Demonstration of an ultrafast causal optical logic gate by interference of coherent transients
by Wilds David Ross

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Physics
Montana State University

Abstract:
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Wilds David Ross

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ABSTRACT

An ultra fast, three port logical gate has been demonstrated using two-pulse photon echo in dye-doped polymer films at low temperatures. Since the coherence time T_2 is much longer than the intervals between the excitation pulses, many more pulses can be added to the system, resulting in coherent interaction (interference) of N inputs. And finally, because of causality, pulses which come later in time do not influence the interference of those applied earlier, yet all pulses within T_2 time interval interfere coherently, similar to quantum computing.

Chapter 1

INTRODUCTION

In the field of optical computing, and most recently quantum computing, logic gates and switches are of great interest. A variety of non-linear optical phenomenon, including saturation of absorption, frequency conversion, Kerr effect etc, have been used in the past to implement such optical switches. Coherent optical transients in frequency selective materials provide a means of constructing analog processing devices such as correlators, delay lines and memories. Because of the non-linear nature of coherent transients, digital elements could also be produced. In this work, we present the principle of a novel, all optical logic gate based on coherent transients. We implement a three-input/three output gate, which satisfies the conditions as a universal gate, a controlled, controlled NOT. This particular type of logic gate is relevant in recent discussions about quantum computing. Although our present realization does not involve quantum mechanical entanglement of atoms, the transient photon echo phenomenon uses quantum mechanical coherence and is in this respect related to coherent and quantum computing. Such a gate is only possible because of the essential causal nature of coherent transients. To illustrate the role causality plays in coherent transient phenomenon; we present in Chapter 1, an elementary discussion of causal coherent optical response. In Chapter 2, we present the theory and implementation of causal interference of coherent transients. We also describe the experimental setup, and the femto-second laser source. We discuss how the system behaves as an optical logic gate, and present the truth tables for both the three input/output system described here, and a four input/output system which could be constructed in a similar manner. Finally, we present a discussion of the importance of the system's causal nature to the gate's logical structure.

Chapter 2

CAUSAL COHERENT OPTICAL RESPONSE

Theory of a Linear Spectral Filter

When mathematically describing optical transient phenomenon, such as photon echoes [1] and time-and-space domain holograms [2,3], it is necessary to insure that the solution satisfies the principle of causality [4]. A causal solution is such that no response (output) appears prior to the excitation (input). If the excitation amplitude is zero for time $t<t_o$, then the response of the system must also be zero for times $t<t_o$. We set the time at which the input function begins taking effect as $t_o=0$. In complicated calculations, there is often the problem of keeping track of causality. Comparison with causal solutions found by analytic calculations can verify that, in the linear limit at least, there are only causal terms in the solution.

The causal character of coherent transients is strongly related to the nature of the system as a passive linear filter. In general terms, the frequency response of a linear system to an input has the form:

$$G(\omega) = T(\omega)\, F(\omega) \qquad \textbf{(1)}$$

Where G(ω) is the output amplitude function, T(ω) is the transmission function, and F(ω) is the input amplitude function. The filter can be described by the time-domain transmission function, or equivalently the impulse response function. Taking the inverse Fourier Transform of the above equation, we can write for the time-domain response function:

$$\begin{aligned} g(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} T(\omega)\,F(\omega)\,e^{-i\omega t}\,d\omega \\ &= \frac{1}{2\pi}\iint T(\omega)\,f(t')\,e^{-i\omega t}\,e^{i\omega t'}\,dt'\,d\omega \\ &= \frac{1}{2\pi}\iint T(\omega)\,f(t')\,e^{i\omega(t'-t)}\,d\omega\,dt' \\ &= \int_{-\infty}^{\infty} I(t'-t)\,f(t')\,dt' = I(t)\otimes f(t) \end{aligned} \tag{2}$$

Where g(t), I(t), and f(t) are the inverse Fourier transforms of G(ω), T(ω) and F(ω) respectively:

$$\begin{aligned} g(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\,e^{-i\omega t}\,d\omega \\ I(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} T(\omega)\,e^{-i\omega t}\,d\omega \\ f(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{-i\omega t}\,d\omega \end{aligned} \tag{3}$$

The relation (3) is, in essence, the convolution theorem, which states that the Fourier transform of the multiplication of two functions is given by convolution of the corresponding Fourier transforms.

Now, since the time response function must be causal, the convolution integral in equation 3 must be zero for $t< 0$, if the temporal input function is zero for $t<0$. This in turn implies that the impulse response must also be zero for $t<0$.

$$I(t) = \begin{cases} 0, \text{ if } & t < 0 \\ I(t), \text{ if } & t > 0 \end{cases} \tag{4}$$

Some mention must be made about zero time $t=0$. This is, of course, an arbitrary assignment of the value zero to some actual time. Time continues passing with no regard to what we label it at any given moment. We use zero here to denote the time at which a distinct event occurs. Figure 1 shows, for instance, an input amplitude function defined as starting at $t=0$, and ending at time $t=t_p$.

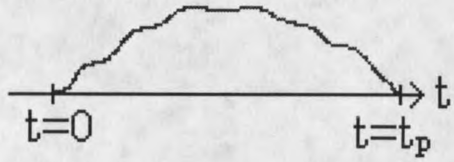

Figure 1: A temporally limited amplitude function, which is non-zero in an interval $0<t<t_p$

Titchmarch's Theorem

The practical aspect of causality is that it can often tell us something useful about the transfer function, which is helpful in performing analytic as well as numerical calculations. In particular, complex analysis provides Titchmarch's theorem [4], which describes certain mathematical properties of causal functions. This theorem relates the real and imaginary parts of a causal function to each other by means of the Hilbert transformation [6]:

$$\begin{aligned} \mathrm{Re}\left[T(\omega)\right] &= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[T(\omega')\right]}{\omega-\omega'}\,d\omega' \\ \mathrm{Im}\left[T(\omega)\right] &= -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Re}\left[T(\omega')\right]}{\omega-\omega'}\,d\omega' \end{aligned} \tag{5}$$

Titchmarch's Theorem:

If a square integrable function $T(\omega)$ *fulfills one of the four conditions below, then it fulfills all four of them:*

(i) *The inverse Fourier transform* $I(t)$ *of* $T(\omega)$ *vanishes for* $t<0$:

(ii) $T(u)$ *is, for almost all u, the limit as v goes to 0+ of an analytic function* $G(u+iv)$ *that is holomorphic in the upper half-plane and square integrable over any line parallel to the real axis:*

(iii) *Re* G *and Im* G *are related by the Hilbert transformation.*

(iv) *Im* G *and Re* G *are related by the inverse Hilbert transformation.*

Relation to Analytic Signal

Titchmarch's theorem is related to the analytic signal, often used in signal analysis theory. Experimentally measured time-dependent signals such as voltage, current, optical amplitude, mechanical displacement, etc. are inherently real valued functions. Their complex Fourier transforms are symmetrical with respect to $\omega=0$, and are therefore non-zero for negative frequencies $\omega<0$. In most cases however, operating with negative frequencies is not practical. Often a new complex function is defined such that the real and imaginary parts are related by the Hilbert transform, called the analytic signal [6]. The Hilbert transform of the real part is called the quadrature function. As an example, the quadrature function of $\cos \omega t$ is $-\sin \omega t$ and the corresponding analytic signal is therefore $e^{i\omega t}$. One could say that the analytic signal bears the same relationship to the real function as the complex function $e^{i\omega t}$ does to the real function $\cos \omega t$.

An optical implementation of an analytic signal can be illustrated by considering the experiment shown in figure 2. First, we place a thin cosine amplitude transmission grating of period $(\Delta\omega)^{-1}$ in the x direction, represented by:

$$T(x) = 1 + \cos(\Delta\omega x) \tag{6}$$

The grating must of course have a DC offset to prevent negative transmission values. We illuminate this grating with a monochromatic laser beam propagating in the z direction. At the output of the grating, there will be a non-diffracted transmitted beam with a Fourier-transforming lens to produce the Fourier image of the diffracted amplitude in the back focal plane ω.

$$\hat{F}[\cos(\Delta\omega x)] = \frac{1}{2}\delta(\omega + \Delta\omega) + \frac{1}{2}\delta(\omega - \Delta\omega) \quad (7)$$

Consider now, that we place a second grating, shifted in the x direction with respect to the first grating such that it experiences a quarter period phase shift. The second grating may be represented by the function $1 + \sin(\Delta\omega x)$, next to the first grating. In addition, we will translate the sine grating by distance d along the positive z direction. The path length difference between the beam diffracted from the first and second grating is

$$\Delta x = d(1 - \cos\theta) \quad (8)$$

This will introduce a corresponding phase shift $\Delta\varphi = \frac{2\pi\Delta x}{\lambda}$. If the distance d is such that:

$$\frac{2\pi d(1 - \cos\theta)}{\lambda} = \frac{\pi}{4} \quad (9)$$

Then the total expression for the diffracted amplitude of these two gratings would be

$$\cos(\Delta\omega x) + e^{i\Delta\varphi}\sin(\Delta\omega x) = \cos(\Delta\omega x) + i\sin(\Delta\omega x) = e^{i\Delta\omega x} \quad \textbf{(10)}$$

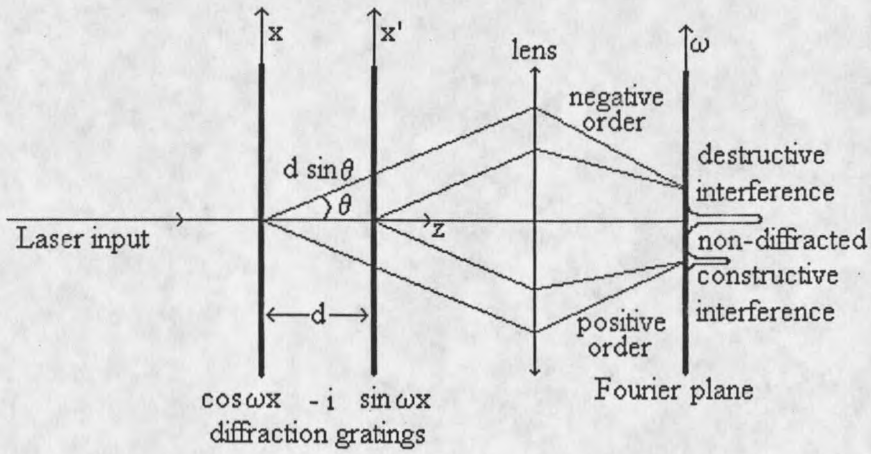

Figure 2: The optical implementation of analytic signal. x, x′-grating planes; ω-Fourier image plane

This is our "old friend" the analytic signal. The diffraction amplitude produced by the second grating combines with the amplitude of the first grating image. In the Fourier plane, the positive diffraction orders will constructively interfere, while in the negative diffraction orders they will interfere destructively. Figure 2 provides an illustration of this thought experiment. The monochromatic laser beam enters from the left to be diffracted by the cosine grating. The non-diffracted portion of the beam continues in the same direction. The beam is then diffracted by the sine grating. Again, most of this beam is not diffracted and continues along the z-axis. The diffracted beams are imaged by a lens, and allowed to interfere in the image plane. This image is the Fourier transform of the combined grating equation.

We may also consider constructing the analytic signal by simply neglecting the negative order and multiplying the amplitude of the positive order by a factor of two. Euler's formula gives for the cosine function.

$$\cos(\omega x) = \frac{e^{i\omega x} + e^{-i\omega x}}{2} \tag{11}$$

Of course, since in practice what we observe is always intensity, the constructive interference gives a four times brighter signal than that produced by one order of the cosine grating alone. This experiment has been performed in the laboratory and the results agree will with the prediction of the theory.

By virtue of Titchmarch's theorem, the Fourier image of such complex signals is identically zero for $\omega<0$. Since the experiment shown in Fig. 2 is performed with monochromatic light which extends in time from $t=-\infty$ to $t=+\infty$, causality does not really play any role here. For example, we can change the x direction phase shift from sin to –sin and then the positive diffraction order will vanish and the negative order will appear. In the case of causal time-domain transfer functions, the real and imaginary parts of $T(\omega)$ should satisfy Titchmarch's theorem, and we can rewrite $T(\omega)$ in the form:

$$T(\omega) = \mathrm{Re}\{T(\omega)\} + i\hat{H}[\mathrm{Re}\{T(\omega)\}] \tag{12}$$

Hilbert Phase

In practice, however, we can measure neither the real nor the imaginary parts of an optical response function. If electric circuits were operated at optical frequencies, then we could perhaps measure separately both the real and imaginary part of a light beam, just as we do electric signals. However, electric circuits are not capable of such an action, and in reality, we can measure only the energy (intensity) profile of a light beam. From the intensity, it is in general easy enough to find the amplitude. However, the problem remains how to find the phase. This is an issue of critical importance in many practical problems, which derive information from light measurements; i.e. astronomy, microscopy, imaging, photonics, etc. Therefore, we would like to find a way to apply Titchmarch's theorem to the complex amplitude and phase of the transmission function, rather than to the real and imaginary parts.

Corollary:

Causal transfer functions, which have Hilbert phase, can be given in the form:

$$T(\omega)=|T(\omega)|\,e^{i\hat{H}[\ln|T(\omega)|]} \qquad (13)$$

Illustration:

Let

$$T(\omega) = |T(\omega)|\, e^{i\Phi(\omega)} \qquad \textbf{(14)}$$

$$B(\omega) = \ln T(\omega) \qquad \textbf{(15)}$$

Now consider

$$B(\omega) = \ln |T(\omega)| + i\Phi(\omega) = \mathrm{Re}\, B(\omega) + i\, \mathrm{Im}\, B(\omega) \qquad \textbf{(16)}$$

We may identify the phase and amplitude of T(ω) *with the real and imaginary part of* B(ω). *And if* T(ω) *is sufficiently well behaved that* B(ω) *also satisfies Titchmarch's Theorem, we may write:*

$$\Phi(\omega) = \mathrm{Im}\, B(\omega) = \hat{H}[\mathrm{Re}\, B(\omega)] = \hat{H}[\ln |T(\omega)|] \qquad \textbf{(17)}$$

$$\Rightarrow T(\omega) = |T(\omega)|\, e^{i\hat{H}[\ln|T(\omega)|]} \qquad \textbf{(13)}$$

<u>Spatial Holography</u>

To better understand how the causality principle applies to time-and-space domain holography, let us first consider the coherent transmission function of ordinary thin space-domain holograms [7].

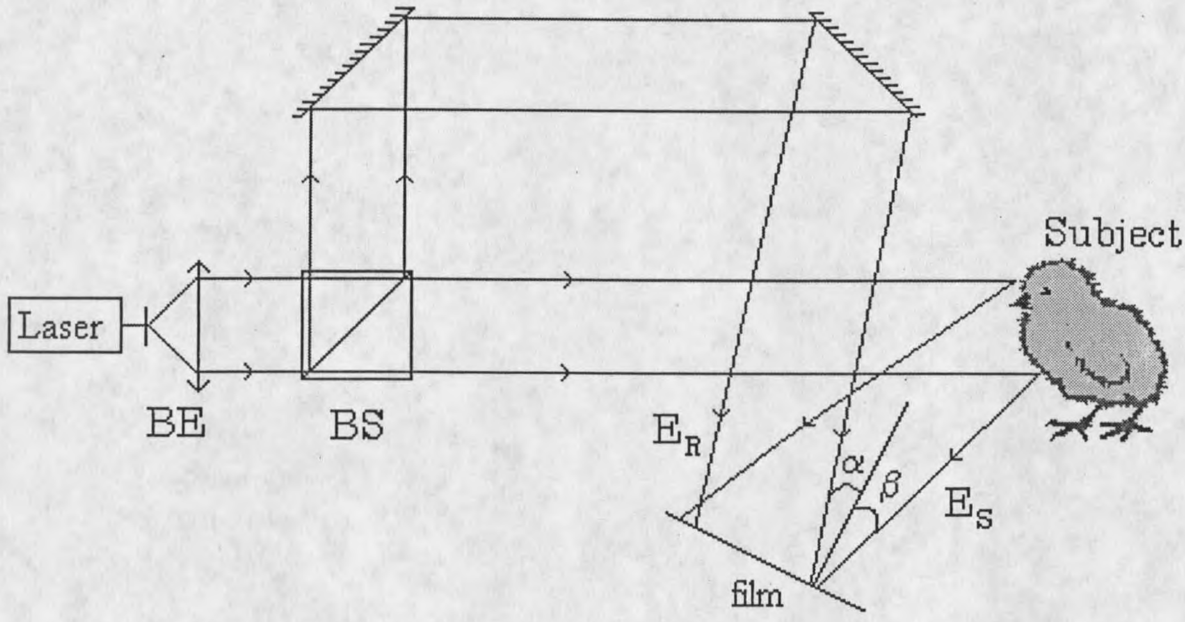

Figure 3: Scheme for recording spatial holograms: BE-beam expander; BS-50% beam splitter; E_R, E_S-amplitude of reference and subject beams.

Holography is a well-known means of recording and later recovering an optical wavefront. It does so by recording the interference pattern of two beams, which spatially overlap at the storage material, usually a photographic film. Figure 3 shows a monochromatic laser beam, which passes through a beam expander before being split 50/50 by a beam splitting cube. One of the beams remains a s a reference, while the other is scattered off the object (chicken). The two beams are then brought to coincide at a photographic film, where they form the interference pattern. The material experiences some effect e.g. a chemical reaction, which causes a change in the absorption profile of the material corresponding to the intensity profile formed by the two beams. We will construct the transfer function from the absorption properties of the material before exposure plus the intensity dependent pattern to be "burned" into the film. Consider figure 3; the reference and subject beams are given by:

$$E_R(x,y) = r\, e^{i(\omega t+\phi)}$$
$$E_S(x,y) = s\, e^{i(\omega t+\theta)} \quad (18)$$

Where r and s are the amplitude functions of the reference and subject beams, and ω is the laser frequency. θ and ϕ denote the phase angles containing the information on the direction of propagation of the beam relative to the normal axis of the film.

$$\phi = \frac{2\pi}{\lambda}\Delta = \left(\frac{2\pi}{\lambda}\right) x \sin\alpha \quad \textbf{(19)}$$

$$\theta = \left(\frac{2\pi}{\lambda}\right) x \sin\beta \quad \textbf{(20)}$$