency beat was detected at the appropriate beat frequency of $2\omega_m = 2.85 \text{ GHz}$ and is shown in figure 74. Here the left side of the figure shows the overall duration of the echo and the rapid oscillations caused by the beat frequency for four different $\delta's = 1.6, 1.52, 1.51, \text{ and } 1.50 \text{ MHz}$ from top to bottom. These $\delta's$ give expected time
delays of $\tau_D = 266.67, 253.33, 251.67, \text{ and } 250.00 \text{ ns}$. The right side of the figure shows a zoomed portion of the left figure. Here several periods of the beat are shown, and each subsequent waveform is shown roughly in phase, $3\pi/2$ out of phase, and $\pi$ out of phase, with the top waveform.

Figure 74. Echo outputs observed on the fast detector showing the beat frequency of the spatially overlapped echoes at $2f_m = 2.85 \text{ GHz}$ for various $\delta$'s.

This technique might be thought to be useful for a beamsteering application in a phased array radar. However, there is a significant drawback. Here the RF carrier for the beam would be the microwave beat frequency (in this case 2.85 GHz) and data or just a CW signal might possibly be steered by changing the programmed delay to
each element of the array. However, it is important to consider the phase of the beat frequency, which changes as

$$\phi = 2\omega_m \tau_d.$$  \hspace{1cm} (6.7)

Note that this is an equation for the phase of the $2\omega_m$ beat signal from the echoes. Each echo was delayed only by $\tau_D$, not $2\tau_D$. Since the phase of the beat and the delay are not synonymous, this method would not steer the beam correctly for broadband phased array radars. In figure 74 the change in time delay, $\Delta \tau_D$, of the top waveform compared to the other waveforms should give a relative phase shift, $\Delta \phi$, for each of the waveforms. The calculated change in delays, $\Delta \tau_D$, are 13.33, 15.00, 16.67 ns for the three shifted waveforms from top to bottom. These delay shifts give expected phase shifts of $\Delta \phi = 38, 42.75$ and 47.5 cycles. These expected phase shifts were observed, as discussed above, experimentally verifying eqn. (6.7).

This technique is not limited to just CW phase modulated probe pulses. In fact, the CW phase modulation was essentially used to create an optical carrier at the center of the spectral gratings. One could modify this sideband carrier by applying some type of modulation that is less than the bandwidth of the grating. One way to modulate the sideband carrier is by having $\beta$ in eqn. (6.1) be a function of time. For example, $\beta(t)$ can be treated as a digital signal, with $\beta = 0$ for an off and $\beta = \beta_o$ for an on, at a bit rate, $\Gamma_D$, which is much less than $f_m$. When this is done, it has the effect of turning on or off the first order sidebands, depending on the appropriate data bit. This technique would then create echoes when $\beta(t) = \beta_o$ and no echo when
\( \beta(t) = 0 \). If one monitored the echo output one would notice that this complicated phase modulation technique is turned into a simple data amplitude modulation for the output echo. An example of this technique is shown in figure 75. Here (a) shows the envelope \( \beta(t) \), (b) shows the digital modulation at \( f_m \), and (c) shows what the expected echoes would look like with the \( 2f_m \) beat frequency and a delay of 0.5 time units.

![Figure 75](image)

Figure 75. An overview of how to use the single laser method to create data pulses discussed in the text.

This modulation technique was experimentally demonstrated using the same setup that has been described above. Here the data rate was \( \Gamma_D = 500 \text{ MBits/s} \) allowing the modulation to be observed with the 1 GHz silicon photodiode on the 3 GHz bandwidth scope. This combination allowed the observation of the envelope
of the echo showing the data bits, but not the frequency beat between the spatially overlapped echoes because it was at a much higher data rate. Here $f_m = 3$ GHz and thus the beat frequency is at 6 GHz. In this case the 1 GHz silicon photodiode was used with the 3 GHz scope to observe the echoes amplitude modulated envelope and not the beat frequency. The echo output of such a modulated sequence is shown in figure 76 as the lower trace. The upper trace is the expected bit sequence for the echo data. Here the programming sequence had $B = 700$ MHz, $f_s = 2.65$ GHz, $\tau_C = 50 \mu s$, and $\delta = 1.4$ MHz, giving an expected echo delay of $\tau_D = 100$ ns. The amplitude modulated echo was observed with this time delay and there are no noticeable bit errors. This experiment shows that this modulation technique can successfully TTD high bandwidth modulated waveforms.

In the end, this technique, while useful for a collinear arrangement, could not provide modulation frequencies to 1 GBit/s because of the requirement that $\Gamma_D$ remain much lower than $f_m$. Ultimately, $f_m$ was limited because the injection locking is limited to amplification of signals below about 3.5 GHz. This puts an upper limit on $f_m$ and for that matter the bandwidth $B$ of LSC pulses using this experimental setup.
Figure 76. A 500 MBit/s amplitude modulated echo sequence (lower) created by modulating $\beta$ in eqn. (6.1) as described in the text. The expected bit sequence is shown as the upper trace.

High Bandwidth Double Laser Experiments

Although data rates of 0.5 GBit/s were shown in the last section, echoes at 1 GBit/s could not be achieved due to the modulation technique used. These echoes were also not spatially isolated from the programming pulses. In order to achieve spatially isolated echoes with modulation frequencies of 1 GBit/s another experimental arrangement had to be used. This was done with the two laser. This method has already been discussed previously and is shown in figure 68. Here the probe pulses are created by another laser, whose carrier frequency is centered on either of the spectral gratings created by the programming LSC pulses. In this situation then, the creation of high bandwidth signals to be delayed by the grating requires that another phase modulator and injection locking system be utilized. This doubles the complexity of the experiment. Along with this, the angled beam geometry used spatially isolates
the echo signal. While nice for background free detection, this spatial isolation also
adds to the complexity of the detection scheme if direct phase modulation is used to
modulate the probe pulse. This is simply because if a phase modulated optical signal
illuminates a detector, that phase modulation is undetectable due to the square law
nature of the detector. This leaves two choices in order to observe the phase modula-
tion. Either use some kind of spectral filtering process to convert phase modulation
into amplitude modulation as discussed in chapter 3, or use a heterodyne detection
 technique that also converts the phase modulation into a detectable amplitude mod-
ulation. Heterodyne detection has been discussed in chapter 4, for the detection of
injection locked BPSK signals. This was done using a delayed-self-heterodyne sys-
tem, where the signal was heterodyned with itself. Heterodyning can be done in other
ways, such as just using a constant heterodyne beam overlapped with the BPSK sig-
nal. Heterodyne techniques are popular in optical communications systems and have
been described elsewhere [54]. In the following experiments both spectral filtering
and direct heterodyne detection are used.

Two main TTD experiments were performed that utilized two separate lasers for
programming and probing. The first experiment was to TTD a CW phase modulated
probe pulse. This was done to examine the functional dependence of the phase as a
function of time delay as in eqn. (1.3).
Figure 77. The experimental setup for the high bandwidth two laser experiments.
The second experiment was to TTD high bandwidth BPSK probe pulses to ensure that the entire LSC programmed spectral grating was behaving appropriately. Figure 77 shows the experimental setup for both of these experiments. In this experiment, each laser would need to be modulated with an EOM. This meant that two injection locking systems had to be used for amplification of the small signal outputs from these EOM's. Much of the bulk of the experimental setup was actually to create these two amplification systems and be able to monitor them both simultaneously with the same SFPC and OSA. This was done by overlapping small portions of the output from each amplifier via a non-polarizing beam splitter. The Ti:Sapphire ring laser (Master 1), which could be stabilized to a spectral hole, was used as the master laser for the LSC programming. The light from the stabilized Ti:Sapphire laser was launched into a fiber that was then connected to EOM 1 for creation of the LSC pulse. The output from EOM 1 was then directed to a slave laser (Slave 1) for injection locking and amplification. The amplified output from the slave laser was passed through a 50/50 non-polarizing beam splitter to produce two beams. These beams then passed into either AOM 1 or AOM 2 for frequency offsetting. The difference in frequency between AOM 1 and AOM 2 is then $\delta$. The beam after AOM 1 is labeled $k_1^*$ and the beam after AOM 2 is labeled $k_2^*$. These two beams constitute the programming pulses for the following experiments. The probe pulse was created by using the ECDL as a master laser (Master 2). In order to create either the CW phase modulation or the BPSK data on the probe pulse, the light from the ECDL was launched into a fiber
and then into EOM 2. This EOM then created the desired modulation and the light was then directed to a slave laser (Slave 2) for injection locking and amplification. The amplified output was then passed through AOM 3, which created the overall temporal duration of the modulated pulse. After AOM 3, the beam is labeled $k_3$. This beam constitutes the probe pulse for the following experiments.

**Delay vs Frequency Offset and $T_2$**

Before any modulation was introduced onto the probe pulse, temporally brief pulses were used to produce echoes. This ensured that the programming was working and that the probe laser was detuned properly. For the following experiments a boxcar geometry was used. In order for the echo to remain at the spatially isolated corner of the box, the phase matching conditions, described by eqn. (6.5), required that the probe pulses be frequency overlapped with the up-shifted LSC grating. Echoes were observed from a 50 ns brief pulse incident upon a $B = 0.5$ GHz grating with $f_s = 1.5$ GHz and $\tau_C = 50 \mu s$. In figure 78, various $\delta$’s are used to produce various time delays, $\tau_D$’s. Here the echoes decay away because of $T_2$ effects that will be discussed below.
Figure 78. Various echoes produced from a temporally brief (50 ns) probe pulse. Here various $\delta$'s were chosen to give several different $\tau_D$'s.

To ensure that the frequency offset was producing the expected time delay, the time delay was measured for each frequency offset. This was done by fitting a Gaussian envelope to each echo. This determined the time delay of the echo pulse. The relative time delay referenced to the first echo pulse was then plotted as a function of frequency offset and is shown as triangles in figure 79. The $\delta$'s for this experiment range from 1.4 MHz to 13 MHz in steps of 0.4 MHz giving expected $\tau_D$'s from 100 ns to 1.3 $\mu$s in steps of 40 ns. This gives an expected linearity of 0.1 $\mu$s/MHz. A linear fit to the data gives a slope of 0.10001 $\mu$s/MHz with a standard error of 0.00002 $\mu$s/MHz and
an RMS deviation of 0.45 ns. The residuals used to calculate this value are shown in the lower portion of figure 79. Due to the fact that the data plotted is relative delay time with respect to the first echo, the intersection of the linear fit is expected to be the time delay of the first echo, in this case 100 ns. However, the linear fit gives an intersection of 101.37 ns. This error in expected intersection is likely the result of slight path length differences as well as the relative positioning of the optical pulses with respect to the actuator in their respective AOM’s. Here the important parameter is the linearity of the fit, as this is representative of how well the delay can be tuned using the frequency offset. Here the tuning responds very linearly, showing that while the overall time delay is slightly off, the expected echo tuning is very precise. In fact it is quite amazing that a temporally brief pulse, with a width of 50 ns, can be resolved with respect to the first echo pulse to within 0.45 ns. This is a resolution error of less than 0.9% of the pulse width.

As was discussed above, the echoes decay with what appears to be an exponential decay. This decay is attributable to $T_2$ and a best fit to the echo peak heights gives a $T_2 = 1.37 \mu s$. This value is an order of magnitude lower than typical experiments in Tm$^{3+}$:YAG. The question as to whether the shortened $T_2$ was due to the programming method causing instantaneous spectral diffusion, or whether the crystal was at a higher temperature than expected was addressed. This was done by examining the $T_2$ for the case of simple brief pulses. The $T_2$ for that case came out slightly larger at 2.1 $\mu s$, but well below the expected $T_2$’s of 10-20 $\mu s$. This meant that most likely
the temperature of these experiments was well above the measured 4.2 K. This was possibly due to the fact that another very large crystal was mounted on the same cold finger and that rather large windows in the foil heat shield were used to give access to both crystals. After the experiments described in this section, minor adjustments to the mounting configuration were made and \( T_2 \)'s returned to their expected values for brief pulse experiments, so most likely this was a temperature dependent effect for \( T_2 \).

Figure 79. Experimentally determined delay times versus frequency offset (triangles) for the echoes in 78. The solid line is a linear fit to the data giving an RMS deviation of 0.45 ns. This RMS deviation was found from the residuals plotted in the lower portion of the figure.

**CW Amplitude Modulation**

The next experiment that was performed was to verify that TTD echoes of signals
would follow eqn. (1.3), that is

$$\phi(\omega_m) = \omega_m \tau_D. \quad (6.8)$$

Here, one expects that the phase for each modulation frequency would follow this formula. In order to experimentally demonstrate this, the echoes had to have a signal whose phase could be easily determined. Perhaps the easiest type of modulation would be a simple amplitude modulated cosine on the echo. But, the only modulation that could be applied to the probe was phase modulation. The question became, was there a way, using the LSC programming method and a CW phase modulated probe, to create an amplitude modulated CW waveform on the echo?

The process of turning CW phase modulation into CW amplitude modulation was discussed in chapter 3. By using a spectral filter, one of the first order sidebands could be eliminated, producing a beat between the other sideband and the carrier. This beat frequency appears with the same modulation frequency as the phase modulated signal, thus creating an amplitude modulated signal from a phase modulated signal. The spatial-spectral grating can be used as a spectral filter since it only diffracts over a given bandwidth. Spectral gratings, created using either LFC or LSC programming, also have very sharp spectral responses, as shown in figure 80 (a). Here the $r_3$ component of the Bloch vector is plotted as a function of the detuning showing the spectral grating created by two linear frequency chirped pulses. In figure 80 (b) the power spectrum of a CW phase modulated signal is shown. Let us assume for the general case that the CW phase modulation has frequency, $\omega_m$. Then the separation
between the two first order sidebands is $2\omega_m$. The spectral grating has a bandwidth larger than $\omega_m$ and smaller than $2\omega_m$, that is $\omega_m < B < 2\omega_m$. The carrier of the CW phase modulated signal is then detuned from the center of the spectral grating by an amount that allows itself and one of the sidebands to be within the bandwidth of the spectral grating. In this situation, the carrier and one of the sidebands will be diffracted from the spectral grating creating echoes, whereas the other sideband that was not on the grating is not diffracted. Thus the echo signal has the carrier and one of the first order sidebands, creating an amplitude modulation with frequency $\omega_m$.

Figure 80. (a) Shows a spectral grating programmed with two LFC’s. (b) Shows a phase modulated probe signal situated such that only the carrier and higher frequency sidebands will be diffracted from the grating in (a), producing an amplitude modulated echo at the programmed delay.
Figure 81. A plot of several output echoes from a CW phase modulated probe with $f_m = 0.5$ GHz for various frequency offsets.
This experiment was carried out using the LSC programming pulses. Two modulation rates were chosen for the probe pulse, $f_m = 0.5 \text{ GHz}$ and $1.0 \text{ GHz}$. As before, the phase modulation was turned on in the middle of the probe pulse. Here the overall temporal duration of the probe pulse was 200 ns, and the phase modulation had a duration of 100 ns centered within the 200 ns duration. In this case the carrier was always overlapped with the spectral grating, producing an echo whose temporal duration was 200 ns, whereas the phase modulation produced amplitude modulation only on the center of the echo for 100 ns.

The delay time of the echoes was controlled through the frequency offset, $\delta$. Figure 81 shows several amplitude modulated echo pulses using the technique described above. Here $f_m = 0.5 \text{ GHz}$, $\tau_C = 50 \mu s$, $B = 0.7 \text{ GHz}$, and $f_s = 1.5 \text{ GHz}$. The frequency offsets, $\delta$’s, ranged from 3.554 MHz to 3.674 MHz in steps of 6 kHz giving expected time delays from 253.9 ns to 262.4 ns in steps of 0.43 ns. The echoes in this figure were averaged with the oscilloscope 32 times. Notice the amplitude modulation with a frequency of 0.5 GHz, temporally centered within the echo, as expected from the spectral filtering. In this case the echoes were captured on a 1 GHz Hamamatsu avalanche photodiode and recorded with the 3 GHz, 10 GSample/s oscilloscope.

In order to check the timing of the various echoes, the time of the first peak of the amplitude modulated waveform was found for each echo. The subsequent echo delay time, referenced to the first echo, was then plotted as a function of frequency offset and is shown as circles in figure 82. Again, because the delay time is a linear function of the
frequency offset, a linear dependence is expected. A linear fit was made to the data that gave a slope of $71.8 \mu s/\text{GHz}$ with a standard error of $0.3 \mu s/\text{GHz}$ and an intercept of -255.1. The expected intercept was -253.9 ns, a difference of approximately 1.2 ns due to similar pulse timing effects as in the previous experiment. However, as before, the important parameter is how well this delay can be tuned. The slope was within 0.4 $\mu s/\text{GHz}$ expected value of $71.4 \mu s/\text{GHz}$. The RMS deviation found from the linear fit was 52 ps.

\[ \tau_D = 71.82 \delta - 255.1 \]
\[ \text{RMS Deviation} = 52 \text{ ps} \]

Figure 82. The relative delay of the first peak in the amplitude modulated echoes from figure 81 as a function of frequency offset (circles). The solid line is a linear fit to the data. The expected slope is $71.42 \mu s/\text{MHz}$. The residuals are plotted in the lower figure from which an RMS deviation of 52 ps was found.
In order to ensure that this beat frequency was at 0.5 GHz their Fourier transforms were taken. Figure 83 shows an example of the power spectrum of one amplitude modulated echo shown in figure 81 (circles). The peak value of the power spectrum can be seen at the expected value of $f_m = 0.5$ GHz. A theoretical plot of the expected peak is also shown, matching the experimental data nicely. The FWHM of the peak was found to be 13 MHz, which matches the expected time window of 77 ns.

In the case where the CW RF signal has been delayed in this fashion, one can easily find the functional dependence of the phase of the CW RF signal. This phase dependence, given by eqn. (6.8), should be a linear dependence on the time delay as discussed in the introduction to this thesis. Here eqn. (6.8) is repeated here for convenience

$$\phi = \omega_m \tau_d.$$  \hfill (6.9)

Note that the phase should change as a linear function of the delay time and the modulation frequency. To find the phase of the data from the Fourier transform, one must use

$$\phi(\omega) = \arctan\left( \frac{\text{imag}(\tilde{F}(\omega))}{\text{real}(\tilde{F}(\omega))} \right).$$  \hfill (6.10)

Here $\phi(\omega)$ is the phase of the CW amplitude modulated waveform, $\tilde{F}(\omega)$ is the Fourier transform of the detected waveform, and $\text{real}$ and $\text{imag}$ stand for the real and imaginary parts. This expression is evaluated at the frequency component of interest, which for this case is $\omega_m$. 
Figure 83. An example of the power spectrum of the amplitude modulated data in figure 81 (circles). The theoretical shape of this power spectrum is also plotted (solid line).

This expression was used to find the phase as a function of the time delay relative to the first delayed signal. In figure 84 the phase is plotted as a function of the relative time delay (circles). In this case, the correct modulo $\pi$ has been added for viewing. As can be seen from the data, the phase appears to be following a linear trend. When this data was linear fit, a slope of 3.16 rad/ns with a standard error of 0.01 rad/ns was found with an RMS deviation of 0.15 radians. According to equation (6.9) the expected slope is the angular frequency $\omega_m = 2\pi f_m$. Here $f_m = 0.5$ GHz giving $\omega_m =$
3.14 rad/ns. The close agreement to the expected value and the small RMS suggests that the frequency tuning of the delay is indeed changing the phase of the waveform in a linear fashion.

Figure 84. The phase of the Fourier transformed data from figure 81 plotted against the relative delay (circles). A linear fit to the data is shown (solid line). The expected slope is 3.14 rad/ns. The residuals are also shown and give an RMS deviation of 0.14 radians.

As was discussed in the introductory chapter, a TTD acts to delay all frequencies of a signal with the same delay. This is unlike a simple phase shifting device that acts only on a single frequency. In order to demonstrate that TTD indeed works to impart different phase shifts for different frequencies, another modulation frequency
was explored. Figure 85 shows echoes for various δ’s from a CW phase modulated probe with $f_m = 1$ GHz. Here $B = 1.25$ GHz, $\tau_C = 50 \mu s$ and $f_s = 1.25$ GHz. The δ’s ranged from 6.2 MHz to 6.61 MHz in steps of 10 kHz giving expected time delays of 248 ns to 264.4 ns in steps of 40 ps. From the figure, the 1 GHz amplitude modulation is apparent as well as the changing time delays for the various δ’s. Here the signals were averaged four times on the oscilloscope.

Figure 85. A plot of several output echoes from a CW phase modulated probe with $f_m = 1.0$ GHz for various frequency offsets.
As for the last modulation frequency, the 1 GHz sequences were Fourier transformed and the phase of the waveforms found. A plot of the phase versus the relative delay is shown in figure 86 (circles). Again a linear fit was made to the data and shown as the solid line.

![Graph of phase shift versus relative delay](image)

\[ \phi = 6.31 \tau_d + 2.577 \]

Figure 86. The phase of the Fourier transformed data from figure 85 plotted against the relative delay (circles). A linear fit to the data is shown (solid line). The expected slope is 6.28 rad/\( \text{ns} \). The RMS deviation was calculated to be 0.38 radians from the residuals plotted in the lower portion.

The expected slope for an \( f_m = 1 \text{ GHz} \) is \( \omega_m = 6.28 \text{ rad/\( \text{ns} \)} \). The linear fit value of 6.31 rad/\( \text{ns} \) with a standard error of 0.01 rad/\( \text{ns} \) is within 0.03 of the expected value.

The RMS deviation was 0.38 rad, which gives a delay resolution of \( \sim 60 \text{ ps} \). This
again shows good temporal resolution and linearity of tuning. But along with this, the linear phase dependence of eqn. (1.3) has been determined for another \( \omega_m \).

This two laser method was successful in producing amplitude modulation from phase modulated probe pulses. This was done by noting that spectral gratings can be used effectively as spectral filters. Precise tuning resolution and linearity was experimentally demonstrated on echoes with strong CW amplitude modulation. Figures 85 and 81 give a sense of how OCT TTD beamsteering would be utilized. Clearly if these RF signals were sent to an array of emitters the beam lobe would be steered. However, even though the experiments were highly successful at proving the linearity of eqn. (1.3), they only did so for two distinct \( \omega_m \)'s. Another approach must be utilized to determine if the entire bandwidth of the spatial-spectral grating created with the LSC programming method would perform similarly.

**TTD of Broadband Data**

In the last section, CW phase modulated probe pulses were diffracted from spectral gratings that acted to TTD the signals as well as spectrally filter them. While the modulation frequencies in the experiment above ranged from 0.5 GHz to 1.0 GHz, the bandwidths, or linewidths of the spectral features, was only 10 MHz. This was because the signal used was a CW phase modulated waveform that lasted for 100 ns. While the spectral grating existed over a wide bandwidth (up to 1.25 GHz), its whole portion was not utilized in the last experiment. This also means that although TTD
was shown at different frequencies and that the phase created by the TTD followed
the linear prediction of eqn. (6.9), the phase dependence as a function of broad band-
width must still be experimentally verified. In order to verify this, large bandwidth
probe signals had to be TTD. The bulk of this thesis has given the background and
the techniques required to produce TTD of a broad bandwidth signal. In this section,
experimental verification of TTD up to 1 GHz bandwidth signals is shown.

Actually, TTD of a 0.5 GBit/s modulated data sequence was demonstrated in the
single laser experiments (shown in figure 76), so it was hoped that with the use of two
lasers, the data rate of the probe could be pushed even higher. In order to do this,
BPSK signals were used to modulate the probe pulse with data rates as high as 1
GBit/s. In these experiments, a boxcar geometry was utilized to spatially isolate the
echo. In order to detect the BPSK modulation, the echo was heterodyned as shown in
figure 77. This was accomplished by spatially overlapping some of the probe pulse’s
master laser with the echo signal on the 1 GHz Hamamatsu avalanche photodiode.
Using this approach, the BPSK modulation on the echoes was successfully detected for
broad bandwidth probe pulses. Typical optical powers of a few 100 μW were used to
heterodyne the echo signal. This technique was used in all of the experiments shown
in this section. As was stated earlier, the bulk of this thesis has given the needed
techniques to actually achieve broadband TTD. The next set of figures exemplifies
the actual achievement of broadband TTD.
Figure 87. A 1 GBit/s BPSK modulated probe pulse (lower) is shown with the heterodyned echo signal (upper). The echo signal has the expected 80 ns delay.

In figure 87 an example of a 1 GBit/s BPSK modulated probe pulse is shown, as well as a heterodyned echo signal. Here LSC’s were used to program an efficient grating and had $\tau_C = 250 \mu s$, $B = 1.25 \text{ GHz}$, $f_s = 1.5 \text{ GHz}$, and $\delta = 0.4 \text{ MHz}$. These parameters gave a programmed time delay of $\tau_D = 80 \text{ ns}$. The long programming time allowed efficient spectral gratings to be produced, resulting in the strong echo signal shown. As stated above, the echo signal has been heterodyned with some of the master laser, allowing detection of the pseudo-random 200 bit BPSK data sequence. Here the echo without modulation was made to destructively interfere with the heterodyne
beam so that the $\pi$ phase shifts from the BPSK would be apparent as constructive interference. These BPSK signals are apparent as the fast modulation on the echo in figure 87. As stated above, the data rate was $\Gamma_D = 1 \text{ GBit/s}$ leading to the fastest TTD BPSK modulated echo sequence ever. The evidence of the BPSK signal time delayed by the appropriate 80 ns required closer examination of the bits to ensure that the echo truly represented the full bandwidth of the input probe.

In figure 88 a zoom of the delayed 1 GBit/s BPSK echo signal is compared to the expected echo signal. As can be seen in the figure, the echo signal follows the expected pattern quite well, especially considering that the bandwidth limit of the detector used was 1.0 GHz. In fact no bit errors can be detected in this echo sequence. This ensures that the LSC programming method was producing the expected spatial-spectral gratings over the 1.0 GHz bandwidth of the probe pulse. This is an important plot, as it shows that broadband TTD echoes can be created, detected and thus utilized in practical systems. Through the use of the high bandwidth LSC programming method, broadband TTD has been realized.
Figure 88. Zoom of the delayed 1 GBit/s BPSK echo signal from figure 87 is compared to the expected echo signal. There are no bit errors in the echo sequence.

Now that broadband TTD has been achieved, the two laser technique was further used to analyze the ability to use the frequency offset, $\delta$, to tune the echo signal's time delay. In figure 89, a sequence of heterodyned echoes is shown for various $\delta$'s from 0.2 MHz to 1.2 MHz in steps of 0.2 MHz. Here $\tau_C = 250 \mu$s, $B = 1.0$ GHz, and $f_s = 1.5$ GHz. This gave time delays, $\tau_D$, from 50 ns to 300 ns in steps of 50 ns. $\Gamma_D = 666$ MBits/s in these sequences. The time axis in figure 89 is referenced to the BPSK data on the first echo. The echoes in this figure are out of phase resulting in positive bits for the $\pi$ phase shifts of the BPSK data. The other echo signals can be seen with the proper time delays with respect to the first echo, again suggesting that the broad bandwidth spectral gratings are giving the proper time delays versus the frequency tuning found in 6.3. This plot is an excellent example of what is desired for phased array radars. Here several broadband signals have been TTD and could be sent to individual RF emitters, thus steering this broadband signal into the far field.
Finally, the material's ability to handle long data sequences was tested. This was done by sending in a 10 μs probe pulse with BPSK modulation at Γ_D = 666 MBits/s. Here, the parameters are the same as the previous Γ_D = 666 MBits/s sequence except δ = 0.8 MHz resulting in τ_D = 200 ns. The echo was again heterodyned to observe the BPSK signal on the echo. The result is shown in figure 90. In the figure, the first upward signal is the start of the heterodyne signal, and the echo signal is out of phase with this heterodyne signal. The temporal axis is with respect to the start of the BPSK on the echo, so that the BPSK data can be referenced. The lower bandwidth of the detector was 10 MHz, resulting in the overall shape of the detected signal.
Figure 90. A 10 $\mu$s long heterodyned BPSK echo with $\Gamma_D = 666$ MBits/s. Here the detector's lower bandwidth is 10 MHz producing the high pass filtered appearance.

In figure 91 several zooms are shown at various echo times for the data shown in figure 90. This data was high pass filtered so that the low bandwidth effects of the detector were eliminated. 200 ns sections were chosen roughly every 2 $\mu$s to help show that the echo BPSK signal is behaving as expected. No bit errors were apparent on this long 10 $\mu$s sequence. Long temporal duration data streams may be necessary in a broadband TTD applications such as for phased array communications. The material clearly has the capability to TTD these broadband signals, even for temporally long data pulses.

These last experiments have successfully demonstrated TTD of broad bandwidth probe pulses. Ultimately, these experiments have verified that each frequency of the waveform gets a TTD and thus the phase obeys eqn. (6.8) for broad bandwidth signals. TTD of BPSK signals with data rates of 1 GBit/s were reached, the highest data
rate yet achieved for BPSK echo signals, with no observable bit errors on the output echoes. Broadband waveforms were TTD using various frequency offsets, giving an example of how this system could be utilized for phased array radar applications. The echoes were observed at their expected time delays with good signal fidelity. Echo outputs for data sequences as long as 10 μs were also shown. This demonstrates the materials capability to TTD temporally long data pulses.

These results conclusively demonstrate that LSC programming pulses combined with either a single laser or dual laser technique can be an effective method for producing and probing TTD spectral gratings. By combining LSC pulses with an injection locking amplifier, these pulses can be significantly amplified allowing their use in programming OCT TTD spectral gratings. By utilizing the temporally overlapped frequency offset programming methods, long τ_c's were employed to produce efficient spectral gratings. The time delays produced from these gratings could be tuned easily by using a frequency offset created by low bandwidth acousto-optic modulators. The high precision in chirp rate, due to digital construction of the signals from pulse pattern generators, and the ability to use stabilized laser sources would also make this technique useful for accumulation. RMS timing resolution has been shown to be better than 52 ps and the dependence of the phase on the TTD (eqn. (6.9)) has been observed. Due to all of these advantages, the LSC programming method could be utilized and is an attractive approach for OCT steering of conventional phased array radar systems. But this technique goes further than this. If it is utilized in
conjunction with a multiple frequency offset, it is possible to create arbitrary analog optical waveforms using LSC pulses (see appendix 3 for a discussion of LSC programmed arbitrary waveform generation). Overall, the future for this technique is very promising as it utilizes commercially available telecommunications equipment such as integrated optics phase modulators and pulse pattern generators. Along with this, the laser systems can be cost effective ECDL's that are easily constructed or purchased, have narrow lines, are compact and easy to use.

Figure 91. Several different 200 ns sections of the echo output in figure 90. No bit errors are observable for the echo output.
CHAPTER 7

SUMMARY

This thesis presented the different techniques and methods needed to produce TTD of high bandwidth probe signals. This was accomplished by using optical coherent transient techniques, specifically the stimulated photon echo process, to effectively TTD high bandwidth signals. Experiments were carried out in Tm$^{3+}$:YAG, a suitable rare-earth ion doped crystal with a transition at 793 nm, large inhomogeneous broadening (\(\sim20\) GHz), narrow homogeneous broadening (\(\sim10\) kHz), and TBP's of approximately $10^6$. Several difficulties had to be overcome in order to reach the desired goals of this thesis.

Due to the advantages of linear chirped pulse programming over conventional brief pulse programming, this method was explored and utilized to produce efficient spatial-spectral gratings over large bandwidths. High bandwidth linear frequency chirps were created using chirped external cavity diode lasers. Due to the need for high efficiency, long programming times were needed. This required that a novel method of temporally overlapping the linear frequency chirps be used to produce the spatial-spectral gratings. This method used a frequency offset of the linear frequency chirps to produce the required time delay. Along with this, new theoretical tools had
to be developed in order to properly model the results of linear frequency chirp programming, taking into account the possibility of optically thick crystals. The major success of these theoretical efforts resulted in a Maxwell-Bloch simulator that could handle arbitrary phase as well as arbitrary pulse envelope. This simulator was used to model the behavior of the Tm$^{3+}$:YAG crystal for various types of input pulses and programming schemes. This simulator was also utilized to examine chirp efficiencies for TTD programming schemes in optically thick crystals. It was shown that crystals with an $\alpha L = 3.0$ can produce echoes with power efficiencies of approximately 60% using the linear frequency chirped programming method. This is approximately twice as high as a conventional brief pulse programming methods.

High bandwidth frequency chirps from chirped external cavity diode lasers suffer from non-linearities and the inability to be frequency stabilized. They are also not currently commercially available and difficult to build. Spurred by these difficulties, another programming approach was examined. This method, called linear sideband chirp programming, used an integrated optics phase modulator to produce a chirped sideband that acted just like a linear frequency chirp. By driving these modulators digitally with a pulse-pattern generator, linear phase chirps with bandwidths greater than 1 GHz were demonstrated with the possibility to exceed 40 GHz. Due to this feature, and the ability to use stabilized laser sources, these chirps suffer less non-linearities. Because of these features, these chirps can be utilized in accumulated programming.
In order to produce high bandwidth probe pulses and subsequent broadband TTD echoes, binary phase shift keying from integrated optics phase modulators was used. The output powers from these integrated optics modulators was low requiring a suitable high bandwidth amplifier be constructed, since no high bandwidth optical amplifiers existed at 793 nm. An injection locking technique was used as this amplifier and was suitably characterized. Different detection methods for these high bandwidth phase modulated probes was also examined. By utilizing linear filter theory, it was shown that phase modulation can be filtered to produce amplitude modulation. Spatial-spectral gratings were shown to be ideal at this filtering, producing a TTD amplitude modulated signal from a phase modulated probe. These spectral filtering techniques may subsequently improve the efficiency of detection of phase modulated signals for TTD applications.

Ultimately, a demonstration of TTD of 1 GBit/s signals was shown using the stimulated photon echo process. These results were accomplished with commercially available equipment including integrated optics phase modulators and compact diode lasers. The results are directly applicable to the steering of phased array radars as well as future possibilities in arbitrary waveform generation. A summary of the major results shown in this thesis are given below.

Maxwell-Bloch Theory and Simulations

Previous work had shown that proper modeling of the stimulated photon echo
process should take into account the optical thickness of the medium. This is done by using the coupled Maxwell-Bloch equations. A simulator utilizing arbitrary phase functions had to be developed in order to model the linear frequency chirped programming and the linear sideband chirped programming shown throughout this thesis. The background and the theory for this simulator were shown as well as the successful modeling of the programming methods used throughout this thesis.

These simulations were instrumental in verifying theoretical predictions of the programming methods. Plots of how the delay times could be tuned versus the frequency offsets of the pulses were examined as well as how the programming strength of the chirps resulted in the dynamics of accumulated spectral gratings. These unique and important problems could not have been examined without the use of this simulator.

Highly efficient stimulated photon echoes were also predicted with this simulator. Power efficiencies for TTD echo sequences programmed from linear frequency chirped pulses can exceed 60%. Many have disregarded the stimulated photon echo process because in the past it has given poor efficiencies. However, these newly predicted echo efficiencies for TTD, as well as recent demonstrations of highly efficient photon echoes for optical storage schemes, should begin to change this perception.
Throughout this thesis, various high bandwidth signals had to be created using integrated optics phase modulators, including linear sideband chirps and BPSK modulation. The outputs of these EOM’s were power limited due to possible photorefractive damage to the modulators. Thus, a technique was needed to amplify high bandwidth signals created with these EOM’s. In order to amplify these signals, an injection locking system was developed and characterized. Due to the complicated dynamics of semiconductor laser injection locking, regions of chaos and undamped relaxation oscillations had to be avoided. This required a detailed characterization of the injection locked amplifier as a function of input powers and detuning.

Injection locking of optical signals produced from the integrated optics phase modulators using both CW and broadband signals were demonstrated. Master laser input signals of 400 $\mu$W were amplified using an AR coated single mode diode laser as the slave. Small signal gains of up to 23 dB were found with good signal fidelity up to 3 GHz for CW and 6 GBit/s BPSK modulated signals. There were also no logic errors observable in the delayed-self-heterodyne measurements of BPSK signals up to the measurement limits of 3 GHz. This reliable amplification technique was utilized throughout the thesis in almost every experimental setup to increase the optical powers of both programming and probe beams.
Temporally Overlapped LFC's

New methods of programming TTD gratings had to be developed to allow longer programming pulses to create efficient spatial-spectral gratings. It was shown that frequency offset temporally overlapped linear frequency chirps could program efficient TTD gratings in inhomogeneously broadened absorbers. Both single-shot and accumulation experiments were performed. The advantages of this technique over previous techniques were shown and included 1) the ability to use chirps longer than the coherence time of the crystal, 2) relaxed laser requirements, 3) lower power requirements, 4) the ability to produce broadband delays over a wide dynamic range, and 5) a simplified system design.

The experimental results, backed by simulations and theory, show the expected traits of this programming method. An analytic calculation showed that in the linear regime the grating efficiency as a function of the programmed bandwidth drops like $1/B^2$ (versus $1/B^4$ for brief pulse programming) and can be completely compensated for by a corresponding increase in the chirp duration. The simulations and experiments have both shown that efficient TTD gratings can be produced using the temporally overlapped LFC method and that the temporally overlapped LFC method will properly accumulate these gratings. Experimental verification of these gratings was shown for both low bandwidths (40 MHz) and high bandwidths (2.0 GHz) using CECDL's.
Linear Sideband Chirp Programming

Another novel programming method, utilizing commercially available integrated optics phase modulators was explored. This technique, known as linear sideband chirping, chirps the sidebands produced using a broad bandwidth BOM. The theoretical background and simulations were shown for this programming technique.

Single laser experiments showed true-time delay of 500 MBit/s BPSK data sequences with a unique $2\omega_m$ beat frequency that could be used as a carrier signal for RF antennas.

Two laser techniques utilizing the LSC programming method were used to spatially isolate the echo and push the data bandwidths on these echoes. The RMS deviation of tuning linearity was found to be better than 52 ps. Along with this, the phase as a function of delay was shown to be linear for delayed echo sequences. Finally, a successful demonstration of true-time delay of broad bandwidth probe pulses was accomplished. Echoes were observed from a 1 GBit/s BPSK modulated probe pulse. No bit errors were detected on these echo pulses. The true-time delays as a function of the frequency offset for 666 MBit/s BPSK modulated echoes came as expected and the signal fidelity of the echo pulses remained high. Echoes from 10 $\mu$s long 666 MBit/s BPSK modulated were also shown.

Future Research Directions

As was stated above, stimulated photon echoes have become widely regarded as
weak. As has been stated in this thesis, and demonstrated in the past, the photon echo can be extremely efficient. This thesis predicted extremely high echo power efficiencies for TTD programming sequences using linear frequency chirps. Experimental demonstrations of these high efficiencies should be top on the list of future research efforts. More and better demonstrations of highly efficient echoes are needed to bring OCT applications to fruition. Along with directly showing these high efficiencies, amplification of photon echoes using the OCT crystal themselves through inversion of the transition should be explored. These rare-earth ion doped crystals are full of transitions, and thus full of possibilities.

Another direct line of research is to continue to increase the bandwidth of the programming and probing methods shown in this thesis. To increase the programming bandwidth, linear sideband chirps driven by YIG oscillators should be examined. Along with this, accumulation of gratings should begin to be explored with linear sideband chirps. While optical power is always limited, through the use of multiple optical sources and accumulation these problems may be overcome. In order to push the bandwidth of the probing, better detection methods are needed. The possibility that more efficient gratings may be able to be programmed gives some hope that broadband, real time capture of echoes can be achieved. Also, if these experiments are continued in Tm$^{3+}$:YAG, the bandwidths of the injection locked amplifier should be examined and pushed. The current limit is 3.5 GHz, limiting BPSK signals to below this bandwidth.
Injection locking of semiconductor diode lasers itself is a ripe field, ready for more research. As higher power single mode lasers are created, and as injection locking of multi-mode diode lasers is shown, interest in this technique will grow. While phase modulation was initially explored in this thesis, more research would be worthwhile. Through the use of numerical modeling of the injection locking equations, the optimal regions for amplification of phase modulated signals can be examined. Also, the physical mechanisms that limit the bandwidth can be explored. Ultimately, by understanding these mechanisms, higher modulation bandwidths may be reached.

Finally, the temporally overlapped programming method is in this author's opinion, a robust and highly practical method for programming. The results shown in this thesis are for simple TTD gratings, where a single frequency offset programmed a single delay. However, this need not be the case. Multiple frequency offsets can be used, each with different programming weights. This gives rise to pulse shaping and arbitrary waveform generation. Initial low bandwidth demonstrations have been achieved and are presented in appendix 3. But, the extension of the methods presented in that chapter to high bandwidths is where true excitement will be found. By utilizing high bandwidth linear sideband chirps in conjunction with multiple frequency offsets created with acousto-optic modulators, broadband arbitrary waveform generation can be achieved, filling a regime between femtosecond and current state of the art electronic waveform generation. Along with this, OCT’s bring their many
benefits along for the ride. Spatial multiplexing, multi-frequency transitions, quantum information processing, extremely large inhomogeneous broadenings, extremely narrow homogeneous lines...OCT's may rule the world some day!
APPENDICES
APPENDIX A

DYNAMICS OF BROADBAND ACCUMULATED GRATINGS
Dynamics of broadband accumulated spectral gratings in Tm$^{3+}$:YAG

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High-bandwidth accumulated spectral gratings are experimentally studied in Tm$^{3+}$:YAG by the stimulated-photon-echo technique with a mode-locked picosecond Ti:sapphire laser system. The experimental results show that the spectral grating builds up and decays on the time scale of the metastable-state lifetime (~10 ns), provided that the time interval of accumulating shots is of the order of the excited-state lifetime (800 μs).

An echo efficiency of the order of 0.1% was achieved with pulse intensities 2 orders of magnitude less than those needed for a single-shot process. These results fit well an analytic solution of the Bloch equations and a three-level system relaxation model. © 2001 Optical Society of America

1. INTRODUCTION

The accumulation of a spectral grating is an efficient way to enhance the strength of stimulated-photon-echo signals. This process involves the building up of a spectral grating in an inhomogeneously broadened absorbing medium by repetitive application of a pair of temporally separated programming pulses. To achieve substantial accumulation requires that the repetition period be shorter than the relaxation time of the spectral grating but longer than the coherence dephasing time. Long relaxation times can result from either a long-lived excited state (two-level systems) or a metastable bottleneck state (three-level systems). In an extreme case of persistent media, there is no upper limit on the repetition period. After the repetitive programming pulses have been applied for several relaxation times, the accumulated grating reaches a steady-state value.

Optimizing the pulse intensities and material parameters for a given repetition rate produces strong echoes with programming pulses that are weak compared with those required for single-shot echo generation. Most significantly, the accumulation processes may be carried out with temporally complex pulses and therefore have applications in optical true-time delay, processing, and memory systems.

For broadband operation, the lower input intensity requirement of accumulated gratings avoids problems with medium damage. Single-shot echo efficiency is maximized for pulse areas of $\pi/2$ on each programming pulse. For the rare-earth-doped crystal used in our experiments the intensity needed for a $\pi/2$ pulse with a 20-GHz bandwidth is of the same order as the bulk damage threshold of the laser-polished crystal. The intensity required for efficient accumulated echoes is more than 2 orders of magnitude less. In addition, achieving efficient operation with low-power laser sources is a necessary step toward developing commercial optical coherent transient (OCT) devices.

The accumulated-photon-echo technique has been used in time-domain spectroscopy of rare-earth-doped crystals as well as in demonstrations of OCT storage and processing devices. In these experiments, as in all previous demonstrations of OCT devices, the programming and processing stages were separated in time. The processing ability lasted only for the lifetime of the spectral grating, and the dynamics of the grating while it was accumulating were not significant. Recently, the use of a continuously programmed continuous processor to achieve continuous, real-time processing capability in nonpersistent hole-burning medium was proposed. Instead of separate programming and processing stages, the continuously programmed continuous processor has programming and data pulses applied simultaneously to the medium. The recorded pattern is accumulated and maintained by repetition of the structured programming pulses while the data stream passes through the medium continuously, generating a continuously processed output signal. Unlimited by the bottleneck lifetime, the processing ability in a nonpersistent medium can last indefinitely. Most significantly, the echo efficiency, defined as the ratio of the output signal power to the power of the input data stream, can be of the same order of magnitude for high-bandwidth accumulated OCT processors as for low-bandwidth single-shot OCT processors. To realize practical devices based on continuous programming, appropriate nonpersistent spectral hole-burning media must be developed and their accumulation and decay mechanisms well understood.

In this paper we report what we believe to be the first experimental study of the dynamics of high-bandwidth spectral grating accumulation and decay in a nonpersistent material. In the research presented in this paper we studied the $^3H_4\rightarrow^1H_6$ transition of Tm$^{3+}$:YAG because of the following properties that make this material a promising candidate for continuous programming: (1) convenient operating wavelength at 703 nm, where commercial diode lasers as well as solid state (Ti:sapphire) continuous-wave and mode-locked pulsed lasers are available, (2) favorable temporal and spectral parameters, in particular a coherent dephasing time, $T_2$, of tens of microseconds at liquid-helium temperature and an inhomogeneous spectrum of 17 GHz, which give a time-bandwidth product of the order of 10$^4$ at a projected data rate over 10 GHz, and (3) the presence of a metastable state of Tm$^{3+}$:YAG because of the Jahn-Teller effect of Tm$^{3+}$$^6^3$.
state \((3F_4)\) in the population relaxation path with an \(\sim 10\) ms lifetime.7-9

This paper is organized as follows: In Section 2 we present an experiment with accumulated spectral gratings that uses picosecond programming and read pulses. In Section 3 an analytical solution to the Bloch equations, including empirical decay constants, is presented and is used to simulate the experiment. The complication of combined spectral gratings composed of programming and probe pulses is also discussed. Section 4 details the second experiment designed to eliminate the complication of combined spectral gratings. The dynamics of the desired grating, formed only by programming pulses, is observed and fitted with the theoretical model. A summary is given in Section 5.

2. EXPERIMENT

Spectral hole-burning memory and signal-processing devices in rare-earth-doped crystals have been studied primarily with acousto-optically modulated cw lasers, which limits their operational bandwidth to less than a gigahertz. Whereas practical hole-burning based devices will eventually employ compact diode and solid-state lasers, we used a picosecond Ti:sapphire mode-locked pulsed laser system with a regenerative amplifier to investigate the high-bandwidth capabilities of \(\text{Tm}^{3+}:\text{YAG}\). An important characteristic of the laser used is its ability to deliver Fourier-transform-limited, 7-ps 300-\(\mu\)J pulses at a repetition rate greater than 1 kHz. To avoid crystal damage, we use an external etalon to spectrally filter the laser pulse to achieve 30-ps 50-\(\mu\)J laser pulses. The resultant pulse bandwidth is approximately a factor of 2 greater than the medium bandwidth (17 GHz). A schematic of the experimental setup is shown in Fig. 1. The optical pathway is split into three beams. The two programming pulses, separated by delay time \(T_{21}\), propagate along beams 1 and 2, respectively, and the probe pulse propagates along beam 3 with a delay \(T_{32}\) with respect to beam 2 (see Fig. 1). The desired spectral grating formed by programming pulse pairs has a period of \(2T_{21}\). Laser-frequency shifts result only in shifts in the grating envelope but have no effect on the phase of the spectral grating, which is determined solely by the difference in the two optical paths. Therefore the Fourier-transform-limited pulse pairs can constructively accumulate gratings over many laser shots, provided that the laser-frequency fluctuations are less than the inhomogeneous linewidth of the medium and the path-length jitter is much less than an optical wavelength.

In our experiment we used a 5.5-mm-long \(\text{Tm}^{3+}:\text{YAG}\) crystal with a 0.1% doping concentration corresponding to an optical density of \(-0.43\) at 793 nm. The crystal was held at 4 K in a liquid-helium cryostat. The three beams were roughly equal (within a few percent) in power and were focused by a lens to cross in the crystal with a spot size \(\sim 0.25\) mm in diameter. The wave vectors of the two programming beams, the probe beam, and the echo beams, labeled \(k_1\), \(k_2\), \(k_3\), and \(k_e\), respectively, satisfy the phase-matching condition: \(k_e = k_3 + k_2 - k_1\). In the continuous programming scheme the processed output is spatially isolated from the transmitted inputs by phase matching with a box geometry. The transmitted inputs were blocked after the cryostat. The output power of the echo signal was detected by a photodiode and recorded by a digital oscilloscope. The delay between the pulses on beams 1 and 2 was set to \(\tau_{21} = 0.7\) ns, and that between beams 2 and 3 was \(\tau_{32} = 0.2\) ns; both times were much shorter than \(T_H\), the homogeneous lifetime of the material. The time interval between two shots, \(\tau_R\), was...
of the same order as excited-state lifetime $T_1$, which is much longer than $T_2$ and shorter than the bottleneck-state lifetime, $T_3 \sim 10$ ms.

We studied the dynamics of the accumulated grating by inserting an optical chopper between beam splitters 1 and 2 to periodically block and unblock the programming pulses while beam 3 with the probe pulses remained unblocked. The chopper was synchronized with the laser at a subharmonic of the laser repetition frequency. A recording of echoes is shown in Fig. 2, for which the laser frequency was 1 kHz and the chopper frequency was set to $(1/32)$ kHz, yielding 16 accumulation shots with programming pulses unblocked and 16 grating probe shots with them blocked. A neutral-density filter with an O.D. of roughly 0.5 was placed on beam 3 to prevent saturation from the probe. The output data in Fig. 2 show a pulse train at 1 kHz of stimulated photon echoes spatially isolated from the input beams and occurring 0.7 ns after the corresponding pulse on beam 3. When the programming pair is unblocked (at $t = 5$ ms), the echo intensity increases with each application of the programming pulses and tends to a steady state for large numbers of accumulation shots. After the programming beams are turned off (at $t = 22$ ms), the echo signal stimulated from the residual grating drops dramatically within 1 ms and then decays away on a time scale of 10 ms. The pulses before 5 and after 39 ms are from the preceding and the following dynamic cycles, respectively.

3. ANALYTIC MODEL

The grating dynamics depend on material parameters $T_2$, $T_1$, and $T_3$, the absorption length, and branch ratio $\beta$ (the percentage of excited-state atoms that decay to the bottleneck rather than straight to the ground state). It also depends on the timing parameters of the experiment, $\tau_{21}$, $\tau_{31}$, and $\tau_R$, and on the input pulse areas. In our experiment, the absorption length in our material was 1.0, the angles between the beams were small (1/50 rad), and the spectrum of the brief pulses used was much broader than the inhomogeneous band and thus the temporal profile of the pulses could be treated as square. To derive an analytical model for the observed dynamics we made the following assumptions: an optically thin medium, a collinear beam configuration, and square temporal shape of all pulses. Under these conditions the accumu-
also use this matrix to describe the coherent evolution of the absorbers between pulses for time intervals short compared to $T_2$ by setting $\alpha = 0$ and $\tau$ equal to the time interval. For $T_2 > \tau$, $\tau_1$, $\tau_2$ the result of a sequence of input pulses from a single laser shot is obtained by application of Eq. (2) sequentially with the appropriate timings and Rabi frequencies. The relaxation that occurs between laser shots is described by the matrix $^{11}$

$$B(t) = \begin{bmatrix} 1 & 0 & 0 & 1 - \beta \exp(-t/T_3) + (\beta - 1)\exp(-t/T_1) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \exp(-t/T_3) \\ 0 & 0 & \exp(-t/T_1) & \end{bmatrix}.$$ 

The coherences ($\rho_{22}$ and $\rho_{12}$) are lost between two laser shots because $T_2 > T_2$. The density matrix at the time of the stimulated echo generated after $n$ programming pulse sequences (with beams 1 and 2 unblocked) and $m$ readout sequences (with beams 1 and 2 blocked) is given by

$$\rho_{m+n}(\Delta, t_3 + \tau_2) = A_2A_3^{\dagger}B(\tau_2)^nA_2A_3A_2A_2A_1 \times [B(\tau_2)^nA_2A_3A_2A_2A_1]^+(\Delta, \bar{\Delta}) \times \rho(\Delta, 0),$$

where $A_i = A(a_i, \tau_i), A_{ij} = A(0, \tau_{ij}),$ and $t_3$ is the time of the last probe pulse. The echo amplitude at time $t_3 + \tau_2$ after $n$ programming shots and $m$ probe shots is obtained by integration of $i[B(\Delta)^{-1}A_{ij}(t_3 + \tau_2) - \rho_{ij}(\Delta,t_3 + \tau_2)]\rho(\Delta)$ over all $\Delta$. This method was used to simulate the grating dynamics in Fig. 2. The calculated and experimental results are plotted in Fig. 3 and are represented by a solid curve and filled circles, respectively. The fit parameters are $\theta_1 = \theta_2 = 0.08\pi$ and $\theta_3 = 0.044\pi$, and $T_2 = 13$ ms.

4. SINGLE-GRATING DYNAMICS

The accumulation and decay of the combined gratings built by beams 1, and 2 and by beams 1 and 3 do not mimic what is expected in an OCT processor. Typically, the phase relationship between pulses 1 and 2 would be the same for all programming pulse pairs to yield coherent accumulation, whereas pulse 3 would be an uncorrelated data stream and incoherent with respect to pulses 1 and 2. To demonstrate the dynamics of the single grating under these conditions we substituted an acousto-optic modulator (AOM) in place of the chopper. The AOM has the effect of adding a random phase equally to both beams 1 and 2, as the AOM’s frequency is not synchronized with the laser repetition rate. Therefore pulses 1 and 2 had a constant phase relation with respect to each other but a random phase relation with respect to pulse 3. Thus the grating that is due to beams 1 and 2 accumulated coherently, whereas the grating that is due to beams 1 and 3 accumulated incoherently. The ratio of the signals from these two gratings drops roughly with the number of laser shots. The echo signals generated by the coherent and incoherent gratings could add constructively or destructively, but on average the combined signal would be that of only the coherent grating. Thus, we can simulate the data from our AOM experiment by assuming that the grating that is due to pulses 1 and 3 is not formed. We do this by introducing into Eq. (4) a coherence loss $[\rho(0)]$ between pulses 2 and 3.

$$p_{m+n}(\Delta, t_3 + \tau_2) = A_2A_3^{\dagger}B(\tau_2)^nA_2B(0)A_2A_3A_3A_2A_1 \times [B(\tau_2)^nA_2A_3A_3A_2A_1]^+(\Delta, \bar{\Delta}) \times \rho(\Delta, 0).$$

The introduction of the AOM also allowed for longer accumulation times (no longer limited by the timing jitter of the chopper that increased with decreasing chopper frequency).

Figure 4 shows the results for 38 laser shots with all 3 beams unblocked followed by 26 laser shots with only the contributions are roughly equal in amplitude, and the signal detected is the square of the output amplitude. We see a drop of roughly a factor of 4 because of this effect after beams 1 and 2 are blocked. The decay after this sudden drop and after the excited state has decayed completely is due to the decay of the bottleneck state back to the ground state. The lifetime of the bottleneck state, $T_3$, is $\approx 10$ ms.
probe beam unblocked. The data represent the average of 512 cycles (filled circles) and simulation (curves).

Fig. 4. Echo efficiencies at a 1-kHz repetition rate within the programming cycle of 64 ms with 38 programming and probe laser shots and 26 probe-only laser shots: experimental results for the average of 512 cycles (filled circles) and simulation (curves).

Fig. 5. Echo efficiencies at a 4-kHz repetition rate within the programming cycle of 31 ms with 66 programming and probe laser shots and 62 probe-only laser shots: experimental results for the average of 512 cycles (filled circles) and simulation (curves).

5. SUMMARY

In summary, the dynamics of high-bandwidth accumulated spectral gratings has been experimentally observed in Tm3+:YAG by means of stimulated photon echoes. The accumulation dynamics were studied under several conditions, and the results matched the analytic model well in all cases. An echo intensity efficiency of the order of 0.1% has been observed. These results are an important step in the demonstration of a continuously programmed true-time-delay processor that works on broadband signals.

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254
APPENDIX B

NUTATION OF STIMULATED PHOTON ECHOES
Temporal and Spatial Behavior of Photon Echoes Stimulated from Long Pulses

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Abstract

To evaluate the coherent saturation effects of continuous optical processing using optical coherent transients we studied the time-dependence of the photon echoes stimulated by probe fields with constant amplitudes and durations comparable to the coherent dephasing time. The propagation directions of the long echoes were also found to be different from brief pulse echoes. Echoes in the non-causal direction were predicted with a theory based on Maxwell-Bloch equations, observed experimentally, and explained analytically.

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\textit{Key Words}: optical coherent transients; stimulated photon echo; coherent saturation; optical nutation.

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1. Introduction

Stimulated photon echo processes in inhomogeneously broadened absorptive media have been considered possible tools for the realization of optical memory and optical processing devices with high bit rates and large time bandwidth product [1,2]. The basic procedure involves two programming pulses that record their amplitude and phase information as a spatial-spectral hologram. This is done by exciting the inhomogeneously broadened absorbers from the ground state to the excited state. Next a third pulse probes the medium, which generates a time-delayed output optical field, which contains information about the input programming and probe pulses. Some of the applications, such as optical coherent transient true-time delay and optical analog correlation [3,4], require the third pulse (the probe) to be temporally long or even continuous. This requirement results in two major differences from the situation with a brief probe. First, an angled beam configuration has to be employed, instead of a collinear one, to spatially distinguish the output echoes from the inputs since the transmitted inputs are temporally overlapped with the echoes. Second, material saturation effects must be considered. Although the long probe pulse can be amplitude or phase modulated to reduce the saturation at a particular frequency the output echo suffers from the saturation effects when the probe time eventually exceeds certain limit at a given optical power. To study the echo's temporal behavior with a long probe field a stimulated photon echo (SPE) process with two brief programming pulses followed by a constant quasi-continuous probe pulse is a simple case to begin with. As is well known, an electric field with an amplitude of a step-function interacting with an inhomogeneously broadened two-level atomic system gives rise to an oscillatory
behavior of the transmitted field and is referred to as optical nutation. The optical nutation signal has been used as an efficient spectroscopic measurement tool to study gaseous atomic and molecular system, as well as rare-earth ion doped crystals [5,6]. However, less attention has been paid to the interaction of a long probe with the medium at the presence of a spatial-spectral grating and the behavior of the resulting echoes.

In this paper, we will focus on the coherent saturation effects of the SPE from a long probe by studying the optical nutation signals and the spatial and temporal behavior of the echoes generated by two brief programming pulses and a long probe pulse with a duration comparable to the medium's coherent dephasing time, $T_2$. We will start with a theoretical analysis using Maxwell-Bloch equations with an angled beam configuration in section 2. The experiments in Tm$^{3+}$:YAG crystal will be described in section 3. The nutational behavior of the stimulated photon echoes was observed at different delays and probe powers. For a fixed grating the echoes generated by the long probes propagating in both directions were also observed, which is completely different from the brief pulse echo exiting in only one direction. The experimental results will be discussed in section 4 along with the numerical simulations from a newly developed Maxwell-Bloch simulator. This simulator considers the beams as having a spatial Gaussian mode and the experimental results are consistent with the simulations.

2. Theoretical Analysis with Bloch-Maxwell Equations

A theoretical modeling of OCT processes with angled beam configuration has been recently developed based on Maxwell-Bloch equations.[7] The schematic of the approach is illustrated in figure 1. In a thin layer of medium, $dz$, located at $z$, the fields
are assumed plane wave propagating along two directions, $\mathbf{k}_+ = \mathbf{k}_z + \mathbf{k}_x$ and $\mathbf{k}_- = \mathbf{k}_z - \mathbf{k}_x$, where $|\mathbf{k}_+| \gg |\mathbf{k}_-|$. The state of the atoms driven by the combined field of $\Omega^+(z, t)$ along $\mathbf{k}_+$ and $\Omega^-(z, t)$ along $\mathbf{k}_-$ can be described by the Bloch vectors as the atomic population inversion, $r_3$, and the two atomic polarization components, $r_1$ and $r_2$, in phase and in quadrature with the field, respectively. The Bloch equations are modified as,

$$
\frac{dr_1(x, z, t, \Delta)}{dt} = -\Delta r_1(x, z, t, \Delta) + \eta_1(x, z, t, \Delta) \Omega_c(x, z, t) - \frac{r_5(x, z, t, \Delta)}{T_2},
$$

$$
\frac{dr_2(x, z, t, \Delta)}{dt} = -\Delta r_2(x, z, t, \Delta) + \eta_2(x, z, t, \Delta) \Omega_c(x, z, t) - \frac{r_5(x, z, t, \Delta)}{T_2},
$$

$$
\frac{dr_3(x, z, t, \Delta)}{dt} = -r_3(x, z, t, \Delta) \Omega_c(x, z, t) + \frac{r_5(x, z, t, \Delta)}{T_3},
$$

where the field components,

$$
\Omega_c(x, z, t) = \Omega^+(z, t) \cos(k_x x) + \Omega^-(z, t) \cos(k_x x)
$$

$$
\Omega_c(x, z, t) = -\Omega^+(z, t) \sin(k_x x) + \Omega^-(z, t) \sin(k_x x),
$$

and $\Delta$ denotes the frequency detuning of the atomic resonance from the driving frequency, $T_2$ is the coherent dephasing time and $T_3$, the population decay time. The propagation effects of the fields through the medium can be derived from the Maxwell equations as

$$
\frac{d\Omega^+(z, t)}{dz} = \frac{\alpha}{4\pi^2} \int_0^{\pi} \int_0^{2\pi} \int_0^{\pi} \left[ r_5(x, z, t, \Delta) \cos(k_x x) + \eta_1(x, z, t, \Delta) \sin(k_x x) \right] g(\Delta) d\Delta d(k_x x)
$$

$$
\frac{d\Omega^-(z, t)}{dz} = \frac{\alpha}{4\pi^2} \int_0^{\pi} \int_0^{2\pi} \int_0^{\pi} \left[ r_5(x, z, t, \Delta) \cos(k_x x) - \eta_1(x, z, t, \Delta) \sin(k_x x) \right] g(\Delta) d\Delta d(k_x x).
$$

Here the variables $z$ and $t$ are decoupled in a retarded time frame by assuming the wave travels forward along the $z$-axis without any back reflection, $g(\Delta)$ represents the inhomogeneous line shape. Numerical solution of equations (1) through (5) are usually required to study angled beam OCT processes involving arbitrary input fields, including...
the SPE from a continuous probe. Simulations will be discussed in section 4 along with the experimental results.

With the equations shown above, the temporal shape and the propagation of a continuous field induced SPE can be predicted with analytical solutions for a simplified case. This case considers the interaction between a continuous input field and with an initial spatial-spectral grating distribution of the atomic population. We make the assumptions as following.

1) The medium starts from a previously programmed initial state at time, \( t = 0 \), as,

\[
\begin{pmatrix}
\rho_0(\Delta) \\
\rho_{10}(\Delta) \\
\rho_{20}(\Delta)
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
\omega_0 + g_0 \cos(2\pi \Delta \tau_d + 2k_x x)
\end{pmatrix},
\]

which can be created by two programming pulses incident along \( k_+ \) and \( k_- \), respectively, the second pulse temporally delayed by \( \tau_d \) with respect to the first one.

2) The constant probe field corresponds to a constant input Rabi, \( \Omega_0 \), provided the atoms have single dipole moment.

3) The medium is optically thin, \( g(\Delta) = 1 \) over the detuning range of interest, and the population decay can be ignored for the time scale comparable to \( T_2 \) since \( T_2 \ll T_3 \).

Under these conditions, equations (1) to (3) can be solved for a probe incident along \( k_+ \) as,

\[
\begin{align*}
\rho_1(x,t,\Delta) &= e^{-i\Omega_0 t} \rho_{10}(\Delta) \{1 - \cos(\sqrt{\Omega_0^2 + \Delta^2} t)\} / \sqrt{\Omega_0^2 + \Delta^2} \\
&+ \Omega_0 \sin(k_x x) \sin(\sqrt{\Omega_0^2 + \Delta^2} t) / \sqrt{\Omega_0^2 + \Delta^2} \\
\rho_2(x,t,\Delta) &= e^{-i\Omega_0 t} \rho_{20}(\Delta) \{1 - \cos(\sqrt{\Omega_0^2 + \Delta^2} t)\} / \sqrt{\Omega_0^2 + \Delta^2} \\
&+ \Omega_0 \cos(k_x x) \sin(\sqrt{\Omega_0^2 + \Delta^2} t) / \sqrt{\Omega_0^2 + \Delta^2}
\end{align*}
\]
By then using these results in equations (4) and (5), we can find the output fields propagating in both directions as,

$$\Omega^+(t) = \Omega_0 + D e^{-i\pi T} W_0 \Omega_0 J_0(\Omega \alpha T)$$  \hspace{1cm} (6)

$$\Omega^-(t) = 0.5 D e^{-i\pi T} \Re[\pi e^{-\alpha T} + \pi J_0(\Omega \alpha \sqrt{T^2 - \tau_0^2}) - F(t)]$$  \hspace{1cm} (7)

Where $D = \alpha L / 2\pi$, determined by the absorption length $\alpha L$, and $J_0(y)$ denotes the 0-th order Bessel function. The calculation of $F(t) = 2 \int_{\Delta} \frac{\Delta}{\Omega^2 + \Delta^2} \cos^2(\sqrt{\Omega^2 + \Delta^2}) d\Delta$ still relies on numerical integration. However, the temporal patterns are already revealed in the expressions (6) and (7). The transmitted field, $\Omega^+(t)$, has a damped oscillatory term in the form of Bessel function, which is the pattern of the optical nutation. $\Omega^-(t)$ represents the echo field consisting of a DC and two time-varying terms. Using expressions (6) and (7) we plot the output fields in both directions in figure 2. The transmitted probe pulse travels through the medium in the $k_+$ direction and is described by equation (6). This is standard optical nutation and is indicated on the plot as such. The echo, described by equation (7), exits the medium in the $k_-$ direction with $\tau_d = 0.5 \mu$s and $\Omega_0 = 0.4$ MHz and is labeled as echo $k_-$ in figure 2. Besides the optical nutation on the transmission, one can see that the delayed SPE also shows damped oscillation with the period determined by the probe Rabi frequency. $T_2$ was assumed infinite in figure 2 to show the oscillations with less damping effect. The nutation on both the transmission and echo is a direct result of coherent saturation becoming significant at a probe duration, $\tau_p >> \Omega_0^{-1}$. 
The echo propagation direction is governed by phase-matching conditions and causality, which makes the asymmetrical diffraction from the spatial-spectral grating [9,10]. This results in that a typical brief pulse SPE can only exit the medium along \( k_- \) provided a brief probe is along \( k_+ \). In that sense we call the direction along \( k_- \) the causal and \( k_+ \), the non-causal direction. A continuous probe, however, gives different results. Setting a probe field along \( k_- \) and using equations (1) to (5) with a similar derivation, one can find the expressions of the output fields as,

\[
\Omega^+(t) = 0.5De^{-i\frac{\pi}{2} \Omega_0 \Omega}[-\pi e^{-\Omega_0 \Omega} + \pi J_0(\Omega_0 \sqrt{t^2 - \tau_0^2}) + F(t)], \tag{9}
\]

\[
\Omega^-(t) = \Omega_0 + De^{-i\frac{\pi}{2} \omega \Omega_0 \omega} J_0(\Omega_0 \omega) \tag{10}
\]

While the transmitted field (10) is exactly the same as (7), a delayed echo (9) exists and now exits the medium in the non-causal direction. Figure 2 shows this echo in the \( k_+ \) direction having the same nutation period and the same DC level as the echo in the causal direction \( k_- \) and is labeled as echo \( k_+ \).

3. Experiments

The experiment, as shown in Figure 3, was designed to observe optical nutation and the nutational SPE's from a long probe and a preprogrammed medium. The laser source is a cw Ti: Sapphire laser frequency stabilized to tens of KHz by locking to a spectral hole in a Tm: YAG [8]. Two acousto-optic modulators (AOM) were used to generate programming and probe pulses on two beams. The two beams were overlapped, with an angle of \( \approx 0.05 \) radians, and focused to a spot of \( \approx 75\mu m \) (1/e waist) in a Tm\(^{3+}\):YAG crystal with absorption length of 1.4. The AOM's also controlled the timing, direction
and the power of the inputs. The duration of the two programming pulses was set to 100 ns each and the probe duration, $\tau_p = 10 \mu$s. The delay between the programming pulses, $\tau_d$, was adjusted from 0.15 $\mu$s up to 2 $\mu$s to see the echo nutation effects. The direction of the first/second pulse was fixed to $\pm \vec{k}_p$. The probe pulse was along one of these two directions to stimulate echoes in causal or non-causal directions. The peak power on each programming pulse was ~200mW while the probe was attenuated to a desired level between 200 to 7.6 mW by the AOM in the probe direction. Two photo diodes (PD) and a digitizing oscilloscope were used to measure and record the output powers as functions of time in both directions.

4 Results and discussions

4.1 Echoes in Causal Direction

In the first experiment, a 10 $\mu$s-long pulse with input power, $P_p = 28$ mW was sent along $\pm \vec{k}_p$ to probe the gratings with different delays. The transmission and the echoes for $\tau_d = 0.15, 0.3, 0.5, 1, 2$ $\mu$s are plotted in figure 4. We see the nutation effects on both transmission and echoes as predicted. The echoes peak at $\tau_d$ after the leading edge of the probe, then, start decaying as the probe field saturates the atomic population and oscillate with the atoms' Rabi oscillation. The peak echo power and the DC level decreases with $\tau_d$. Figure 5 gives the echo efficiency varying with the probe power at fixed delay, $\tau_d = 0.15 \mu$s. The high power corresponds to a high Rabi and results in the fast oscillation and damping on the echo signal. The peak echo efficiency also deteriorates at high power.

These observations are consistent with the trends from the analytical solutions. However, the experimental results are obtained with the situation being more complicated
than the simplified theory. The laser beam we used is a spatial Gaussian beam rather than a plane wave, which makes the optical power vary across the beam, and thus so does the Rabi frequency. We also used a crystal with an absorption length of 1.4 rather than thin medium in order to obtain higher efficiencies. The absorption changes the optical power and the Rabi frequency as the field propagates. The oscillations of the echoes and the transmissions in the experiments are the collective effects of the local Rabi frequencies in the medium. This can be simulated by numerically solving the Maxwell-Bloch equation set (1)-(5). Figure 6 gives the simulation results corresponding to the experiments in figures 4. From the optical nutations in figures 4a, we can estimate the input probe Rabi frequency, $\Omega_0 \sim 0.8$ MHz, at the center of the Gaussian beam assuming a thin medium [11]. The simulations result in a good fit to the experiment at $\Omega_0 = 0.9$ MHz due to the thick medium absorption. Besides the coherent dephasing, the multi-Rabi components also cause the rapid damping to the DC level since the oscillations at different Rabi frequencies average out after the first few periods.

4.2 Echoes in Non-causal Direction

The second experiment is to switch the probe direction to $\overline{k}_-$ and monitor the output looking for an echo in the non-causal direction, $\overline{k}_+$. Figures 7 and 8 give the results under the same conditions as in figures 4 and 5, respectively, except the probe directions. The optical nutation was again observed on the transmission (figure 7). Comparing the transmission results in figure 7 with figure 4 one can see that the optical nutation on the transmissions in both directions are the same. Nutational echoes were also seen in the non-causal direction, in which a brief pulse SPE usually would not exist. The oscillations on the echoes in the non-causal direction are similar to those in the
causal direction except the echoes start from zero power at the delay \( \tau_d \). This experiment can also be simulated with the angled beam Maxwell-Bloch simulator. Figure 9 gives the simulation fit to the experimental results in figure 7. The two figures show good agreement between the theory and the experiment.

The SPE from the long probe exiting in the non-causal direction does not violate the phase-match conditions or causality. The reason that an echo appears in the non-causal direction is that the probe pulse is longer than the delay \( \tau_d \). The front part of the probe pulse interacts with the grating and creates a coherence, which cannot generate a propagating field. However, the probe field incident into the medium anytime \( \tau_d \) later than the front part re-phases this coherence into the propagating direction \( k_+ \) and emits an echo. To demonstrate this, we did a third experiment. Two programming pulses were set along \( k_+ \) and \( k_- \) with a fixed delay, \( \tau_d = 0.3 \mu s \), and two 100ns pulses separated by \( \tau_{pd} \) were used as probes along \( k_- \). The output in the non-causal direction \( k_+ \) was measured as \( \tau_{pd} \) was increased from 0.25 \( \mu s \) up to 0.75 \( \mu s \) (shown in figure 10). The first two pulses in the traces are scattered light from the probe pulses showing the time delay \( \tau_{pd} \). The trace marked programming indicates the timing of the programming pulse plotted with a time shift backwards by 40\( \mu s \). The two brief probe pulses do not generate SPE's directly from the spatial-spectral grating in the non-causal direction because of causality. This is seen in the top trace where \( \tau_{pd} < \tau_d \). The probe, however, creates coherences in the medium. The first probe pulse interacts with the medium and generates the coherence so that the second probe pulse "sees" the medium as if a pulsed field had existed at \( \tau_d \) after the first pulse with a wave vector along \( 2k_- - k_+ \). The second probe pulse rephases this
coherence as a two-pulse echo. When $\tau_{pd} > \tau_d$ ($\tau_{pd} > 300$ ns in figure 10) the rephased coherence generates a real echo along $k_+ \hat{z}$ at $\tau_{pd} - \tau_d$ after the second probe. This is the echo observed in the non-causal direction and explains why long quasi-continuous probes can create such an echo.

5. Conclusion

We have studied the time-dependence of the SPE's from long probe fields with durations comparable to the coherent dephasing time. Nutational effects on the transmission and echoes have been observed experimentally. The theoretical simulations fit well with the experimental results. The propagation properties of the long SPE have been investigated for both causal and non-causal directions. The echoes in the non-causal direction have been predicted by the theory and observed in experiments. The experiment demonstrates that the long, quasi-continuous echoes still obey causality.

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Captions

Figure 1 (a) The angled beam configuration and (b) input timing.

Figure 2 Analytical results from equations (7), (6), and (9) corresponding to the traces from top to bottom for $w_0 < 0$ and $w_0 = -g_0$, $\tau_d = 0.5 \mu s$, $\Omega_0 = 0.4$ MHz and $1/T_2 = 0$.

Figure 3 Schematics of experimental setup and timing of the input pulses.

Figure 4 Experimental results of the output powers vs. time of the transmission in $k_s$ and echoes in $k_-$ for probe in $k_+$ with $P_p = 28$ mW.

Figure 5 Experimental results of the output powers vs. time on the transmission in $k_s$ and echoes in $k_-$ with $\tau_d = 0.3 \mu s$.

Figure 6 Simulation results for the experimental results in figure 4.

Figure 7 Experimental results of the output powers vs. time on the transmission in $k_-$ and echoes in $k_+$ in for probe in $k_-$ with $P_p = 28$ mW.

Figure 8 Experimental results of the output powers vs. time on the transmission in $k_-$ and echoes in $k_+$ with $\tau_d = 0.3 \mu s$.

Figure 9 Simulation results for the experimental results in figure 7.

Figure 10 Echo output in non-causal direction $k_+$ from double pulse probe with various time separation $\tau_{pd}$ for fixed delay, $\tau_d = 0.3 \mu s$. 
Fig. 1. J Lumin. Tian et. al.
Fig. 2. J Lumin. Tian et. al
Fig. 3. J Lumin. Tian et. al
transmission

Experiment
Probe Power-28mW
Echoes

\[ \tau_d = 0.15 \, \mu s \]

0.3 \, \mu s
0.5 \, \mu s
1 \, \mu s
2 \, \mu s

Time (\mu s)

Fig. 4 J Lumin. Tian et. al
Fig. 5 J Lumin, Tian et. al
transmission

Simulation
Rabi=0.9 MHz
echoes

$\tau_d = 0.15 \mu s$

0.3 $\mu$s
0.5 $\mu$s
1 $\mu$s
2 $\mu$s

Time ($\mu$s)

Fig. 6 J Lumin. Tian et. al
Experiment

transmission

echoes

Power (a.u.)

\[ \tau_d = 0.15 \, \mu s \]

\[ \tau_d = 0.3 \, \mu s \]

\[ \tau_d = 0.5 \, \mu s \]

Time (\mu s)

0 2 4 6 8 10 12 14 16

Fig. 7 J Lumin. Tian et. al
Fig. 8 J Lumin. Tian et. al
Simulation transmission

Echoes

\tau_d = 0.15 \mu s

\tau_d = 0.3 \mu s

\tau_d = 0.5 \mu s

Time (\mu s)

Fig. 9 J Lumin. Tian et. al
Fig. 10 J Lumin. Tian et. al
APPENDIX C

ARBITRARY WAVEFORM GENERATION USING LINEAR SIDEBAND CHIRPS
Analog optical signal processing (AOSP), with the ability to beat out conventional digital signal processing in both power budget and processing power, is becoming a topic of interest. OCT's inherently, with their large TBP's and wide bandwidths, lead the field in their abilities. With the high bandwidth methods developed in this thesis, and a few simple additions, two AOSP applications, pulse shaping and arbitrary waveform generation (AWG), could be researched on high bandwidths. A pulse shaper takes an input pulse and typically rearranges the energy in the pulse to reshape the pulse into a different sequence of pulses. Figure 92 (a) shows the simple case of equally weighted pulse shaping. Here a single input pulse is diffracted off of a spectral grating producing the three pulses on the output. In (b) the weighting is changed so that the output pulses have arbitrary height. This type of weighting leads into AWG which is the creation of any kind of generalized arbitrary waveform. These methods rely upon the fact that OCT's can act as temporal lenses, in much the same way that spatial diffraction works [37, 92].
The following paper details a low bandwidth demonstration of pulse shaping and how AWG can be accomplished using OCT's. By utilizing this technique in conjunction with either the linear sideband chirps described in chapter 6, or a CECDL described in chapter 5, high bandwidth AWG and pulse shaping can be demonstrated.
Optical Pulse Shaping Using Optical Coherent Transients

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Abstract: Optical coherent transient signal processing offers a novel method of optical pulse shaping at current optical communication bandwidths (GHz). Using multiple temporally-overlapped, frequency offset and phase-tuned, linear frequency chirps, we propose optical pulse shaping and processing in inhomogeneously broadened absorbers. Rare-earth doped crystals have the ability to perform pulse shaping in the frequency regime between the femtosecond pulse shaping and current analog electronics. Demonstrations of this pulse shaper on low bandwidths (~20 MHz) are shown. These include pulse train creation, self-convolution and auto-correlation, and chirped pulse compression.

OCIS codes: (070.4550,300.6240,320.5540)

Optical pulse shaping of femtosecond pulses has been thoroughly studied, and applications such as coherent control of simple molecular processes have already utilized pulse shaping in experiments. Other applications have been proposed such as production of dark solitons for long haul fibers, optical time domain multiplexing (OTDM), optical
code-division multiple access (CDMA) multiplexing, and chirped pulse compression. Most methods of optical pulse shaping in the femtosecond to picosecond regime involve gratings to spatially disperse the frequency components of a pulse, which is then modulated using spatial light amplitude and phase modulators. Due to the limited resolution of gratings and spatial light modulators these techniques have frequency resolution of ~10 GHz making 100ps the maximum temporal width of the pulse packet. Other methods of pulse shaping utilize beamsplitters and physical delay lines to create a desired temporal shape. These approaches have the disadvantage that it is very difficult to adjust or change the shape of the output pulse on a reasonable time scale. Previous investigations of picosecond pulse shaping have utilized spectral hole burning and the photon echo process in the organic materials, but these results relied upon physical delays and could only achieve temporal durations ~100 ps. Very few methods of optical pulse shaping in the nanosecond regime exist.

In this paper we propose and experimentally demonstrate a method to perform pulse shaping using spectral hole burning and optical coherent transients (OCT) in inhomogeneously broadened absorbers such as rare-earth ion doped crystals. These crystals have inhomogeneous linewidths up to a 100 GHz, homogeneous linewidths as narrow as 1 kHz, and inhomogeneous to homogeneous ratios (time-bandwidth product) of $10^5$-$10^8$. These crystals offer an excellent platform for pulse shaping in the gap between traditional femtosecond pulse shaping techniques and the capabilities of electronics to create analog optical waveforms.

In previous papers, we demonstrated experimentally that two temporally overlapped linear frequency chirped pulses (TOLFC) could be utilized to create periodic
A probe pulse incident upon this grating produces an echo output with a delay

\[ \tau_d = \frac{\delta}{\alpha}. \]  

(0.1)

Here \( \delta \) is the frequency offset between the two chirps and \( \alpha \) is the chirp rate. The TOLFC method has many advantages over brief pulse programming and programming with two temporally separated linear frequency chirps. These advantages include chirp durations longer than the coherence time of the rare-earth ions, the ability to use a single chirp source for the programming pulses, and large delay tuning range (~\( \mu \)s) using small frequency offsets (~MHz). With the development of high bandwidth (>40 GHz) chirped lasers the TOLFC method offers an attractive approach to high bandwidth OCT processing. In this paper we propose a novel method utilizing multiple TOLFCs to produce complex spectral gratings for optical pulse shaping. This method offers a quickly adjustable (~ms), versatile, fully programmable way to perform pulse shaping in the picosecond to submicrosecond regime.

Using multiple frequency offset TOLFCs one creates several time delay gratings given by (0.1) instead of only one. Figure 1a diagrams the multiple TOLFC programming method for use as a pulse shaper. A reference linear frequency chirp, \( C_1(t) \), and control chirps, \( C_2(t) \), with the desired frequency offsets are temporally and spatially overlapped in the inhomogeneous broadened material to create a complex spectral grating. Later the pulse to be shaped, \( E_3(t) \), is diffracted off the grating producing multiple echoes. In general the desired output signal of a pulse shaper can be written in the form,
Here $\tau$ is the desired sampling period, $A_n$ are complex amplitudes, and $b_\tau$ is a temporally brief pulse with bandwidth, $B_\tau$. The output echo field of the stimulated photon echo process has a Fourier transform $E_i(\omega) \propto E_i^*(\omega)E_k(\omega)E_\ell(\omega)$, where $E_i(\omega)$ for $i = 1, 2$ and $3$ are the spectra of the reference, the control and the probe fields, respectively. By choosing the reference and the control pulses properly, a probe pulse can be shaped into an output pulse with an arbitrary shape. If the pulse to be shaped is brief, $E_2 = b_\tau(t)$, the action needed to create the desired output signal, $E_\ell(t)$, is equivalent to the creation of multiple delayed copies of the input pulse with the proper complex amplitudes. If the reference pulse is a linear frequency chirp, $C_1(t) \propto \exp\left(\omega_0 t + i \alpha t^2\right)$, of a bandwidth, $B > B_\tau$ with a chirp rate, $\alpha$, and a start frequency, $\omega_0$; then, the control pulse should take the form, $E_2 \propto \sum_n A_n C_1(t - n\tau)$, which is a superposition of delayed copies of the reference chirp weighted by $A_n$.

\[ E_\ell(t) \propto C_1(t) \sum_n A_n \exp\left(-i\alpha n^2 - \omega_0 n \tau + i \alpha n^2\right) \]  

If the delays are much shorter than the chirp duration, the delays ($n\tau$) can be replaced with frequency shifts of $n\delta = \alpha(n\tau)$ giving

\[ E_\ell(t) \propto C_1(t) \sum_n A_n \exp\left(-in\delta - i\omega_0 n + i \alpha(n\delta)^2\right) \]  

This produces multiple TOLFCs with a different frequency offset and weighted by a complex amplitude $A_n$. The additional phase term,

\[ \phi_n = -\omega_0 n\delta + \frac{(n\delta)^2}{\alpha} \]
ensures the echo has the same phase regardless of delay, with relative phase between echoes controlled by $A_n$ on each control chirp of the pulse shaper programming. In addition, the beam paths of reference chirp and the control chirps are angled, to separate the pulse shaped output from the probe and spurious echoes.

The pulse shaper is not limited to shaping just a single brief pulse. If an arbitrary input pulse is to be shaped into an arbitrary output with a given bandwidth and sampling rate, one only needs to work out the right set of the complex weighting factors, $A_n$, needed on the control pulse. For example, to turn the pulse train, $E_3(t) = \sum_{n=1}^{N} A_n \delta_n (t - n\tau)$, into its self-convolution, one can use the reference chirp and control pulses as discussed above. To get the auto-correlation, the control pulse needs to be,

$$E_3(t) \propto C(t) \sum_{n=1}^{N} A_n \delta_n \exp \left( -\frac{in\delta t - i\alpha n \frac{\delta}{\alpha} + i \frac{(n\delta)^2}{\alpha}}{\alpha} \right),$$

which is just the control pulse needed to create the time reverse of $E_3(t)$ in the basic pulse shaping process described above.

Another application of pulse shaping commonly used is chirped pulse compression. This multiple TOLFC method also has this ability. Previous studies have shown that OCT's have the ability to compress chirped pulses. Here we show that the linear frequency chirps need not be temporally separate, and generalize chirp compression to the multiple TOLFC method. Chirp compression can be accomplished by using a reference chirp and a frequency offset control chirp with different chirp rates $\alpha_1$ and $\alpha_2$, respectively as shown in Figure 1b. One can solve $\alpha_2$ analytically if $|E_5(\omega)|$, $|E_5(\omega)|$ are made uniform over the bandwidth of interest by assuming larger time bandwidth products. To compress the chirped pulse to its Fourier transform limit,
\( E_z(t) = \delta_z(t - \tau_d) \), one can set the control chirp as, \( E_z(\omega) = E_1(\omega)E_2^*(\omega)E_z(\omega) \), where \( E_1(\omega)E_2^*(\omega) \) is equivalent to a linear frequency chirp with a chirp rate, \( \alpha_z = \frac{\alpha_1 - \alpha_2}{\alpha_2 - \alpha_1} \). In the time domain, the control chirp then takes the form of

\[
C_z(t) = \exp\left( \frac{i}{\omega_0} (t - \tau_d) + \frac{\alpha_z}{2} (t - \tau_d)^2 \right),
\]

which is a temporally delayed chirp. Here \( \tau_d \) is the delay time of the echo pulse with respect to the leading edge of the probe pulse and is confined to less than \( T_2 \) and greater than the probe chirp duration plus \( 1/B \). Since the delay is a function of the frequency, \( \tau_d \) is also the delay of the probe chirp's start frequency. For the case of a limited time bandwidth product, the control chirp takes the same form, and the compressed pulse becomes a bandwidth limited \( \delta \)-function. For temporally overlapped reference and control chirps one can expand equation (0.1) to calculate the delay as a function of rf frequency \( \omega \), frequency offset \( \delta \), and chirp rates \( \alpha \), as

\[
\tau_z(\omega) = \frac{\omega}{\alpha_1} \frac{\omega}{\alpha_2} + \frac{\delta}{\alpha_z}.
\]

Using this method it should be possible to compress a multi-GHz chirp with a temporal duration of a few microseconds to its bandwidth limit.

Proof-of-concept demonstrations of the TOLFC pulse shaper for the different processes described above were done. Acoustic-optic modulators (AOM's) were used to create the linear frequency chirps, limiting these initial demonstrations to 20 MHz bandwidth. Experiments were performed using an external cavity diode laser with an injection locked amplifier lasing at the \(^{3}H_4 - ^{3}H_6\) transition in \( \text{Tm}^{3+}\text{YAG} \) (~793 nm). The laser beam was split with a 50/50 beam splitter, passed through two separate AOM's driven by arbitrary waveform generators to create the reference and control chirps. The two beams were then focused and overlapped in the crystal, which was held at liquid
helium temperatures (~4.5 K). Powers before the crystal were ~15mW on each path and the beams were focused into the crystal using a 75mm lens giving a spot diameter of ~60µm. The echo output of the pulse shaper was then incident onto an amplified silicon photodetector with 50 MHz of bandwidth or an avalanche photodiode with frequency response from 1MHz to 1GHz. The RF waveform used to program the reference chirp was \( \sin(2\pi f_1 t + \gamma t^2) \) where \( f_1 \) is the start frequency of the chirp. The control chirps were frequency offset copies of this reference chirp as in equation (0.4).

First we tested the pulse shaper’s ability to produce pulse trains with arbitrary times and phases. The goal was to program a complex spectral grating to produce the 11 bit Barker code (11100010010) at a 10 MHz data rate. Programming chirps were 5µs long with 20MHz bandwidth giving a chirp rate of 4MHz/µs. The frequency offset between control chirps needed to produce the 10 MHz sampling rate was 400kHz. The phase control was performed with (0.5), using a negative amplitude for a control chirp corresponds to an echo with a π phase. Figure 2 shows the echo output from the 100ns probe pulse processed by the two different complex gratings producing the expected code in both a binary amplitude modulated coding (Fig 2a) and a bi-phase (0, π) coding (Fig 2b). The phase encoding (0,π) is observed by nulls in the output of the bi-phase representation whenever there is a phase transition. Due to coherence dephasing effects the echo output amplitude usually decays exponentially with the delay time. To compensate for this, the relative amplitude of each chirp of the control pulse was adjusted to create uniform echo amplitudes across the data sequence. 

Next we tested the ability of the pulse shaper to shape an arbitrary pattern. To demonstrate this we programmed the bi-phase 5-bit Barker code (1,1,1,-1,1) and the time-
reverse (1,-1,1,1,1) versions of the code into the material. We then probed each grating with (1,-1,1,1,1), which yielded the operations of self-convolution and auto-correlation of the probe pulse, respectively. This is also an excellent test of the phase control of the pulse shaper. In the first experiment, only the intensity of the output waveform could be detected with only the nulls to suggest a phase flip. When performing a correlation or convolution, phase is an important factor in determining the shape of the expected output. The theoretical outputs, compensated only for decay, are shown in Figure 3 with the experimental convolution and correlation output of the pulse shaper. The agreement between the experiment and calculated outputs confirms that we have phase control over the echo output. This scheme can be used to realize convolution and correlation operations between two arbitrary waveforms where one or both of the waveforms can be controlled.

Finally we used this TOLFC pulse shaping method to perform linear frequency chirped pulse compression. A 1 μs pulse with a -20 MHz/μs chirp rate was to be compressed. For a 6μs long 20 MHz up chirped reference pulse the chirp rate for the second chirp of 4 MHz/μs was required for compression. An offset of 0.3 MHz was added to the second chirped pulse giving a 90ns delay between the end of the chirped probe and the echo. Figure 4 shows the probe chirp and the compressed echo output. The temporal width of the echo measured at ¼ the intensity was 64 ns. This gives a compression factor of ~16 with compressed echo duration close to the expected of 50 ns for a 20 MHz bandwidth limited pulse. As the chirped bandwidth of the pulses is increased, the compression factor should also increase. This ability to compress chirps can also be combined with the basic multiple-TOLFC process described above by adding
more control chirps creating multiple chirp compressing gratings with different time delays. By choosing the proper delays and phases of these compressed echoes and with the use of a high bandwidth chirp laser, high bandwidth analog optical signals and arbitrary waveforms can be created in the picosecond to submicrosecond regime.

We have demonstrated the ability to perform pulse shaping in rare-earth ion doped crystals using multiple, temporally overlapped linear frequency chirps. The versatility of this method was evident in its ability to not only shape brief pulses, but to perform pulse shaping on arbitrary signals, producing convolved outputs or compressed pulses. With the use of a high bandwidth chirped external cavity diode laser to create the chirps and AOM's to control the frequency offsets and phases, one could perform these processes at high bandwidths with low bandwidth electronics and low bandwidth acoustic-optic modulators.
References

List of Figure Captions

Figure 1.

(a) Schematic of the TOLFC pulse shaping process. Multiple linear frequency chirps with different starting frequencies (dotted lines) are temporally overlapped with a single higher frequency reference chirp (dashed line). Later a brief probe pulse is diffracted off the grating producing multiple echoes. In the real programming process, the maximum frequency offset is much less than the bandwidth of the chirps.

(b) Schematic of the TOLFC chirp compression process. Two chirps with different chirp rates are temporally overlapped, creating a linearly chirped time delay grating that compresses the probe chirp to a delayed Fourier limited pulse.

Figure 2. The echo output of the pulse shaper probed with a 100ns pulse. Output is the binary representation of the 11 bit Barker code (11100010010) in (a) binary amplitude modulated (0,1) format and (b) a Bi-phase modulated (0,π) format.

Figure 3. The calculated and experimental output of the pulse shaper programmed with the (a) time reverse of the bi-phase 5 bit Barker code (1,-1,1,1,1) and (b) the time forward (1,1,1,-1,1) then probed with (1,-1,1,1,1) producing the self-convolution and the auto-correlation.

Figure 4. Test of chirped pulse compression. A 1 μs, 20 MHz chirp was diffracted off a chirp compressing grating producing the narrow echo. The full width half max of the echo measured at one quarter the intensity is 64 ns, close to the bandwidth limit. The probe and echo are plotted on different scales.
Figure 1.
Figure 2
Figure 3

Figure 4

FWHM = 64 ns
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