THE INFLUENCE OF SOLAR RADIATION IN SNOW ON NEAR SURFACE ENERGY BALANCE IN COMPLEX TOPOGRAPHY

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

MONTANA STATE UNIVERSITY
Bozeman, Montana

November 2015
I would like to thank my committee, Dr. Ed Adams, Dr. Dan Miller, and Dr. Ladean McKittrick for their continual help, support, and guidance through this scholastic process. Ladean, thank you for teaching me basic coding in two software language and discussing the minutia and inner workings of this project. I am grateful for the Montana State University Mechanical Engineering Department and the Civil Engineering Department for making this opportunity possible.

Thank you to Yellowstone Club Ski patrol for keeping up study plots and sharing their field knowledge. A special thanks to Tom Leonard.

Thank you to my office mates Tony LeBaron, David Walters, Jeremiah Johnson, and Brad Stanton for time in the field, discussions in the office, and quality days skiing.

I would also like to thank my closest friends and family for all of their love and support. Laura Pflum, Loribeth Evertz, Ryan Christianson, Eric Hansen, and Jeff Burkhartsmeyer, you are all amazing. Jeff, thanks for editing. To my parents, Dana and Sandy, I would have never done this without all the early morning texts, phone calls, hugs, and the occasional free trip to the grocery store. To my wonderful siblings, thank you for the editing, saving my thesis from cyberspace, and helping me rejuvenate with scrabble, gardening, and outdoor adventures. To my manfriend, Carson, I love you. Thank you for your unwavering confidence and support, especially while living in Rocket City. You are the toughest, nicest, smartest person I know.
# TABLE OF CONTENTS

1. INTRODUCTION ........................................................................................................... 1

2. BACKGROUND ............................................................................................................ 3

   2.1 Avalanches ............................................................................................................... 3

   2.2 Snow, Snow Types, and Weak Layers .................................................................. 3

     2.2.1 Snow .................................................................................................................. 3

     2.2.2 Snow Types ....................................................................................................... 5

     2.2.3 Weak Layers ..................................................................................................... 6

   2.3 Numerical Modeling Software ............................................................................. 7

   2.4 First Principle Energy Balance ............................................................................ 9

     2.4.1 Radiation .......................................................................................................... 13

     2.4.2 Longwave Radiation ....................................................................................... 16

     2.4.3 Shortwave Radiation ....................................................................................... 18

     2.4.4 Latent Heat ....................................................................................................... 21

     2.4.5 Convection ....................................................................................................... 22

     2.4.6 Conduction ....................................................................................................... 22

3. METHODOLOGY ........................................................................................................... 24

   3.1 Cold Lab Experiments ......................................................................................... 24

   3.2 Instrumentation and Data Collection ................................................................... 27

   3.3 Defining the Terrain Model ............................................................................... 31

   3.4 Meteorological Input ......................................................................................... 35

   3.5 RTRT Coding ....................................................................................................... 35

4. RESULTS AND DISCUSSION .................................................................................... 42

   4.1 Attenuation Parameter Results ......................................................................... 42

   4.2 Effects of Weather Inputs .................................................................................. 52

   4.3 Lab Results .......................................................................................................... 64

   4.4 Thermal Imaging and Spatial Variability ............................................................ 74

   4.5 Winter Radiation Recrystallization Event ......................................................... 84

     4.5.1 Event Parameters ............................................................................................. 85

     4.5.2 Temperature Results ..................................................................................... 86

     4.5.3 Spatial Variation ............................................................................................. 96

     4.5.4 Mass Flux ....................................................................................................... 102

5. CONCLUSIONS AND RECOMMENDATIONS ..................................................... 105

REFERENCES CITED .................................................................................................... 110

APPENDICES ............................................................................................................... 115
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Albedo for snow, ice, and sand from Gray and Male (1981)</td>
<td>19</td>
</tr>
<tr>
<td>2.</td>
<td>The energy available to be absorbed at the surface of the snowpack. The total energy available sums to 1</td>
<td>38</td>
</tr>
<tr>
<td>3.</td>
<td>Density and layer thickness inputs for RTRT. Density is in kg/m$^3$ and thickness is in meters</td>
<td>39</td>
</tr>
<tr>
<td>4.</td>
<td>An example of the absorption, out of 1. Note that the thicknesses are not equal and layer 0 (surface) considers only half of the layer thickness</td>
<td>39</td>
</tr>
<tr>
<td>5.</td>
<td>Density and thickness profile for RTRT YCS numerical analysis. Density is in kg/m$^3$, and thickness is in meters</td>
<td>53</td>
</tr>
<tr>
<td>6.</td>
<td>Density and layer thickness profile for modeling lab data. The position in qdensity corresponds to the qthickness position. Grain size was 1 mm</td>
<td>67</td>
</tr>
<tr>
<td>7.</td>
<td>Layer density and thickness profile used in RTRT for winter RR event. These parameters were not measured, and they are an estimation. The assumed total depth of the snowpack is 2 meters, which is an over estimation</td>
<td>85</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Simple model of a column of snow (left) is the surface of the column, which is 1 m(^2). The right figure represents the model nodes going through the snow. RTRT can have up to 20 layers.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Schematic of energy exchange on a given slope. Shortwave (SW) radiation is incoming from the sun and is absorbed in the top few centimeters of the snow (see LHS of figure.) Sensible heat in the form of convection as well as latent heat can either be added or removed from the snow surface depending on temperature. Conduction occurs within the snowpack. This model assumes the conduction coefficient between snow and air is small enough to be negligible. Longwave interactions occur between the surface, surrounding topography (such as rocks, cliffs, and other snow surfaces), and clouds. There is also some SW interaction associated with complex topography.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>The electromagnetic spectrum. Shorter wavelengths are higher energy. (Extension 2008)</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>The snow box used in the lab (a) Shows the box with the sliding front and sliding top closed, (b) is the top sliding off, (c) is the box with the top removed, (d) is the front of box being removed, and (e) is the box without its front or top.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Google Earth image of Yellowstone Ski Club on Pioneer Mountain. The north study plot can be seen pinned in the upper left corner. The American Spirit study site is shown with the pin towards the center of the image. And the south weather station is the pin at the bottom center of the image.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Location of the three weather stations used on Pioneer Mountain in the Yellowstone Club, near Big Sky Montana. (a) YC South (YCS) weather station at 45.2307°N, 111.4422°W (b) American Spirit (YCAS)</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>7. YCS study plot (i) looking north from weather station. (ii) The south weather station with MSU student and Yellowstone Club ski patrol collecting data. (iii) Looking south on YCS study site.</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>8. (i) RTRT rendering of 30 meter DEM of Pioneer Mountain where all three study sites are located. This model covers just short of 20 km$^2$. (ii) Zoomed in rendering of YCS weather station meadow</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>9. RTRT rendering of 1 m LiDAR data for south facing study plot.</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>10. National Renewable Energy Lab (NREL) irradiance data with a two fitted curves. These curve fits were used to calculate the percentage of energy available in each bin before snow albedo is considered.</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>11. Albedo for a grain radius of 100 um, solar zenith of 30 deg, and a semi-infinite depth (Wiscombe &amp; Warren 1980).</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>12. Modeled surface temperature of a lab experiment for which shortwave was and was not accounted. This model was run for 24 hours at steady-state before the graphs shown. Therefore, the shortwave result starts warmer due to an iterative time step between hours 23:45 and 00:00.</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>13. Depth profile with and without SW turned on for a one element, lab based model. SW00 and NoSW00 are the depth profiles after the steady-state initiation of the model. SW12 and NoSW12 is the depth profile 12 hours after SW00 and NoSW00.</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>14. Absorptivity for 100 kg, 1mm snow. Absorptivity in this range (0.1-0.07) all represent what would be classified as fresh snow.</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>15.</td>
<td>Temperature gradient through depth of snow that is modeled as 100 kg/m³ density, 1mm snow. The range of absorptivities shown all fall within the fresh snow classification for albedo.</td>
<td>46</td>
</tr>
<tr>
<td>16.</td>
<td>RTRT results of the surface temperature for theoretical homogenous snow with densities of 50, 100, 150, 200, and 250 kg/m³, respectively. At the surface, lower density snow has much lower solar radiation absorption with comparable latent heat, long wave radiation, and convection values.</td>
<td>47</td>
</tr>
<tr>
<td>17.</td>
<td>The effect of snow density on temperature gradient throughout a snowpack. One mm grain size 12 hours into the experiment is shown here.</td>
<td>49</td>
</tr>
<tr>
<td>18.</td>
<td>Surface temperature results for one element homogenous snow for four different densities.</td>
<td>50</td>
</tr>
<tr>
<td>19.</td>
<td>Temperature vs depth results from RTRT for four different grain sizes at the last numerical time step, i.e. 12 hours into the experiment.</td>
<td>51</td>
</tr>
<tr>
<td>20.</td>
<td>(i) RTRT rendering of 1 meter resolution of YCS study plot. (ii) RTRT rendering of 24 meter resolution of Pioneer MTN. The white triangles in each represent the element chosen to plot surface temperature of each.</td>
<td>53</td>
</tr>
<tr>
<td>21.</td>
<td>Numerical and measured results using 1m LiDAR data using various weather inputs.</td>
<td>55</td>
</tr>
<tr>
<td>22.</td>
<td>Numerical and measured surface temperature results using 1m LiDAR data using various weather inputs.</td>
<td>56</td>
</tr>
<tr>
<td>23.</td>
<td>Measured longwave for south facing study plot (YCS), north facing study plot (YCN), and the American Spirit study plot (YCAS). The unobstructed view of the sky at YCAS results in lower longwave readings compared to the two study sites surrounded by trees.</td>
<td>57</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>24.</td>
<td>Measured shortwave for YCS, YCN, and YCAS. As expected the most radiation from the sun is seen at the south facing weather station with the north facing weather station reading lowest shortwave radiation.</td>
<td>58</td>
</tr>
<tr>
<td>25.</td>
<td>Calculated Pioneer Mountain (30 m resolution) surface temperature results and measured temperature at north and south study plots.</td>
<td>59</td>
</tr>
<tr>
<td>26.</td>
<td>Depth vs temperature profiles for 6:00 AM, 2:00 PM, and 6:00 PM for the 1 m LiDAR at the south study site on March 21. The maximum subsurface temperature is seen at 2:00 PM with RTRT assuming a maximum temperature of 0°C.</td>
<td>61</td>
</tr>
<tr>
<td>27.</td>
<td>Depth vs temperature profiles for 6 AM, 2 PM, and 6 PM for the 1 M LiDAR at the north study site on March 21.</td>
<td>62</td>
</tr>
<tr>
<td>28.</td>
<td>Surface temperature variation using one weather file input and 30 m DEM. On the day when an event was modeled, (the second day), a temperature difference of a few degrees is seen. However, one would expect a greater difference during all day light hours due to aspect, self-shadowing, and elevation.</td>
<td>63</td>
</tr>
<tr>
<td>29.</td>
<td>Spatial depth profile on March 20 at 2:00 PM using RTRT with 30 meter resolution.</td>
<td>63</td>
</tr>
<tr>
<td>30.</td>
<td>Microscope images of lab experiment on June 10, 2014. (i) and (ii) are radiation recrystallization crystals grown during the experiment. (iii) is a representative snow crystal from beneath the radiation recrystallized layer. Grid seen is 2 mm.</td>
<td>66</td>
</tr>
<tr>
<td>31.</td>
<td>Hyperspectral image of buried radiation recrystallization layer and graph of brightness. On the graph, perturbation just after -20 mm corresponds to the radiation recrystallized layer seen in image.</td>
<td>66</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>32.</td>
<td>Percent of energy absorbed per layer. Each layer is 0.5 cm until the last four layers, which are 1, 3, 9, and 9 cm thick, respectively for a 30cm thick snow pack.</td>
<td>67</td>
</tr>
<tr>
<td>33.</td>
<td>Surface temperature of lab experiment that was carried out on June 10, 2014. TC0 represents the thermocouple closest to the surface of the snow in the lab. STSensor represents the IR temperature sensor pointed at the snow, and RTRT represents the surface temperature as calculated by RadThermRT.</td>
<td>68</td>
</tr>
<tr>
<td>34.</td>
<td>June 10, 2014 RR metamorphism in lab. TC02 and RT02 represent the hours lapsed. TC is thermocouple array in the snow sample and RT in the numerically modeled results.</td>
<td>69</td>
</tr>
<tr>
<td>35.</td>
<td>Calculated mass flux from RTRT. The model assumes that the surface of the snow is saturated with respect to water vapor.</td>
<td>70</td>
</tr>
<tr>
<td>36.</td>
<td>Surface temperature reading on Dave Walter’s experiment. These results are within 2˚C of each other.</td>
<td>71</td>
</tr>
<tr>
<td>37.</td>
<td>Depth profile of Dave Walter’s experiment. TC02 is the measured result two hours into the experiment corresponding to the calculated result RT02. TC06 and RT06 are the measured and calculated results six hours into the experiment, respectively. TC12 and RT12 are the results at twelve hours into the experiment. The results through the first centimeter do not align well, below the first centimeter one can see good congruency in the shape of the results with a small constant temperature offset.</td>
<td>72</td>
</tr>
<tr>
<td>38.</td>
<td>Calculated mass flux from RTRT. The model assumes that the surface of the snow is saturated.</td>
<td>73</td>
</tr>
<tr>
<td>39.</td>
<td>RTRT rendering of Pioneer Mountain on March 20, 2014 at noon using American Spirit data. The white is the trees that are scaled out. The results show a maximum temperature difference of approximately</td>
<td></td>
</tr>
</tbody>
</table>
List of Figures - Continued

Figure Page

1.5°C, where the measured results (not shown) show a difference of about 5°C at the two study plots.................................................................75

40. Researcher preparing to download data from thermal image at south study plot in tree stand. Photographer: Tony LeBaron ........................................................................................................76

41. Thermal image of south facing study plot. Image (i) was taken at 13:00. The back element is -3.1°C and the lower left element is -4.0°C. Image (ii) was taken at 16:00. The back temperature element is -7.4°C and the lower left element is -11.2°C. And image (iii) was taken at 19:00 on March 20, 2014 with node temperatures of -22.9 and -26.7, respectively ..............................................................77

42. Complex topography and surface temperature from RTRT. The white triangle represents the location of the YCS (element 8789), and the white square represents element 18282. A representation of March 20, 2014 at 4:30 PM is shown. This image shows approximately the same area as shown in the thermal images in Fig. 3 ........................................................................78

43. Results of thermal imaging (Img-1 and Img-2) compared to the surface temperature results calculated in RTRT (RTRT-1 and RTRT-2). The calculated temperature runs about 2°C warm during the heat of the day ................................................................................................................79

44. RTRT rendering of the north study plot on March 20, 2014 at 1:00 PM. Maximum temperature difference seen on the snow is approximately 5°C ........................................................................................................80

45. Thermal imaging, RTRT solution, and surface temp measured at the south weather station. The measured data and RTRT solution are not at the same precise location but have the same aspect, slope, and snow depth ........................................................................................................81

46. RTRT and Thermal imaging plots of approximately the same respective locations. Just before this period, the weather was consistently warm and the snow definitely
went isothermal. Here it was also quite warm so that there was not as much temperature variation over the slope because the entire slope is approaching zero. RTRT results (left) show a few degrees of variation in the heat of the day. The thermal imaging results (right) do not show this variation. This is probably due to the fact that the camera was tilted a little too high, and was just missing the shadowing from the tree where it was located.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>47. Thermal images for March 24 and 25. Images are shown at midnight, 7 AM, 2 PM, and 9 PM on March 24, and then 4 AM, and 11 AM on March 25.</td>
<td>82</td>
</tr>
<tr>
<td>48. Radiation recrystallization photos from Yellowstone Club ski patrol at (I) 0 cm, (II) 1 cm, (III) 2 cm, and (IV) 3 cm below the surface taken on January 11, 2011.</td>
<td>83</td>
</tr>
<tr>
<td>49. Percent energy absorbed per layer. Energy absorbed is a function of density, thickness, and position. Since the densities and thicknesses are not the same throughout the snow pack, an exponential decay is not seen.</td>
<td>85</td>
</tr>
<tr>
<td>50. Surface temperature for three days at the south study plot location: measured temperature at YCS weather station (YCS-Measured) along with numerical results using YCS/AS data and AS only data for the mountain model. The AS/Pio result is consistent with the overall temperature trend, but magnitude for global temperature maximums and minimums is too small. Also, the AS/Pio data does not converge well on local maximums and minimums. The YCS data, however, follows the measured trend very well and includes the local temperature perturbations pretty well, but it does not reach the global maximum well.</td>
<td>86</td>
</tr>
<tr>
<td>51. Surface temperature of three days at the north study plot location: measured temperature at the YCN weather station (YCN-Measured) along with numerical results using YCN/AS for the YCN model and AS for</td>
<td>88</td>
</tr>
</tbody>
</table>
the mountain-scale model. The AS/Pio result is consistent with the overall temperature trend, but magnitude for global temperature maximums and minimums is too small. Also, the AS/Pio data does not converge well on local maximums and minimums. The YCN data, however, follows the measured trend very well and includes the local temperature perturbations pretty well. However, it also does not reach the global maximum well.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.</td>
<td>89</td>
</tr>
<tr>
<td>53.</td>
<td>90</td>
</tr>
<tr>
<td>54.</td>
<td>91</td>
</tr>
<tr>
<td>55.</td>
<td>91</td>
</tr>
<tr>
<td>56.</td>
<td>92</td>
</tr>
<tr>
<td>57.</td>
<td>93</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>58.</td>
<td>South facing small scale, 1 m, (YCS) and large scale, 30 m, (Pio) depth versus temperature profiles on January 2.</td>
</tr>
<tr>
<td>59.</td>
<td>South facing small scale, 1 m, (YCS) and large scale, 30 m, (Pio) depth versus temperature profiles on January 3. The large scale result does not show the positive knee-shaped temperature gradient that is seen in the small scale.</td>
</tr>
<tr>
<td>60.</td>
<td>South facing small scale, 1 m, (YCS) and large scale, 30m, (Pio) depth versus temperature profiles on January 4.</td>
</tr>
<tr>
<td>61.</td>
<td>Depth vs temperature profile of YCN (1M) and Pioneer Mountain Scale (30M)</td>
</tr>
<tr>
<td>62.</td>
<td>RTRT rendering showing 4 elements chosen to show 1 m scale spatial variability.</td>
</tr>
<tr>
<td>63.</td>
<td>Spatial surface temperature results for south study plot.</td>
</tr>
<tr>
<td>64.</td>
<td>Spatial results for south slope on January 3.</td>
</tr>
<tr>
<td>65.</td>
<td>Locations of RTRT results shown below.</td>
</tr>
<tr>
<td>66.</td>
<td>Surface temperature of six locations on the mountain scale.</td>
</tr>
<tr>
<td>67.</td>
<td>Depth vs temperature profile of 6 locations on Pioneer Mountain. These results are for January 2, at 2 PM. They show no spatial variation greater than 1/100 °C.</td>
</tr>
<tr>
<td>68.</td>
<td>Depth vs temperature profile of 6 locations on Pioneer Mountain. These results are for January 3, at 2 PM. Despite not seen variation at the surface, there is strong differences in the gradients here.</td>
</tr>
<tr>
<td>69.</td>
<td>1 m YCS surface mass flux.</td>
</tr>
<tr>
<td>70.</td>
<td>North facing mass flux for 1 m YCN data.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>71.</td>
<td>Opening ESRI's ArcMap 10.1 to begin processing data.</td>
</tr>
<tr>
<td>72.</td>
<td>Open blank map in ArcMap 10.1</td>
</tr>
<tr>
<td>73.</td>
<td>Connect to folders containing wanted data.</td>
</tr>
<tr>
<td>74.</td>
<td>Expected spatial error for unfiltered data.</td>
</tr>
<tr>
<td>75.</td>
<td>Unfiltered data without spatial reference.</td>
</tr>
<tr>
<td>76.</td>
<td>Finding ArcToolbox icon.</td>
</tr>
<tr>
<td>77.</td>
<td>Define projection in GUI.</td>
</tr>
<tr>
<td>78.</td>
<td>Data with new spatial reference applied.</td>
</tr>
<tr>
<td>79.</td>
<td>GUI to clip or clip to extent.</td>
</tr>
<tr>
<td>80.</td>
<td>Turning on Spatial Analyst Toolbar if necessary.</td>
</tr>
<tr>
<td>81.</td>
<td>Saving raster data set to .asc not .txt. After using “save in folder” icon, make sure to delete the “.TXT”.</td>
</tr>
<tr>
<td>82.</td>
<td>RTRT GUI units setting. All work was done in meters.</td>
</tr>
<tr>
<td>83.</td>
<td>Text file for running RadThermRT with layer thickness and density profiles.</td>
</tr>
</tbody>
</table>
Once snow reaches the ground it begins to metamorphose. It may thermodynamically metamorphose into a weak layer, which could lead to slab avalanches. The effect of local weather, topography and snow depth on this process can be estimated with a first principle one-dimensional energy balance equation in conjunction with a mesh topographic model. To do this, the commercially available software RadThermRT (RTRT) was used. This work focused on the effect of solar radiation on surface and near surface temperatures as well as the effect of varying the resolution of the topographic model. Three main components were completed. A solar radiation attenuation coefficient was developed based on wavelength, snow grain size, and snow density from published literature. Then this code was used to calculate results from twelve hour radiation recrystallization experiments carried out in a cold lab with homogenous snow. Finally, conditions for metamorphic events were calculated and qualitatively affirmed in the field at the Yellowstone Club ski area. This work demonstrates that solar radiation has a significant effect on the surface temperature as well as temperature at depth, and weak layer metamorphic events can be modeled. Based on RTRT calculations with 100 kg/m³ density snow, shortwave radiation increased the temperature at the surface by approximately 5°C and at 2.5 centimeters below the surface by 9°C. During the 2013/14 and 2014/15 seasons, diurnal weather data was collected at the Yellowstone Club ski area, and events around the mountain were recorded with the help of the Yellowstone Club ski patrol and thermal imaging. For radiation recrystallization events, strong positive-knee-shaped gradients were successfully modeled on congruous slopes. RTRT and measured results agreed within 2°C. Spring events were also calculated and measured but there were some false positives. In the winter, spatial variation over the mountain was greater than in the spring where snow temperatures were ubiquitously high. Overall, this work is useful for modeling snow surface and depth temperatures to project the occurrence of weak layer metamorphic events. Going forward from this work, projecting longevity of weak layers and including a layer history of the snow would further improve the model.
1. INTRODUCTION

Metamorphism on the microscale affects snow stability at the macroscale (Adams et al. 2011). After snow falls to the ground, it begins to metamorphose based on its physical surroundings and local climate. The effect of climate and topography can be modeled with a one-dimensional energy balance in conjunction with a topographic grid. This combination allows the calculations to include complex topography, time of year, latitude, longitude, elevation, slope angle, aspect, and the surrounding surface profile.

The one-dimensional energy balance equation is comprised of six terms representing the effects of the following: specific heat, convection, conduction, latent heat, longwave radiation, and shortwave radiation. Extensive proof of concept work has been done previously (Staples 2008; Morstad 2004; Adams & Curran 1999). This work focuses on the effect solar radiation has on snow metamorphism based on a one-dimensional energy balance equation.

The effect of electromagnetic waves on surface and subsurface temperatures is dependent on the wavelength of the incoming and outgoing radiation as well as the grain size and density of the snow pack. Longwave and shortwave radiation play a significant role in metamorphism of snow on the ground. This metamorphosis may lead to weak layer formation. If these weak layers persist, they can lead to slab avalanches.

Using RadThermRT, a commercially available thermodynamic modeling software package (ThermoAnalytics 2003), the effect of electromagnetic energy absorption of different wavelengths was explored for varying snow densities and grainsize. Radiation attenuation code was written to optimize and improve the results. Each energy balance
term is discussed, and the numerical results of three radiation recrystallization events from the field are shown based on weather, snow density, and topographic modeling resolution.
2. BACKGROUND

2.1 Avalanches

There are two basic types of avalanche classifications: loose-snow avalanches and slab avalanches. Loose-snow avalanches start at a point and propagate into avalanches occurring in snow that has relatively low cohesion. Slab avalanches are initiated by a trigger causing a crack failure in a weak layer at depth below an existing slab-(McClung & Schaerer 2006). The resulting separated slab then slides, picking up mass as it goes. This weak layer is often observed between slabs of snow, but it can also be found at the ground. This work will focus on the metamorphic processes that cause weak layer growth—the condition which facilitates slab avalanches.

2.2 Snow, Snow Types, and Weak Layers

2.2.1 Snow

Snow on the micro level is not a homogenous material. Several layers of different types of crystals are almost always observed in the field. Even in the lab, it is difficult to achieve a homogenous snow pack because of uneven settling and an increase in density at the bottom due to overburden.

New snow can fall as needles/columns, plates, or stellar crystals depending on temperature and humidity. After snow is on the ground, it metamorphoses because it is a granular material existing near its melting point. It has a large specific surface area and is often in transition between two of its three phases if not in all of its phases. The basal layer of snowpack is almost always warmer than the surface layer, which leads to a
consistent temperature gradient through the snow (Colbeck 1983). Also, old snow without overburden increases in strength over time as bonds grow between ice particles, which is largely driven by vapor pressure with only a small temperature gradient. In addition to increasing strength over time, there is also an increase in density due to material deposition and diffusion through the ice (Hobbs, 1974).

The most commonly accepted system, described by Fierz, et. al. (2009), is comprised of nine primary physical characteristics. This classification includes microstructure, grain shape, grain size, snow density, snow hardness, liquid water content, snow temperature, impurities, and layer thickness. Grain shape is broken down into nine additional classifications: precipitation particles, machine made snow, decomposing and fragmented precipitation particles, rounded grains, faceted crystals, depth hoar, surface melt forms, and ice formations. Grain size, which will be discussed further in the methodology as well as the results and discussion chapter, is developed into six classifications: very fine (less than 0.2mm), fine (0.2-0.5 mm), medium (0.5-1.0 mm), coarse (1.0-2.0 mm), very coarse (2.0-5.0 mm), and extreme (greater than 5.0 mm) (Fierz et al. 2009).

Machine made snow, surface melt forms, and ice are outside of the scope of this work, and the international classification does not include age or temperature gradient within the basic grain type classification but as an addition to the grain shape classification. However, there is a common simple six part classification method that nicely concatenates the temperature gradient within the scope of the grain type by Colbeck in “The International Classification System for Seasonal Snow on the Ground”
(1990). This classification system coincides with the temperature and grain size
discussion within this work and is discussed below.

2.2.2 Snow Types

Even though all ice crystals develop hexagonally, there is a constellation of grain
sizes and shapes. Snow can be described and classified a number of ways, but this
discussion will focus on parameters related to snow on the ground. One common
parameterization for snow on the ground is a 6-type classification system based on age,
shape, and temperature gradients. Class 1 encompasses precipitation particles, i.e. new
snow fall. Class 2 contains decomposing and/or fragmented precipitation particles,
which is usually the result of aging snow. Class 3 and 4 are rounded grains and faceted
crystals, respectively. Class 3 snow is formed when a snowpack maintains a temperature
gradient less than 5°C/m, with grains remaining rounded. Temperature gradients greater
than 5°C/m and less than 15°C/m will yield class 4, faceted crystals. Temperature
gradients greater than 15°C/m result in class 5 crystals (Jordan et al. 2008). Class 5
represents cup-shaped or fluted crystals and depth hoar. And class 6 contains wet grains
(Colbeck et al. 1990). Except for class 1, all of these classes occur from or during
metamorphism, and metamorphism of the microstructure directly affects the stability of
the snowpack at the macrostructure level (Adams et al. 2004). Results in this work
include modeling and verifying class four and five temperature gradients. One of three
metamorphosed crystal classes are seen at propagation sites of slab avalanches: depth
hoar, near-surface facets, or surface hoar (Birkeland 1998).
These metamorphic events are driven by surface temperature and temperature gradients (Adams et al. 2004; Morstad 2004; Colbeck 1983). In the snowpack, temperature gradients greater than $10^\circ$C/m (class 4) are considered to be above equilibrium threshold so that metamorphic processes are occurring (Akitaya 1974; Armstrong 1987; Colbeck 1982; Marbouty 1980). In order to numerically model these temperatures, a one-dimensional first principle energy balance equation is used, see Section 2.4.

2.2.3 Weak Layers

Usually, weak layers are associated with faceted crystals. However, there can be weak layer development from falling snow that is classified as either low density flakes or graupel. Near surface metamorphism processes include surface hoar and near surface facets. Energy exchange between the snow and the atmosphere play an important role in the formation of all faceted snow types (McClung & Schäerer 2006). Resulting weak layer crystals are usually identified as class 4 or 5 according to Fierz’s international classification system.

In Montana, Birkeland states that there are three predominant near surface faceting occurrences: diurnal recrystallization, melt-layer recrystallization, and radiation recrystallization. Diurnal recrystallization occurs with clear cold nights and followed by clear sunny days. When this type of event occurs, temperature gradients can be greater than $200^\circ$C/m with large vapor gradients. With these parameters, the temperature gradient in the snow switches from having a relatively cool surface and warm basal layer at night to having a relatively warm surface and cool basal layer during the day, which
results in bi-directional faceting. Melt-layer crystallization is formed by either incoming radiation followed by snow or a rain-turned-into-snow event, which results in an ice crust covered by near surface facets (LaChapelle 1970; Colbeck 1982; Armstrong 1985).

Radiation recrystallization is found at the surface of a snowpack and often occurs during clear, calm weather. This metamorphoses is driven by a temperature gradient, which is predominately caused by the net change of incoming solar radiation at the near-subsurface and outgoing longwave radiation at the surface (Mock & Birkeland 1986; McClung & Schaerer 2006). On south facing slopes, solar radiation can have a warming effect below the surface of a few degrees (McClung & Schaerer 2006; Fierz et al. 2008).

2.3 Numerical Modeling Software

There are many different numerical modeling packages that have been developed to physically describe snow and snow metamorphism. The most well known and widely used are Snow Thermal Model (SNTHERM) (US Army Corps of Engineers, Engineering Research and Development Center), Crocus (Vionnet et al. 2012), and SNOWPACK (Bartelt & Lehning 2002), which all explicitly take into account layering (Vionnet et al. 2012; Bader & Weilenmann 1992) At Montana State University, RadThermRT was developed in conjunction with Thermoanalytics (Adams & Curran 1999).

SNTHERM uses a layered snowpack with constant parameters in each layer, thus finite difference method is used. These layers include snow, soil, and air. The governing finite differencing equations use a Crank-Nicolson center differencing method. Elements are added for snowfall or rain, then compaction is added. Next, solar radiation and surface fluxes are calculated resulting in water flow modeling. Sublimation and diffusion
of water are then calculated with an estimation of grain diameter. A thermal balance is then solved and any necessary liquid water or grain size adjustments are made to balance mass (Jordan 1991). A close counterpart to SNTHERM was DAISY before it was updated to SNOWPACK (Jordan et al. 2003; Vionnet et al. 2012).

SNOWPACK, developed in Switzerland, is used to predict snow settling, surface energy exchange, and mass balance. The governing differential equations are solved using an implicit Lagrangian Gauss-Seidel finite-element method. These differential equations deal with the conservation of mass, energy, and momentum. This finite energy balance allows new snow to be created by adding finite elements. Settling is described by treating snow as a viscoelastic material, and microstructure parameters are also modeled using temperature gradients and metamorphism routines (Bartelt & Lehning 2002).

Crocus, developed in France, incorporated with SURFEX (an algebraic image and animation visualization software (Labs & Holzer 2008) models mass and energy exchange between ground surfaces and the atmosphere (Vionnet et al. 2012). This model has a range of simulation types available, including modeling complex topography over an entire mountain range. Crocus is also a one-dimensional, layered, finite difference scheme. Each layer includes thickness, enthalpy, density, age of snow, dendricity, sphericity, grain size and slope angle. The numerical operation order is: input snowfall updates snow layering, which then leads to metamorphism; followed by compaction; and then wind drifting. Then, snow albedo is considered to calculate solar radiation transmission and surface energy balance. After the surface energy balance and
temperature profiles are updated, snow melt is calculated, water flow and refreezing is modeled and mass sublimation or deposition is calculated (Vionnet et al. 2012).

RadThermRT (RTRT) does not deal with melt. It only models snow temperature up to 0°C, but like SNTHERM, Crocus, and SNOWPACK a 1-D center difference energy balance is used to model temperature with complex topography. A complete discussion of RTRT is found in the next chapter.

2.4 First Principle Energy Balance

To numerically model first principle energy balance of snow pack so that surface temperature and near surface temperatures can be modeled, RadThermRT (RTRT) was used. RTRT is a commercially available program by ThermoAnalytics, Inc. that simulates energy exchange for complex parts, which has been modified for snow and complex terrain by Adams and McDowell (Adams & McDowell 1991; Adams & Curran 1999; ThermoAnalytics 2003; Adams et al. 2004). In RTRT, a terrain model is defined as a surface of triangle elements or facets with an underlying grid of nodes, see Figure 1.
Figure 1: Simple model of a column of snow (left) is the surface of the column, which is 1 m². The right figure represents the model nodes going through the snow. RTRT can have up to 20 layers.

These elements are defined as specific parts with each respective part having specific material properties. The definable material properties within terrain parts include part type: asphalt, foliage, snow, swamp, concrete, layered, soil, and water, which each have their own definable properties. In the scope of this work, foliage, soil type and snow were used. The parameters available here include foliage type, growth factor, cover factor, solar absorptivity, core temperature, surface moisture, and bulk moisture for foliage. For soil, parameter options include soil type, surface moisture, solar absorptivity, bulk moisture, core temperature, and the fraction of quart content. Snow parameters include type, condition, depth, and core temperature. The core temperature refers to the lower boundary condition, the bottom of the basal layer.

A view factor is computed and applied to each triangular facet. This view factor is determined by taking into account the following factors: geometric influences, including slope, elevation, terrain shadowing, obstructions by objects, surface interactions and reflections; as well as calculation of the zenith and azimuth of the sun;
and sky and air temperatures. This view factor is a constant and is used to calculate radiation exchange (Staples 2008). Other parameters include latitude and longitude coordinates, time zone, and elevation of the area.

For each triangular node, one-dimensional finite difference energy flux, $Q$, is calculated. The first law of thermodynamics states that energy is neither created nor destroyed. It can be described as the change in energy over time is equal to the sum of all the heat transferred into and out of the system plus power into the system as seen in equation 1:

$$\frac{\partial E}{\partial t} = \sum Q + \sum P. \tag{1}$$

However, work is not done on our system, so the problem is simplified to,

$$\frac{\partial E}{\partial t} = \sum Q. \tag{2}$$

$E$ is the total energy stored in the system, $t$ is time, and $Q$ is the total heat flux transferred to and from the system (Incropera et al. 2007). Therefore,

$$mc_p \frac{\partial T}{\partial t} = \sum Q, \tag{3}$$

where $m$ is the mass of the node, $c_p$ (kJ/kg·K) is the specific heat of the material, $T$ (K) is temperature, and $t$ is time (Marttila 1999). Total heat flux is:

$$Q = Q_{lw} + Q_{sw} + Q_{th} + Q_{sh} + Q_{cd}. \tag{4}$$
Figure 2: Schematic of energy exchange on a given slope. Shortwave (SW) radiation is incoming from the sun and is absorbed in the top few centimeters of the snow (see LHS of figure.) Sensible heat in the form of convection as well as latent heat can either be added or removed from the snow surface depending on temperature. Conduction occurs within the snowpack. This model assumes the conduction coefficient between snow and air is small enough to be negligible. Longwave interactions occur between the surface, surrounding topography (such as rocks, cliffs, and other snow surfaces), and clouds. There is also some SW interaction associated with complex topography.

\[ Q \text{ is the total heat exchange, which is the energy available to change the} \]
\[ \text{temperature of the snowpack (Morstad 2004) (Staples 2008), Figure 2.} \]
\[ Q_{lw} \text{ is longwave radiation, } Q_{sw} \text{ is shortwave radiation, } Q_{lh} \text{ is latent heat due to phase change, } Q_{sh} \text{ is} \]
\[ \text{convective heat, and } Q_{cd} \text{ is conduction (Adams, 2004a).} \]

\[ \text{The following discussion will predominately occur in energy flux [W/m}^{2}\text{].} \]

Therefore, equation (2) can be written as:

\[ \rho c_p z \frac{\partial \tau}{\partial t} = q_{lw} + q_{sw} + q_{lh} + q_{sh} + q_{cd}, \]
where $\rho$ is the density of snow, $c_p$ is the specific heat of snow, and $z$ is length when discretized. In this work, energy into the control volume is considered positive.

$c_p$ is calculated from Dorsey (1940)

$$c_p = 2.115 + 0.0077 * T, \tag{6}$$

c$_p$ is in kJ/(kg °C), and $T$ is in °C.

2.4.1 Radiation

All objects emit and receive radiation. Thermal radiation is considered to be between 100 nm and 100,000 nm, which includes ultraviolet (UV), all visible, and infrared (IR) spectrum, see figure 2 (Incropera et al. 2007). Frequency corresponds proportionally to energy carried by each photon:

$$E = h \nu. \tag{7}$$

$E$ is the energy per photon, $\nu$ is the frequency of the wavelength, and $h$ is Planck’s constant, $6.62 \times 10^{-34}$ m$^2$·kg/s. Distribution of energy from the sun can be described by Planck’s law:

$$E_\lambda(\lambda, T) = \frac{C_1}{\lambda^5 \left[ \exp \left( \frac{C_2}{\lambda T} \right) - 1 \right]}, \tag{8}$$

which describes the spectral emissive power as radiation varies with wavelength ($\lambda$) (Incropera et al. 2007). $C_1 = 3.742 \times 10^8$ W·μm/m$^2$ is the first constant of radiation and
the second constant, $C_2$, is $1.439 \cdot 10^4 \mu m \cdot K$. $T$ (K) is the absolute temperature of the black body (sun) at 5800 K.

It is estimated that at least 50% of longwave radiation is absorbed at the surface and 100% has been absorbed at 1cm (McClung & Schaerer 2006). Most shortwave radiation is reflected, but a non-negligible amount penetrates the snow surface and is absorbed, dependent on wavelength, density, grain size, and depth. Some of the shortwave is transmitted by light reflecting forward off of ice crystals. This can be seen at the bottom of any snow pit by noticing the blue hue of snow as well as seen by near surface heating of the snow.

Shortwave radiation penetrates the deepest in snow with large snow grains and high density (McClung & Schaerer 2006). And absorption increases at longer wavelengths as well as with old snow/snow melt.

Ninety-nine percent of shortwave radiation affecting the snow pack is solar radiation, and ninety-nine percent of terrestrial radiation consists of longwave radiation. The amount of absorbed shortwave radiation which is diffuse is 10 percent for bright days, 50 percent for cloudy days, and 100 percent for completely overcast days (Gray & Male 1987). These potential changes in the energy balance due to solar radiation absorption at depth along with outgoing longwave at the surface are what makes radiation recrystallization a unique and pertinent phenomena.

The energy delivered by radiation into the snow affects microscale metamorphism. A good description of this energy for shortwave radiation is an
exponential decay model, especially within a few centimeters of the surface (Meirol-Mautner 2004).

The total shortwave interaction in snowpack can be described as acting two ways in the snowpack: it is scattered or absorbed. Together, these sum to the extinction coefficient:

\[ \chi = \sigma + \kappa. \]  

\( \chi \) is the extinction coefficient, \( \sigma \) is the scattering coefficient, and \( \kappa \) is the absorption coefficient (Bohren, Craig F., Barkstrom 1974). \( \kappa \) is dependent on wavelength, grain-size, density, and impurities (Brun et al. 1989). This absorption constant for shortwave is an acceptable way to infer the spectral absorption through snowpack (Warren et al. 2006; Herbert Curl et al. 1972). The absorption is described with exponential decay model.

The base of this model is a Beer-Lambert-Bouger exponential decay equation (Herbert Curl et al. 1972; Bohren, Craig F., Barkstrom 1974; Schwerdtfeger & Weller 1971; Grenfell & Perovich 1981; Warren et al. 2006; Fierz et al. 2008):

\[ q_{sw} = q_{sw0}e^{-k_s z}, \]

where \( q_{sw0} \) is the incoming shortwave radiation, \( k_s \) is the attenuation coefficient of snow, and \( z \) is the snow depth. There is some contention that this model lacks accuracy at the very top of the snow because it does not take into account the increased scattering at the surface (Meirol-Mautner 2004). The shortcoming of this absorption description is
that it does not inherently take into consideration different absorptive properties for
different wavelengths of radiation, grain size, or density of snow, all of which are
imperative to describing attenuation. An empirical attenuation constant was developed
by Bohren and Barkstrom (1974),

$K_s = 0.84 \times \frac{\rho_s}{\rho_i} \sqrt{\frac{k_i}{d}},$  \hfill (11)

where $\rho_s$ (kg/m$^3$) is density of snow, $\rho_i$ is the density of ice, $k_i$ (m$^{-1}$) is the
attenuation coefficient of ice, and $d$ (m) is grain size, and the grains are assumed to be
rounded. Meirold-Mautner (2004) indicate that this model yields results that are too
high. However, this empirical constant does not take into consideration the dependence
that $k_i$ has on radiation wavelength nor the amount of energy of each wavelength that is
emitted from the sun. Also, the equation needs to be discretized so that it can be used in a
center differencing method format. A description of this process can be found in the
Methodology section.

2.4.2 Longwave Radiation

Absorption of longwave radiation, represented in eqn. (13) below as $q_{lw}$ (W/m$^2$),
occurs at the surface of all objects. The range of wavelengths for this type of radiation is
generally between 6,800 and 100,000 nm (Gray & Male 1987). However, there is some
variation in the definition of range for these wavelengths. For example “Snow and
Climate” cites the longwave range to be between 5,000 and 40,000 nm (Jordan et al.
2008). At the spectrum defined by Gray and Male, snow is nearly a black body
signifying that all incoming longwave is absorbed and the maximum thermal radiation
available is emitted (Wiscome & Warren 1980; Armstrong & Brun 2008). Based on the acquisition capabilities of sensors in the field, longwave is considered to be between 4,500 and 44,000 nm in this work. Warren, Brandt, and Grenfell note that absorption between these wavelengths are some of the highest, i.e. almost all energy is absorbed at the surface (Warren et al. 2006).

Emissive power of longwave radiation is described by an altered Stefan-Boltzmann’s law for materials that do not act as a perfect black body, see eqn. 12

\[ E = \varepsilon\sigma T_s^4. \]

\( T_s \) is the absolute temperature in Kelvin, and \( \sigma \) is the Stefan-Boltzmann constant (\( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \)), and \( \varepsilon \) is emissivity which is between 0 and 1 (Incropera et al. 2007; Staples 2008). Longwave radiation exchange between an element and the sky or other elements is described as:

\[ q_{lw_i} = F_{is}(q_{Li} - \sigma\varepsilon_i T_i^4) + \sigma \sum_{j=1}^{n} F_{ij}(\varepsilon_j T_j^4 - \varepsilon_i T_i^4), \]

\( q_{lw_i} \) is the longwave radiation flux, \( q_{li} \) is the incident longwave radiation from the atmosphere, \( \sigma \) is Stefan-Boltzmann’s constant, \( \varepsilon_i \) is the emissivity of the element \( i \), \( F_{is} \) is sky view factor of element \( i \), \( \varepsilon_j \) is the emissivity of element \( j \). \( F_{ij} \) is the view factor between elements \( i \) and \( j \). \( T_j \) and \( T_i \) are the absolute temperatures of element \( i \) and \( j \), respectively.
2.4.3 Shortwave Radiation

$q_{sw} \ (W/m^2)$ is the heat flux from shortwave (solar) radiation. Absorption of shortwave radiation is sometimes assumed to be negligible within the snow, but in Habicht, Ambach, and Grenfell, this was disproved (1981). Absorption is the greatest in the infrared spectrum (Grenfell & Perovich 1981). Computational details of shortwave will be discussed further in the methodology chapter, but the basic background will be discussed here. Within RTRT, $q_{sw}$ is described by:

$$q_{sw} = \alpha \left( \frac{A_p}{A} I_n + F_i s I_d \right) + \sum_{j=1}^{n} (B_{ij}(1 - \alpha_j) q_{sw} j).$$

(14)

$\alpha$ is absorptivity from 0 to 1. $A_p (m^2)$ and $A(m^2)$ are the projected apparent area and area of element $i$, respectively. $A_p$ is calculated for each time step to account for the sun moving across the sky. $I_n (W/m^2)$ is direct solar radiation, $F_is$ is the sky view factor of element $i$, and $I_d (W/m^2)$ is diffuse solar radiation. $B_{ij}$ is the view factor for elements $i$ and $j$ (Adams et al. 2004). Within RadThermRT, shortwave radiation is treated as a heat source at each finite difference volume of snow.

Absorptivity is equal to one minus albedo: $\alpha = 1 - \text{albedo}$. Fresh snow in RadthermRT is considered to have an albedo of about 95%, old dry snow has an albedo of 85%, and snow that has been rained on is estimated to have an albedo of around 40% . Within these generalizations, snow tends to have a higher albedo for shorter wavelengths, larger solar zenith angles, fewer impurities, smaller grain sizes, and deep snow depths (Adams et al. 2004). To give an idea of albedo, Table 1 shows the albedo of several materials (Gray & Male 1987). Particulate in the snow from soot, soil, etc. also
drastically affects the albedo/absorptivity. Any increase in foreign particulate manifests as greater increases in the absorptivity.

Table 1: Albedo for snow, ice, and sand from Gray and Male (1981).

<table>
<thead>
<tr>
<th>Material</th>
<th>Albedo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresh Snow</td>
<td>.75-.95</td>
</tr>
<tr>
<td>Old Snow</td>
<td>.40-.70</td>
</tr>
<tr>
<td>Sea Ice</td>
<td>.30-.40</td>
</tr>
<tr>
<td>Sand</td>
<td>.24-.30</td>
</tr>
</tbody>
</table>

Shortwave radiation is usually considered to be electromagnetic waves from about 200 nm to 2200 nm (Gray & Male 1987). Again, there are some agreement discrepancies here and another example of the variation in the published ranges considered shortwave radiation to be from 300 to 2,800 nm (Jordan et al. 2008). Shortwave radiation in this work will be considered to be between 300 and 2,800 nm, which is compatible with our field instrumentation. This covers the high end of the ultraviolet through to the low end of the infrared spectrum, Figure 3. The shorter wavelengths, higher energy, are absorbed less than the longer wavelengths, lower energy.
Energy absorption in this range is dependent on wavelength, size of snow grains, density, and age of snow. The lowest absorption occurs between 300-600 nm (the visible range) with 400 nm being the absolute lowest (Warren et al. 2006). What is considered a “moderate” amount of absorption is seen between 1,000 and 3,000 nm. The highest absorption occurs near the red and near-infrared wavelengths (600-1200 nm). Wavelengths in this range do not penetrate below a depth of about 45 cm (Warren et al. 2006). The radiation absorbed inside of the snowpack affects the energy absorbed for snow metamorphism, therefore both absorption and reflection are needed for energy balance (Meirold-Mautner 2004). Planck’s energy distribution describes the energy emitted from the sun. In turn, irradiance describes the radiant energy flux that is normal to the snow surface.

A ray tracing algorithm is used in order to calculate the area involved in radiation exchange. There is a constant component for the view factor, including the apparent area of the sky and surrounding elements that are seen. There is also an apparent direct solar component which is calculated at each time step, defined in RTRT. This algorithm
places a sphere at the centroid of each element that casts rays out with the sphere maximizing the number of rays normal or near normal to the direction of the element. The number of rays coming from the sphere can be changed in the RTRT GUI to affect accuracy and inversely the efficiency of the model. Shortwave radiation emitted from the sun is modelled as a black body.

2.4.4 Latent Heat

$q_{lh}$ describes latent energy flux due to phase change. This term is used to estimate the amount of energy expelled from or added to the control volume at the surface. This flux is only represented up to 0°C:

$$q_{ lh } = L_m F_m. \tag{15}$$

$L_m (J/g)$ is the latent heat of phase change, i.e. the change in enthalpy. $F_m$ is the mass flux. Mass flux can be described as:

$$F_m = 0.662 \rho_a U_a K^c \left( \frac{e_a - e_s}{p_a} \right). \tag{16}$$

$F_m (g/m^2 \cdot s)$ is mass flux, 0.662 is the ratio of the dry air gas constant over the water vapor gas constant. $p_a$ is air pressure (Pa). $U_a (m/s)$ is wind speed. $K^c$ is a dimensionless heat transfer coefficient. $e_a$ and $e_s$ (Pa) are the vapor pressure of air and the vapor pressure over snow, respectively (Adams et al. 2004). An in-depth discussion of the vapor pressures can be seen in Adams’ et al paper, “Modeling Snow Temperature in Complex Topography” (2004).
2.4.5 Convection

Sensible heat due to convection describes the flux that is a result of the temperature difference between the snow and air, and can be described as:

\[ q_{sh} = c_{pa}\rho_a U_a K_c (T_a - T_s). \]  

(17)

\( c_{pa} \) (J/g·K) is the specific heat of air, \( \rho_a \) is the density of air (g/m\(^3\)), \( U_a \) (m/s) is measured wind speed, \( K_c \) is a dimensionless heat transfer coefficient. Together \( c_{pa}, \rho_a, U_a, \) and \( K_c \) make up \( h_{avg} \) (W/m\(^2\)·K), which is the average convective heat transfer coefficient, \( T_a \) and \( T_s \) (k) are the temperatures of air and snow, respectively (Adams et al. 2004).

2.4.6 Conduction

Fourier’s law is used to model heat flux through the snow pack:

\[ q_{cd} = -k_{eff} \frac{d^2 T}{dz^2}. \]  

(18)

\( k_{eff} \) (W/m·K) is the effective thermal conductivity of snow, which includes conduction through snow’s ice lattice and pore spaces as well as vapor diffusion and any phase change within snow pack (Adams, personal communication, 2015). \( k_{eff} \) is negative because heat transfer is positive into the control volume but the surface is defined as unit normal out, i.e. positive heat flux requires a negative coefficient (Incropera et al. 2007). \( z \) is depth in the slope normal direction, and it is density that controls \( k_{eff} \) in RTRT. The equation used comes from Bader and Kuroiwa (1962) as,
\[ k_{\text{eff}} = 0.00007 + 0.007 \cdot \rho^2, \]

in units of cal/(cm sec °C), and then it was converted to

\[ k_{\text{eff}} = 419(0.00007 + 0.007(\rho \times 10^{-3})^2). \]

\( k_{\text{eff}} \) is an empirical expression in W/(m²K). The 419 coefficient converts calories to joules.

The first-law energy balance solution using the components discussed above was solved using a centered finite difference method of the resulting partial differential equation. It was solved using the Crank-Nicholson method (Marttila 1999). This method has no limits for the step size and is stable without being too computationally intensive (Kreyszig 2006). The boundary condition at the surface of snow is in terms of flux. \( \frac{dT}{dz} \) at the surface is zero, i.e. it is assumed that the conduction coefficient between the surface of the snowpack and the surrounding air is negligible due to a very small snow/air conduction coefficient.
3. METHODOLOGY

In this analysis, there were three key components. First, numerical code was written within RTRT to better handle attenuation of solar energy through the snowpack. Second, in the Subzero Science and Engineering Research Facility radiation recrystallization was grown and measured. Third, three weather stations were set up on Pioneer Mountain in the Yellowstone Club Ski area near Big Sky, Montana. These were used to collect meteorological inputs for numerical analysis as well as for snow temperature validation.

3.1 Cold Lab Experiments

In order to develop and observe radiation recrystallized layers, a snow box was constructed of 0.635 cm thick sanded birch veneered plywood. Its dimensions were 38.1 cm by 38.1 cm by 40.64 cm tall, and 0.635 cm. The lid was configured to slide over the snow to ensure a smooth and homogenous surface. A removable front face allowed the generated weak layer to be viewed and imaged, with a hyperspectral camera, from the side, Figure 4.
Figure 4: The snow box used in the lab (a) Shows the box with the sliding front and sliding top closed, (b) is the top sliding off, (c) is the box with the top removed, (d) is the front of box being removed, and (e) is the box without its front or top.
To set up a radiation recrystallization experiment, first the snow box was filled with snow sifted through a 2.6 mm mesh to maximize homogeneity. The snow was then allowed to sinter for at least 12 hours at -15°C prior to the 12 hour lab experiments.

Prior to initiation of the experiment, the top of the snow was smoothed in the snow box, and the ceiling of the cold chamber was scraped clear of surface hoar that developed on the very cold ceiling. Removal of this ceiling condensate maintains the coldest exposed surface to maximize longwave energy exchange through the coupling of the snow and ceiling.

Laboratory conditions for the radiation recrystallization growth were set at a -15 °C for the room, a -50°C ceiling, and a metal-halide solar simulation lamp (280-2480 nm) exposed the snow surface to approximately 695 W/m². These values were chosen based on field observations from the Yellowstone Club.

After the twelve hour period, the lamp was turned off, and the snow was allowed to sit for approximately 15 minutes. The newly formed weak layer was then gently buried with 3-5 centimeters of sifted snow and allowed to sit overnight. The next day, this snow profile was imaged with a hyperspectral camera. Photos of crystals from the weak layer as well as from random locations in the snow were taken with a microscope camera. Lastly, density profiles were taken from approximately the top, the middle, and the bottom of the snow profile.
3.2 Instrumentation and Data Collection

Over the 2013-14 and 2014-15 seasons, weather data was collected from three study sites at the Yellowstone Club Ski Resort. These data were used as inputs into the energy-balance numerical model as well as to check the results of the model.

The three study sites were chosen to assess the influence of solar radiation based on aspect. One, the American Spirit site, was chosen for its unobstructed view of the sky. A second plot was chosen for its direct southernly aspect, and a north site was chosen for being on the “shady” side of the mountain. As a result of the southern exposure, radiation recrystallization events occurred at the south study plot, and surface hoar events tended to occur on the north study site.

The site Yellowstone Club South (YCS) was located on a south facing slope of Pioneer Mountain at 45.2307 °N, 111.4422 °W, see Figure 5 & Figure 6. It had a slope of approximately 30 degrees, and the approximate elevation of the weather station was 2770 meters. It was located in a small meadow that was surrounded by trees on three sides with a small cliff band above it. Thermal images were also taken at this site.
Figure 5: Google Earth image of Yellowstone Ski Club on Pioneer Mountain. The north study plot can be seen pinned in the upper left corner. The American Spirit study site is shown with the pin towards the center of the image. And the south weather station is the pin at the bottom center of the image.

Figure 6: Location of the three weather stations used on Pioneer Mountain in the Yellowstone Club, near Big Sky Montana. (a) YC South (YCS) weather station at 45.2307°N, 111.4422°W (b) American Spirit (YCAS) at 45.2400°N 111.4429°W and (c) YCN at 45.247°N and 111.46°W (ACME Mapper 2013)
The second site, American Spirit (YCAS), was located on a ridge to collect unobstructed longwave and shortwave data. It is located at 45.2400 °N 111.4429 °W with an elevation of approximately 2680 meters. This site was chosen because of its unobstructed ridge location, which minimizes the effect of surrounding topography. American Spirit is approximately 1,150 meters north-east of YC South.

The third weather station, Yellowstone Club North (YCN), was located at 45.247°N and 111.46°W. It was a north facing slope at an elevation of approximately 2536 m. This station was in a clearing surrounded by mature trees on three sides that flattens out at the lower end of the site where another forested area begins. It also had a slope of about 30 degrees.

At YCN and YCS weather stations, Kipp and Zonen CMP 3 pyranometers were used to measure shortwave incoming and reflected radiation, between 300 and 2,800 nm. A Kipp and Zonen CGR 3 pyrgeometer was used to measure the longwave between 4,500 and 44,000 nm. Air temperature and relative humidity was measured with Campbell Scientific’s CS215 probe, snow depth was measured by a Nova Lynx 260-700 sensor. Snow surface temperature was measured by an Everest Interscience IR snow surface temperature thermometer. Wind speed and direction were measured by a Met
One 034A-LC anemometer. Data was collected every 30 seconds and averaged over every half hour.

At the lower end of the south study plot, thermal images were recorded using a Flir T440 (spectral range 7500-13000 nm), which was set in a tree stand. Images were taken every half hour. Post processing was done for each image, which included setting the emissivity (0.99), the distance to the object of interest, air temperature, optics temperature for instrument calculation (which was assumed to be the same as the air temperature), and relative humidity. The air temperature and relative humidity were taken from the YCS weather station.

At the American Spirit weather station, an Eppley precision infrared radiometer was used to measure incoming longwave radiation, and an Eppley precision spectral pyranometer was used to measure shortwave radiation. Data collected at this station was averaged over one hour increments. This site was managed by the Yellowstone Ski patrol who chose the longer time step average to save. Therefore, half hour increments between time steps were linearly interpolated.

Weather stations were maintained through collaboration with the Yellowstone Ski patrol and weekly visits to the sites by MSU students and faculty. The pyranometers and pyrgeometers were brushed free of snow, and the thermocouple arrays were adjusted so that the top thermocouples remained near the surface of the snow. This was done as needed.

Simulated weather data was imposed and measured in the subzero science and engineering cold chamber. Surface temperature was collected using an Everest Interscience 4,000 zl infrared thermometer. Eppley Lab Precision Infrared Radiometers
were used to measure incoming longwave radiation. An Eppley Precision Spectral Pyranometer was used to measure incoming shortwave radiation. Omega HMP60 relative humidity sensors and T type thermocouples were also used. During the experiment, data was collected every thirty seconds. When this data was used, the results were averaged over half hour increments.

3.3 Defining the Terrain Model

In RTRT, a terrain model of a slope or mountain of interest was defined in ArcGIS, a Geographic Information System (GIS). Four different types of data were used for data processing. A given model requires two types of information: elevation data and land coverage data.

1 m² data from filtered and unfiltered Light Detection and Ranging (LiDAR) were used to model slopes that were approximately 100 m² or less. The filtered LiDAR returns the elevation of the slope. The unfiltered LiDAR returns the slope with all artifacts found on the slope. In this case, this included trees.

24 m² digital elevation maps (DEM) along with vegetation maps from United States Geological Survey (USGS) Gap Analysis Program (GAP) data were used for mountain scale modeling. GAP data was actually 30 m², so USGS data was remapped to 30 m² data by sampling it down to 1 m² data and then resampling it back up to 30m². This was done so that the two data types would have the same overlay. Each element was assigned a part type and square elements were split into two triangular elements, and these elements are draped over a DEM in RTRT. An RTRT rendering of this can be seen in Figure 8.
A detailed description of data processing in ArcGIS can be found in Appendix A. ArcMap was used to build two ASCII (.asc) files. The first ASCII file contains elevation data in meters. The second ASCII file contains terrain type data. For example, for the LiDAR data, three types of terrain were identified. Type 1 was identified as the default terrain of snow, type 2 was identified as vegetation, and type 3 was identified as cliffs.

To identify vegetation, a raster calculator was used to determine the difference in height between the filtered and the unfiltered LiDAR. Then a conditional statement was built declaring that differences in elevation between the two maps greater than one meter to be considered trees and elevation changes less than one meter were set to remain the default terrain (snow), a rendering of this can be seen in Figure 9.

Cliffs were also calculated using a conditional statement. Here, the unfiltered LiDAR was used to write a conditional statement after the raster slope calculation tool was used. Any slope greater or equal to 50 degrees was declared to represent a cliff, terrain type 3.

For the 30 m data, the geometry of treed terrain is lost. With 1 m data, a height and shape of trees were included. However, for this data, no tree height is included. The only effect trees have in this case are that the part type contains material properties of a treed area.

The 30 m data files were built from 24 m DEM from NRIS from 2002, and 30 m Montana Land Cover Framework (GAP) data from 2010. A limitation to consider for this data is that it has not recently been updated for work done at the Yellowstone Club, and so current ski runs and glades do not show up in this model, Figure 9.
Figure 8: (i) RTRT rendering of 30 meter DEM of Pioneer Mountain where all three study sites are located. This model covers just short of 20 km². (ii) Zoomed in rendering of YCS weather station meadow

Figure 9: RTRT rendering of 1 m LiDAR data for south facing study plot.
The 30 m DEM was used for the base elevation calculation, and the GAP data was used for both tree calculation and for the cliff calculation. The GAP data consisted of 24 data types, including agricultural land; various types of grass land; swamps; cliffs, canyons, and talus; developed land, water, and many types and stages of trees. This was built into a conditional statement to reduce the types down to cliffs, canyons, and talus; trees that would not be covered by snow; and everything else that would be covered by snow, i.e. three terrain parts. After this conditional statement was processed, a second conditional statement was written to define the cliff component of the cliff, canyon, and talus part. This conditional statement was written for 45 degrees in order to include some of the small cliff bands that would otherwise be excluded.

After these three terrain types were built and exported, the two .asc files were exported to RadThermRT using the RTRT GUI. Within the GUI, each part was checked and any discrepancies were filled in or deleted. Terrain parameters, also discussed in the background, were filled in according to each part as well as defining boundary conditions, time steps, and resolution of calculations. Instructions on how to set-up and run RTRT with layer thickness and density profile can be found in Appendix B and Appendix C, respectively.

Twenty layers are assigned for each element of a snow pack. Tree elements have 13 layers. Boundary conditions include a core temperature found at the base of the snowpack, the air temperature at the surface of the snow pack, and insulation of conduction with adjacent elements at the sides (Marttila 1999; Staples 2008).
For modeling the lab in RTRT, a two element model was used. It was assumed that the box was 1 m$^2$, so each element was 0.5 m$^2$. The two elements were built as being flat with no view of each other, and they had no slope.

3.4 Meteorological Input

Weather files were built using data from the three weather stations. The weather data used specifically for this included air temperature, shortwave radiation, long wave radiation, wind speed, and wind direction. There is also a column in the original weather file for rain rate, but precipitation was not considered in this modeling effort.

3.5 RTRT Coding

A center differencing method was used to code the shortwave penetration into snow. A constant albedo for all wavelengths was used, which is an adjustable parameter within the RTRT GUI. A copy of the code can be seen in Appendix D with a sample calculation of energy absorption in Appendix E. Then, considering the shortwave energy that does enter the snowpack, a system of wavelength bins and snow layers were used to define the energy penetration through the snow pack. Thus, the snow attenuation constant was written as a function of ice attenuation coefficients for bins of wavelengths, layers of density, and position into the snow, seen in equation 21:

\[
K_s(\lambda, l) = 0.84 \cdot \frac{n_r(l)}{\rho_i} \cdot \sqrt{\frac{k_i(\lambda)}{d}}.
\]
\( \lambda \) is the wavelength and \( l \) is the layer. \( \rho_s \) is the density of snow, \( \rho_i \) is the density of ice, which is set to a constant 917 kg/m\(^3\). \( k_i \) is the attenuation coefficient of snow and is taken from Warren, Brandt, and Grenfell (Warren et al. 2006) and from Grenfell and Perovich (Grenfell & Perovich 1981). \( d \) is the rounded grain size.

Eight bins were analyzed with twenty density and thickness layers. After the absorptivity coefficient is taken into account, the remaining energy is broken into eight bins based on wavelength and energy. This resulted in an equivalent energy for each bin, which was calculated as a percentage of irradiance from a Planck’s distribution and albedo.

Planck’s distribution, as discussed in section 2.4.1, quantitatively describes how much energy from the sun is being emitted. However, not only is a certain amount of energy available to be absorbed, the wavelengths are not absorbed equally across all of the bandwidths. The irradiation reaching the surface of the earth based on bandwidth has the same shape as Planck’s distribution but does not have the same values, which depend on location, season, and time. Data from the National Renewable Energy Laboratory (NREL) on a day in December in the Big Sky, Montana area was used. This data determined the amount of energy (assuming that it was an average constant amount is a sufficiently close approximation) that was available to be absorbed per bin. Therefore, the amount of energy reflected was calculated as a percentage so that absorption could be changed within the RTRT GUI, and the respective changes would be made within the absorption code written for RTRT. Irradiance, shown in Figure 10, was then considered
with respect to albedo, Figure 11. The resulting layer fraction, an example of 8 bins, is shown in Table 2.

Figure 10: National Renewable Energy Lab (NREL) irradiance data with a two fitted curves. These curve fits were used to calculate the percentage of energy available in each bin before snow albedo is considered.
Figure 11: Albedo for a grain radius of 100 um, solar zenith of 30 deg, and a semi-infinite depth (Wiscome & Warren 1980).

Table 2: The energy available to be absorbed at the surface of the snowpack. The total energy available sums to 1.

<table>
<thead>
<tr>
<th>Bin</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>300-550</td>
<td>550-800</td>
<td>800-1050</td>
<td>1050-1300</td>
<td>1300-1550</td>
<td>1550-1800</td>
<td>1800-2050</td>
<td>2050-2300</td>
</tr>
<tr>
<td>Band (nm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>0.048</td>
<td>0.099</td>
<td>0.247</td>
<td>0.204</td>
<td>0.217</td>
<td>0.118</td>
<td>0.053</td>
<td>0.013</td>
</tr>
<tr>
<td>Available</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, the original shortwave data was broken into bins and $k_s(\lambda,l)$ was applied to each bin through the layer. Thus, these layer fractions drive the temperature calculations of the snow, especially in the first centimeter.

An example of the parameters used for modeling a lab test can be seen below in Table 3. In the lab, density measurements were taken after the experiment. In the field, density profiles were taken to make some estimations for density. A resulting absorption profile by layer can be seen in Table 4.
Table 3: Density and layer thickness inputs for RTRT. Density is in kg/m^3 and thickness is in meters.

<table>
<thead>
<tr>
<th>qdensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>175.0, 175.0, 175.0, 175.0</td>
</tr>
<tr>
<td>175.0, 175.0, 175.0, 175.0</td>
</tr>
<tr>
<td>200.0, 200.0, 200.0, 200.0</td>
</tr>
<tr>
<td>224.0, 224.0, 224.0, 224.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>qthickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005</td>
</tr>
</tbody>
</table>

Table 4: An example of the absorption, out of 1. Note that the thicknesses are not equal and layer 0 (surface) considers only half of the layer thickness.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (m)</th>
<th>Energy Absorbed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.005</td>
<td>0.236067</td>
</tr>
<tr>
<td>1</td>
<td>0.005</td>
<td>0.301037</td>
</tr>
<tr>
<td>2</td>
<td>0.005</td>
<td>0.165571</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>0.093649</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
<td>0.054894</td>
</tr>
<tr>
<td>5</td>
<td>0.005</td>
<td>0.033605</td>
</tr>
<tr>
<td>6</td>
<td>0.005</td>
<td>0.021616</td>
</tr>
<tr>
<td>7</td>
<td>0.005</td>
<td>0.014652</td>
</tr>
<tr>
<td>8</td>
<td>0.005</td>
<td>0.010453</td>
</tr>
<tr>
<td>9</td>
<td>0.005</td>
<td>0.00781</td>
</tr>
<tr>
<td>10</td>
<td>0.005</td>
<td>0.006451</td>
</tr>
<tr>
<td>11</td>
<td>0.005</td>
<td>0.005407</td>
</tr>
<tr>
<td>12</td>
<td>0.005</td>
<td>0.004345</td>
</tr>
<tr>
<td>13</td>
<td>0.005</td>
<td>0.003575</td>
</tr>
<tr>
<td>14</td>
<td>0.005</td>
<td>0.002996</td>
</tr>
<tr>
<td>15</td>
<td>0.005</td>
<td>0.002687</td>
</tr>
<tr>
<td>16</td>
<td>0.05</td>
<td>0.009839</td>
</tr>
<tr>
<td>17</td>
<td>0.09</td>
<td>0.01</td>
</tr>
<tr>
<td>18</td>
<td>0.09</td>
<td>0.005281</td>
</tr>
<tr>
<td>19</td>
<td>0.09</td>
<td>0.010067</td>
</tr>
</tbody>
</table>

There are two other useful aspects to using this layering methodology. First, it allows the user to constrain the concept of homogenous snow to homogeneity within a given layer, and secondly, the layers allow one to control the resolution of the model.
For RTRT, the lab weather input files and the RTRT GUI had to be altered due to the variations between the lab environment and nature. The weather input files were written for 36 hours. The first 24 hours of the file were meant to reproduce the conditions of the snow while it was in the lab before the experiment began. To do this, the first few measurements of the room recorded by the agilent before the full spectrum lamp was turned on were used for the entire 24 hours. This included measurements of the room’s air temperature, the incoming shortwave radiation, longwave radiation, and relative humidity. Then, for the 12 hours that the lamp was on, averaged 30 minute data was applied. In RTRT, the sun always goes across the sky, but in the lab setting, the lamp does not move. In order to approximate this in RTRT, the longest day of the year, June 21, at the North Pole at 90 degrees north 111 west was used to simulate the twelve hour experiment. The ground elevation was set to 1468 meters, which is the elevation of Bozeman, Montana. The YCS RTRT GUI global position at the lower left corner was at 45.23 degrees north and 111.23 degrees west. The ground elevation was set at 2530 meters. The time step was set for 30 minutes. The YCN RTRT GUI global position was set at 45.2 degrees north and 111.3 degrees west with a ground elevation of 2530 meters. The time step for YCN was set to 30 minutes.

The Pioneer Mountain RTRT platform was written to use a one hour time step and the 30 meter data. This large step was chosen because of the larger resolution of the entire mountain data and because of the size of the file. Even at the 30 meter scale, the model size was very large. The boundary condition at the basal layer was -1°C. The lower left hand global position was written to be 45.21 degrees north and 111.47 degrees west with a ground elevation of 2600 m. For the YCS, YCN, and Pioneer Mountain
models, 24 hours of data was run before the day of interest began to ensure temperature equilibrium before processing began.
4. RESULTS AND DISCUSSION

In this section, numerical surface temperature, temperature gradients results with respect to depth, and mass flux will be discussed for lab work, one meter YCS and YCN and the 30 meter resolution Pioneer Mountain RTRT models. First, however, the effect of shortwave radiation is shown as well as the effects of density and grain size in the empirical absorption coefficient equation. The field data discussion encompasses four parts: thermal imaging, a winter radiation recrystallization event, a spring radiation recrystallization event, and spatial variation.

4.1 Attenuation Parameter Results

Shortwave radiation had a drastic effect on surface temperature and temperature gradients. Using laboratory conditions, Figure 12 shows the calculated effect that shortwave radiation can have on the surface temperature of a snowpack. The maximum temperature difference was 5 °C at 11 hours into the experiment. Note that the line describing “no shortwave” does not appear to be at a steady state like one would expect. This variation was due to the shortwave radiation being set to zero in the weather file that was originally compiled from a lab experiment. i.e. there is a slight change in the longwave input due having the lamp heat the ceiling a bit, that still exists in both weather files.
Figure 12: Modeled surface temperature of a lab experiment for which shortwave was and was not accounted. This model was run for 24 hours at steady-state before the graphs shown. Therefore, the shortwave result starts warmer due to an iterative time step between hours 23:45 and 00:00.

Figure 13 shows the effect that shortwave radiation had on the temperature through depth. When the experiment was initialized (at t=0), there was little to no difference between the SW on and the SW off scenarios, but after the experiment ran for twelve hours, the maximum subsurface temperature difference was at 9°C at a depth of 2.5 cm. In addition to the temperature variance, there was also an important difference in the shape of the gradient. With the model where SW was not included, there was no knee-shaped temperature gradient.
Figure 13: Depth profile with and without SW turned on for a one element, lab based model. SW00 and NoSW00 are the depth profiles after the steady-state initiation of the model. SW12 and NoSW12 is the depth profile 12 hours after SW00 and NoSW00.

Absorptivity controls the amount of SW available to be absorbed by the snow pack, and small changes in absorptivity can change the temperature results by several degrees. Figure 14 shows the effect of different absorptions using 1 mm grain size (coarse grain assumption) and 100 kg/m$^3$ density snow assumption. For example, the temperature difference between 10% absorptivity and 7% absorptivity was about 0.5°C at the beginning of this experiment, and it was almost two degrees different 5 hours into the experiment. The absorptivity also affects the magnitude and the shape of the temperature vs depth profile, Figure 15. The maximum gradient was seen using the an
absorptivity of 0.09 at 1.2°C/cm but the maximum gradient seen total is seen using an absorptivity of 0.1 at 1.5 cm below the surface at 0.8°C/cm. The smallest gradient is seen with the absorptivity of 0.07 at only 0.6°C/cm. The trend of these results was to be expected because with less total energy available, the differences in energy absorption are smaller.

Figure 14: Absorptivity for 100 kg, 1mm snow. Absorptivity in this range (0.1-0.07) all represent what would be classified as fresh snow.
Figure 15: Temperature gradient through depth of snow that is modeled as 100 kg/m$^3$ density, 1mm snow. The range of absorptivities shown all fall within the fresh snow classification for albedo.

The attenuation coefficient is what controls shortwave radiation absorption through the depth of the snowpack. Within the scope of the absorption coefficient, estimating material properties—density and grain size— with sufficient accuracy is challenging and impacts the temperature results by several degrees as well as the curve shape. Also, modeling impurities and perturbations effects are a challenge (Askebjer et al. 1997), but homogeneity within the layer is assumed for this modeling effort.
Figure 16: RTRT results of the surface temperature for theoretical homogenous snow with densities of 50, 100, 150, 200, and 250 kg/m$^3$, respectively. At the surface, lower density snow has much lower solar radiation absorption with comparable latent heat, long wave radiation, and convection values.

Figure 16 illustrates the effect of density on a twelve hour energy balance problem. Before the temperature results reach steady state, at one hour into the experiment, the surface temperature for the 50 kg/m$^3$ column of snow was about -8.5 °C while the temperature of the 200 kg/m$^3$ snow column was approximately -7.8°C. The transient solution for the lighter column of snow showed 13.3 watts leaving the surface volume compared to 3.7 watts entering the system. The higher, 200 kg/m$^3$, density column resulted in 16.4 watts leaving the surface volume with 26.88 watt entering the system. The energy balance component that inhibited the heating of the 50 kg/m$^3$ column
of snow was atmospheric convection coupled with smaller input fluxes from solar radiation. Convection comprised 30% of the total flux occurring one hour into this modeled experiment. For the 200 kg/m$^3$ snow column, convection comprises about 21% of the total flux occurring in this surface volume.

Also, most of the heating at the surface as well as the top two centimeters subsurface is due to radiation. For the 50 kg/m$^3$ density snow, 37% of energy flux was due to solar radiation. The flux due to solar radiation for the 200 kg/m$^3$ snow was 57% of the total flux taking place within the surface volume.

Conversely, at the first subsurface node —0.5 cm below the surface— at one hour for the lowest density snow column, the temperature was calculated at -1.8°C the 11.4 watts entered the control volume with 3.8 watts leaving. At the first subsurface node, the temperature was -9.6°C and 26.8 watts were calculated as incoming and 16.4 watts were calculated as outgoing from the control volume. At the first subsurface node, for the 50 kg/m$^3$ snow column 74% of flux is due to solar radiation—11.4 watts. For the higher density column both conduction and shortwave radiation added energy to the control volume during this transient stage for the higher density column of snow. 96% of energy added to this control volume was due to solar radiation—or about 7.25 watts.
Figure 17: The effect of snow density on temperature gradient throughout a snowpack. One mm grain size 12 hours into the experiment is shown here.

The steady state result for increased subsurface cooling for lower density snow can be seen in Figure 17, which represents the final time step in this model and long after the system has reached a steady state. The temperature difference between the 50 kg/m$^3$ density column and the 200 kg/m$^3$ column at the surface is about 4.3°C. The difference at 0.5 cm below the surface is only 1.7°C, but this is due to the modeling restriction that any temperature that goes above zero is set to 0, and the extra energy available from this is considered in the latent heat term. However, at 2.5 cm below the surface, which is the
first control volume available for this scenario where the low density snow column does not reach zero, the difference in temperature when compared to the 200 kg/m$^3$ snow is 10.2°C.

Figure 18: Surface temperature results for one element homogenous snow for four different densities.
Figure 19: Temperature vs depth results from RTRT for four different grain sizes at the last numerical time step, i.e. 12 hours into the experiment.

Optical grain size, which is assumed to be spherical in this work also effects the temperature profiles of the snow pack (Askebjer et al. 1997; Grenfell et al. 2011). Figure 18 and Figure 19 illustrate surface temperature and the depth temperature profile, respectively. The surface temperature difference between the four grain diameters shown is only about 0.9 °C at the last time step. However, the difference in surface temperature results is significant. At 0.5 cm beneath the surface, the temperature difference is about 3.7 °C at the last time step. As grain size increases, net conduction at the surface as well
as subsurface decreases. Surface convection decreases with increased grain size as well as longwave radiation. Solar radiation decreases with increased grain size at the surface but increases subsurface. The difference in solar input at the surface between 0.2mm and 2.0 mm is about 10 watts at the surface and 7 watts subsurface. This increase in subsurface heating due to solar radiation is what drives a strong gradient in the high grain size and low density snow columns.

4.2 Effects of Weather Inputs

RTRT was used to numerically evaluate the surface temperature and the temperature gradient through the snow pack for the field, shown in Figure 20. The RTRT model of a small slope used 1 m LiDAR data: both the north and the south study plots were modeled in this manner, Figure 20 (i). 24 m DEM were rescaled to 30 m and used for mountain scale Figure 20. Here, the importance of the location of weather stations on the mountain, weather data processing, and element size will be illustrated and discussed. The results of the two scales as well as using different weather stations will also be shown throughout the results section, but for this section, data is used from March 20 and 21 of 2014.
Figure 20: (i) RTRT rendering of 1 meter resolution of YCS study plot. (ii) RTRT rendering of 24 meter resolution of Pioneer MTN. The white triangles in each represent the element chosen to plot surface temperature of each

For these results, density measurements were taken from the field and input into RTRT. Table 5 shows the layer thickness and density profile of the RTRT input file. The density was measured at approximately the top, bottom, and middle of the snow pack so the exact layer profile is an estimation.

Table 5: Density and thickness profile for RTRT YCS numerical analysis. Density is in kg/m$^3$, and thickness is in meters.

<table>
<thead>
<tr>
<th>density</th>
<th>100.0, 100.0, 100.0, 100.0, 100.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>056.0, 056.0, 056.0, 056.0, 056.0</td>
</tr>
<tr>
<td></td>
<td>056.0, 056.0, 056.0, 056.0, 075.0</td>
</tr>
<tr>
<td></td>
<td>075.0, 100.0, 100.0, 125.0, 125.0</td>
</tr>
<tr>
<td>thickness</td>
<td>0.005, 0.005, 0.005, 0.005, 0.010</td>
</tr>
<tr>
<td></td>
<td>0.010, 0.010, 0.010, 0.010, 0.030</td>
</tr>
<tr>
<td></td>
<td>0.050, 0.050, 0.050, 0.050, 0.050</td>
</tr>
<tr>
<td></td>
<td>0.050, 0.050, 0.050, 0.250, 0.250</td>
</tr>
</tbody>
</table>
Figure 21 and Figure 22 show RTRT’s numerical results in conjunction with the measured result from the south study plot. Using weather input data exclusively from the study plots (YCS and YCN, respectively) resulted in the best agreement with the measured results. The absorptivity for this was 0.09 and the grain size was 1.5mm, coarse grained, for the YCS results. The temperature results using the local weather stations followed the measured temperature very closely and also represented the local maximums and minimums. For YCS, the result does not quite reach the daily maximums, and this is due to convergence criteria needed for RadThermRT--the grain size was increased by 0.5 mm. If the grain size was not increased, the solution did not converge but the result (not shown) did match up with daily temperature maximums almost exactly. For YCN, the results match almost perfectly with the measured result using a 0.5 mm grain size (medium grain) and a 0.09 absorptivity.

Using the American Spirit data for the longwave and shortwave radiation and YCS for all of the other weather inputs (YCS/AS), the solution is very close to YCS measured data only without the small local perturbations, and it runs cooler at night. The grain size for this experiment was 0.5 mm and the absorptivity was 0.1. The American Spirit and YCN (YCN/AS) result, like the YCN only result, also shows good agreement to the measured data. However, it does have a time lag in the results, which could be due local climate differences over the mountain.

This combination of weather inputs was considered for two reasons. First, based on the location of the weather stations at both the north and the south study plot, it was thought that the longwave and shortwave sensors would be reading energy exchange not only from the sky but also from the topographic surroundings. To combat this, American
Spirit was placed so that radiation interactions due to topography would be minimized. Secondly, longwave and shortwave radiation are not measurements that are usually measured in the field, so if one station could be used for a large area, even just for the radiation components, that alone would make this model more useful.

Finally, the results were calculated using just weather inputs from American Spirit (Am. Spirit). Absorptivity for this solution was set at 0.1, and the grain size used was 0.5 mm. For both the north and the south results, the temperature results run too cool during the day and too warm at night. Some of the local detail is also lost in this calculation due to the one hour averaging rather than the half hour averaging that the north and south study plot stations use.

Figure 21: Numerical and measured results using 1m LiDAR data using various weather inputs.
Figure 22: Numerical and measured surface temperature results using 1m LiDAR data using various weather inputs.

Figure 23 shows the half hour and one hour average longwave results that were measured for the three weather stations used in this work. As would be expected, the LW results from American Spirit consistently show the smallest amount of longwave measured. A lower longwave value equates to a colder calculated sky temperature. Therefore, the higher values of longwave do suggest surface interactions as well as longwave from the sky. This is particularly apparent at the north study plot, but the maximum difference in the measured results is only about 30 W/m². The American Spirit results also have a time lag in the results. This is primarily due to the one hour average time step, but the location may also have an effect due to the movement of cloud cover over the mountain.
Figure 23: Measured longwave for south facing study plot (YCS), north facing study plot (YCN), and the American Spirit study plot (YCAS). The unobstructed view of the sky at YCAS results in lower longwave readings compared to the two study sites surrounded by trees.

The shortwave results have a drastic change at the different locations. The south study plot has the highest readings, and the north study plot has the lowest readings with the American Spirit readings in between. The south readings are almost twice the magnitude of the American Spirit readings, and the American Spirit results are twice as high as the north study plot readings. This explains why, when using radiation from American Spirit, the low south facing maximum temperatures and the high maximum temperatures at the north study plot (refer back to Figure 21 and Figure 22).
Figure 24: Measured shortwave for YCS, YCN, and YCAS. As expected the most radiation from the sun is seen at the south facing weather station with the north facing weather station reading lowest shortwave radiation.

One further illustration of these results can be seen in Figure 25. These results are from the 30 meter DEM results of Pioneer Mountain instead of the 1 meter results discussed above. The first stark result is that there is very little temperature variation between the north and south plots. During the day on March 21st, there is one degree of separation between the two results. However, the north facing result follows the measured north temperature during the day pretty well –within one or two degrees Celsius, but it runs up to five degrees warmer during the night. The south numerical
result runs low during the day, around ten degrees lower during the day on the 21st, and it also runs warmer at night due to variation in measured radiation input.

Figure 25: Calculated Pioneer Mountain (30 meter resolution) surface temperature results and measured temperature at north and south study plots.

These weather inputs also affect the temperature profile through depth. Figure 26 and Figure 27 show depth results using the same parameters as the results discussed above. The highest temperature gradients for the south study plot occurred using YCS, and the smallest gradient was seen using the American Spirit data. As would be expected, the opposite of this was true for the north plot: YCN weather files result in the lowest temperature gradients seen, American Spirit results in the highest temperature gradients seen, and YCN/AS lies in the middle. Another important result seen here were some false positives for the knee-shaped temperature gradients. A positive knee-shaped
temperature gradient is defined by being cooler at the surface, warming subsurface, and then cooling closer to the ground. This was definitely true for the north study plot and was probably true for the south study plot. This will be discussed further in later sections.
Figure 26: Depth vs temperature profiles for 6:00 AM, 2:00 PM, and 6:00 PM for the 1 m LiDAR at the south study site on March 21. The maximum subsurface temperature is seen at 2:00 PM with RTRT assuming a maximum temperature of 0° C.
Figure 27: Depth vs temperature profiles for 6 AM, 2 PM, and 6 PM for the 1 M LiDAR at the north study site on March 21.

Figure 28 and Figure 29 show spatial results for both surface temperature and temperature profiles. For the surface temperature, using American Spirit data with the
Pioneer Mountain platform, there is about 4 degrees of difference. There is also about a 2 degree temperature difference in the depth profiles.

Figure 28: Surface temperature variation using one weather file input and 30 m DEM. On the day when an event was modeled, (the second day), a temperature difference of a few degrees is seen. However, one would expect a greater difference during all day light hours due to aspect, self-shadowing, and elevation.

Figure 29: Spatial depth profile on March 20 at 2:00 PM using RTRT with 30 meter resolution.
Part of the complexity of energy balance in snow is the precision and accuracy of data inputs. Because snow is always at a high homologous temperature relative to its melting point, it has a huge variation in material properties and all three phases of water can often be found in and around snow. This in and of itself makes measurements difficult. In addition, there are many other considerations when taking weather measurements. For example, the south weather station site radiation always reads much higher than the American Spirit site, (locations and data collection to be discussed later), and this is probably due to both its southerly aspect and radiative interaction with the topography around the sensor. In addition to material property considerations, there are also collection considerations. In the lab, measurements were taken every 30 seconds and could be averaged over any time step that one wished. However, the American Spirit weather station was managed by the Yellowstone Ski patrol and was averaged over an hour, and MSU graduate students managed the north and south sites. These complexities also come into play when taking measurements to compare with numerical modeling results.

4.3 Lab Results

Radiation recrystallized facets are consistently grown in the lab. The results of two experiments are shown here: One experiment was executed by myself and one was executed by MSU PhD candidate David Walters. Temperature was difficult to measure accurately in these experiments because the solar lamp heats up the thermocouples (radiation contamination), and therefore, it was expected that the RTRT depth results would run cooler than the measured results. Figure 30 (i) and (ii) shows the resulting
radiation recrystallized crystals of one experiment. For this experiment, 700 W/m\(^2\) was measured at the surface of the snow from the metal-halide lamp, and the air temperature of the room was held at -15°C with a ceiling temperature of -50°C. However, when the measured longwave results were used to calculate the sky temperature, the results ranged from -18 to -12.5°C. This was probably due to the LW sensor also measuring longwave from the walls, which are much warmer than the ceiling; heat of the glass of the lamp; and frost on the ceiling.

The resulting RR layer was very thin. The thickness of the layer was about 2 mm for all of the experiments grown in the lab. Figure 30 (iii) shows a representative crystal sample of the sifted snow. The rounded grains were sifted for a homogenous snow result, but as can be seen in the hyperspectral image in the snow was not completely homogenous, and there is some layering due to how the snow was sifted and spread within the box. This heterogeneity is inherent due to how difficult it is to realistically sift in even layers of snow let alone sift with no layering at all. Despite this heterogeneity in layering, a definite thin layer of radiation recrystallization can be seen, Figure 31.

Figure 31 shows a hyper spectral image of the snow, and the resulting wavelength analysis. The change in brightness in both the image and the graph can be seen showing the change in wavelength absorption, scattering, and transmission at the different resulting grain shapes. The radiation layer has an increase in scattering and transmission when light reaches the RR layer.
Figure 30: Microscope images of lab experiment on June 10, 2014. (i) and (ii) are radiation recrystallization crystals grown during the experiment. (iii) is a representative snow crystal from beneath the radiation recrystallized layer. Grid seen is 2 mm

Figure 31: Hyperspectral (320-960 nm) image of buried radiation recrystallization layer and graph of brightness. On the graph, perturbation just after -20 mm corresponds to the radiation recrystallized layer seen in image.

The layer profile and density profile used in RTRT for the lab experiments can be seen below in Table 6. The density profile came from measurements in the lab, and the thickness profile was chosen to calculate depth temperatures at the same resolution as the thermocouples, i.e. 0.5 cm apart for the first 10 centimeters. The bottom four layers were expanded so that they included the full depth of the box in the calculation.
Table 6: Density and layer thickness profile for modeling lab data. The position in density corresponds to the thickness position. Grain size was 1 mm.

| qdensity  | 175.0, 175.0, 175.0, 175.0, 175.0 |
|           | 175.0, 175.0, 175.0, 175.0, 175.0 |
|           | 200.0, 200.0, 200.0, 200.0, 200.0 |
|           | 224.0, 224.0, 224.0, 224.0, 224.0 |

| qthickness | 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.005, 0.010, 0.030, 0.090, 0.090 |

Figure 32: Percent of energy absorbed per layer thickness corresponding to Table 6. Each layer is 0.5 cm until the last four layers, which are 1, 3, 9, and 9 cm thick, respectively for a 30cm thick snow pack.

When comparing the RTRT results with the measured data there are several challenges to consider. Snow properties are difficult to measure, (as can be seen in Figure 33), the surface temperature was measured both with a thermocouple at the surface of the snow and with an infrared thermometer. The surface thermocouple is consistently about 4°C warmer than the RTRT results. The temperature of the IR sensor
and the RTRT results also differ by 4°C when the IR sensor result is oscillating, but after 6 hours, the calculated and measured results are within 0.5°C. The sensor reading is not stable during the first six hours. Some of this is probably due to going in and out of the cold chamber and also disturbing the snow box by changing its position some for hyper spectral imaging. Thermocouples in direct sunlight in snow warm more than the snow due to different material properties. Thus, it is to be expected that the near-surface thermocouple ran warmer than the either the infrared thermometer or the numerical result. However, even though the general trend of the infrared thermometer makes sense, the temperature variation in the first 6 hours, was not expected.

Figure 33: Surface temperature of lab experiment that was carried out on June 10, 2014. TC0 represents the thermocouple closest to the surface of the snow in the lab. STSensor represents the IR temperature sensor pointed at the snow, and RTRT represents the surface temperature as calculated by RadThermRT.
The first ten centimeters of the lab snow profile are shown in Figure 34. Again, the thermocouples from the lab consistently measure warmer than the numerical model, especially in the first two centimeters of the snow pack. Both the measured and calculated data show the trending knee-shaped gradient at approximately the same time. With the very high shortwave input, it was expected that the positive, knee-shaped temperature gradient would be much greater. However, the LW measurements in the lab proved to be much higher (resulting in a much warmer ceiling) than was expected. This, of course, really affects the shape and size of the temperature gradient within the snow.

Figure 34: June 10, 2014 RR metamorphism in lab. TC02 and RT02 represent the hours lapsed. TC is thermocouple array in the snow sample and RT in the numerically modeled results.
Mass flux was calculated in RTRT. Deposition on the surface of snow as well as sublimation away from the surface effects the microstructure. The results, Figure 35, show that there is some mass loss over the duration of the experiment with loss increasing over time. This is caused by the room temperature being held constant while the surface temperature is warming up. This also coincides with David Walters’ experimental observation of loss of height of samples over the duration of the experiment (David Walters, 2015, personal communication).

Figure 35: Calculated mass flux from RTRT. The model assumes that the surface of the snow is saturated with respect to water vapor.

Figure 36, Figure 37, and Figure 38 show the results of an experiment completed by Dave Walters. His methodology covered the same duration as mine, but it was
conducted in a larger snow box. 101 cm by 81 cm by 40 cm deep with insulated sides. The results of the surface temperature show very similar profiles. As the surface temperature begins to even out, there is about a 2°C temperature difference between the measured and calculated results.

![Surface temperature reading on Dave Walter's experiment. These results are within 2°C of each other.](image)

The depth profile has decent profile agreement, after the first centimeter except for hour 12. The discrepancy of the thermocouple reading at the end of this experiment was an undefined perturbation in the results. The much warmer temperature readings at the first two thermocouples (they are 0.5 cm apart) is due to them being exactly at or just above the surface. This would cause them to warm up even more than if they were buried within the snowpack. This also could be due to there being some extra space around the...
TC when they were dug into position. Figure 38, like Figure 35, shows mass loss over the duration of the experiment. Again, this corresponds to mass loss seen in the lab as shrinkage of the total height of the experiment.

Figure 37: Depth profile of Dave Walter’s experiment. TC02 is the measured result two hours into the experiment corresponding to the calculated result RT02. TC06 and RT06 are the measured and calculated results six hours into the experiment, respectively. TC12 and RT12 are the results at twelve hours into the experiment. The results through the first centimeter do not align well, below the first centimeter one can see good congruency in the shape of the results with a small constant temperature offset.
Figure 38: Calculated mass flux from RTRT. The model assumes that the surface of the snow is saturated.

These results demonstrate good agreement between calculated and measured results with respect to both surface temperature and depth profiles. There were some errant results probably due to user error with instrumentation as well as the inherent error in trying to measure temperature within snow. Overall, the results show good results for modeling a RR event and good ability to model surface temperature.
4.4 Thermal Imaging and Spatial Variability

Part of the scope of this work included RTRT’s ability to calculate spatial variation of temperature based on both YCS/YCN inputs and American Spirit weather input. For the mountain scale, the best way to quantifiably look at spatial thermal results was to apply an American Spirit weather file and then compare these results to the measured results at the north and south weather plot. Some of these results were discussed in the previous section (Material Properties and Weather Stations) and further results for this will be seen in the winter RR event discussed in the next section.

For now, a quick view of Figure 39, shows that using an RTRT input file with American Spirit data shows a difference of only 1.5°C where the measured difference at the two study plots shows a difference of about 5°C for a day in mid-March. Surface temperature results using only AS over the 30 meter data for Pioneer Mountain do not give good results in the spring.
Figure 39: RTRT rendering of Pioneer Mountain on March 20, 2014 at noon using American Spirit data. The white is the trees that are scaled out. The results show a maximum temperature difference of approximately 1.5°C, where the measured results (not shown) show a difference of about 5°C at the two study plots.

For the small scale, RTRT does much better. Thermal imaging was done in the 2013/14 field season as well as in the 2014/15 field season at the south study site. In the 2013/14 season the camera took images at half hour increments for several consecutive days a few times during the season. In the 2014/15 season, the camera, Figure 40, ran for up to two weeks taking consecutive half hour photographs.
Figure 40: Researcher preparing to download data from thermal image at south study plot in tree stand. Photographer: Tony LeBaron.

Figure 41 shows three thermal images from March 20, 2014. Image (i) was taken at 1:00 PM, image (ii) was taken at 4:00 PM and image (iii) was taken at 7:00 PM. The location marked with a triangle in the thermal images and RTRT rendering (Figure 42) corresponds to “Img-1” in Figure 43, and the rectangle corresponds to “Img-2.” Figure 43 shows RTRT and thermal imaging results for the afternoon of March 20. The temperatures disagree by up to 4°C.

It is important to consider the discrepancies in this comparison. First, the thermal images are not averages over half hour increments. They only capture the temperature at the moment imaged. Secondly, not only is the data used in RTRT averaged over half hour increments, it is using 30 minute time steps to calculate the solution, which makes the solution pretty clumsy in its ability to show results on the half hour. Finally, the corresponding of RTRT nodes to areas in the image is only an estimate, and they may differ slightly in actual location thus affecting the results. With these things in mind, the results shown in Figure 43 show the spatial temperature variation, especially in the
middle of the afternoon when one would expect the largest temperature difference between shadowed and non-shadowed areas. These results also show the importance of the complex topography input because more variation is seen on the 1 m scale than is seen on the 30 m scale.

Figure 41: Thermal image of south facing study plot. Image (i) was taken at 13:00. The triangular element is -3.1 °C and the rectangular element is -4.0°C. Image (ii) was taken at 16:00. The triangular temperature element is -7.4°C and the square element is -11.2°C. And image (iii) was taken at 19:00 on March 20, 2014 with node temperatures of -22.9 and -26.7, respectively.
Figure 42: Complex topography and surface temperature from RTRT. The white triangle represents the location of the YCS (element 8789), and the white square represents element 18282. A representation of March 20, 2014 at 4:30 PM is shown. This image shows approximately the same area as shown in the thermal images in Fig. 3.
Figure 43: Results of thermal imaging (Img-1 and Img-2) compared to the surface temperature results calculated in RTRT (RTRT-1 and RTRT-2). The calculated temperature runs about 2 °C warm during the heat of the day.

Both the thermal images and the RTRT rendering show the significant effect of the topography. Another good example of this can be seen in Figure 44, which is an RTRT rendering of the north slope on the same day as seen before (March 20). The figure is oriented so the top is corresponds to higher elevation. The maximum temperature difference seen on this slope is 5°C. It is also a great example of the extent to which trees warm on one side and cool on the other.
In the 2014/15 season, the thermal imaging went much better. Data was collected consecutively for several weeks. A two day window of this data is shown below, Figure 45. The weather during this time period was very warm, so there is not as much temperature variation over the slope because the slope is approaching zero. The RTRT results in Figure 46 show a temperature variation of up to 3°C where the thermal images only show a difference of 1°C. Inspecting the thermal images in Figure 47 shows a distinctive lack of shadowing, which one would expect to see during the afternoon and early evening suggesting the camera was tilted a little too high. Also, the thermal camera
temperature reading drops very low during the night, this may be due to user error for inputting air temperature, reflected temperature, distance, optics temperature, or relative humidity for each image. Realistically, none of these measurements nor calculations are perfect, but they are all within a few degrees of each other, which is sufficient for this modeling effort.

Figure 45: Thermal imaging, RTRT solution, and surface temp measured at the south weather station. The measured data and RTRT solution are not at the same precise location but have the same aspect, slope, and snow depth.
Figure 46: RTRT and thermal imaging plots of approximately the same respective locations. Just before this period, the weather was consistently warm and the snow went isothermal. It was also quite warm so that there was not as much temperature variation over the slope because the entire slope is approaching zero. RTRT results (left) show a few degrees of variation in the heat of the day. The thermal imaging results (right) do not show a much smaller variation. This could be due to error in location correlation.
Figure 47: Thermal images for March 24 and 25. Images are shown at midnight, 7 AM, 2 PM, and 9 PM on March 24, and then 4 AM, and 11 AM on March 25.
Even though the imaging isn’t perfect, the results here show the importance of the complex topography and that RTRT does well at modeling with the small (1 m) grid. However, the large (30 m) grid does not produce very meaningful results. This is particularly well illustrated by comparing the temperature variation of the large and small scale models. The large model showed less temperature variance over the entire mountain than what the small grid showed on just one slope. This work also illustrated how well RTRT was modeling spatial variation. This spatial variation will be revisited in Section 4, showing better large scale results in the winter than in the spring.

4.5 Winter Radiation Recrystallization Event

The following is a discussion of the RR event recorded by the Yellowstone Ski Patrol on January 3, 2011. This event was chosen because of all of the information available for it. The YC wrote an informative daily log for it. They also took photos of the event at several different depths. Finally, this is a great event to model because the results have such a clear delineation between the event and no event.

The discussion will encompass four parts. First, layer densities and thickness choices will be discussed along with the solar energy absorbed and weather input choices. Then the surface temperature and temperature profiles are shown using local slope inputs as well as global (mountain) inputs. Then spatial variation both on the 1 m and the 30 m grids will be discussed, and finally, there will be a brief discussion on the mass flux results for this time period.
4.5.1 Event Parameters

In the Yellowstone Club Ski Patrol reports, a radiation recrystallization event was noted for January 3, 2011.

This event was modeled using a density and thickness profile seen below, Table 7. The top 3 cm of snow was assumed to be lighter than the rest of the snow. These layer densities and layer thicknesses were an estimation and were not measured. The grain size was assumed to be 1 mm. The resulting energy profile can be seen in Figure 49. The irregular shape of the day for energy absorption per layer is due to the density and thickness profile chosen. Absorptivity for this snow was set to 0.09 for the North and South plots and was set to 0.1 when the 30 m Pioneer Mountain model was used.

Figure 48: Radiation recrystallization photos from Yellowstone Club ski patrol at (I) 0 cm, (II) 1 cm, (III) 2 cm, and (IV) 3 cm below the surface taken on January 11, 2011.

Table 7: Layer density and thickness profile used in RTRT for winter RR event. These parameters were not measured, and they are an estimation. The assumed total depth of the snowpack is 2 meters, which is an over estimation.

| qdensity | 150.0, 150.0, 150.0, 150.0, 150.0 |
|          | 200.0, 200.0, 200.0, 200.0, 200.0 |
|          | 200.0, 200.0, 200.0, 200.0, 200.0 |
|          | 200.0, 200.0, 200.0, 200.0, 200.0 |

| qthickness | 0.005, 0.005, 0.005, 0.005, 0.010 |
|           | 0.010, 0.010, 0.010, 0.010, 0.030 |
|           | 0.050, 0.050, 0.050, 0.050, 0.050 |
|           | 0.050, 0.050, 0.050, 0.250, 1.250 |
Figure 49: Percent energy absorbed per layer thickness, (corresponds to qthickness in Table 7.) Energy absorbed is a function of density, thickness, and position. Since the densities and thicknesses are not the same throughout the snow pack, an exponential decay is not seen.

4.5.2 Temperature Results

The day preceding, the day of, and the day after the event were all modeled. The event was modeled on both the 1 m north facing study plot model and the 1 m south facing study plot model as well as on the 30 m Pioneer Mountain model. Surface temperatures measured at the study sites were compared with results from RadThermRT. Three sets of input data were used for modeling: weather from the south study plot with longwave and shortwave from the American Spirit (AS), which will be referred to as YCS/AS; weather from the north study plot with longwave and shortwave from AS, YCN/AS, and a weather file exclusively from AS data. Using only local (YCS or YCN) data is not shown here because the numerical model did not converge--probably due to an extremely abrupt jump in the shortwave YCS data. A thirty minute time step was used for the 1 m model, and a smaller time step could have been used to try to reach
convergence for local only data; however, because of the very high resolution of the one-meter data as well as the number of elements in the model, the computational time for time steps smaller than 30 min were deemed too intensive, especially since the solution with the AS/YCS and AS/YCN data worked so well.

With the YCS/AS (YCS in the legend) data modeled in the 1 m platform, the calculated surface temperature agrees nicely with the measured data, Figure 50. Generally, the calculated temperature runs less than a degree cooler than the measured temperature. However, the calculated temperature does miss both the global maximums and minimums. It decreases beyond the measured temperature when the temperature dips, and it increases beneath the maximum when there is a steep increase in temperature. The YCS/AS result falls about seven degrees short of the measured global maximum.

The numerical results using the 30 m platform and AS data, lacks the detail that is seen with the 1 m platform. It does not converge onto small local perturbations well, in fact, often over shooting them. Also, the magnitude for the sharp temperature increases and sharp temperature decreases is very low, up to about seven degrees for both increases and decreases. The lack of resolution for small perturbations is due to the larger step size (1 hour) used for the model as well as the longer averages used for the American Spirit (AS) data. The general trend, however, still gives a good sense of what is occurring at the surface for temperature.

The trends seen for the south site are also true for the north site. YCN/AS weather data with 1 m RTRT platform, tends to run less than a degree cooler, and the 30 m platform has difficulty with local perturbations. For both locations, some of the convergence issues may have been better if computation time had been increased with a
decrease in step size, but usability is a consideration for this work and decreasing step size substantially increases computation time.

![Graph](image)

**Figure 50:** Surface temperature for three days at the south study plot location: measured temperature at YCS weather station (YCS-Measured) along with numerical results using YCS/AS data and AS only data for the mountain model. The AS/Pio result is consistent with the overall temperature trend, but magnitude for global temperature maximums and minimums is too small. Also, the AS/Pio data does not converge well on local maximums and minimums. The YCS data, however, follows the measured trend very well and includes the local temperature perturbations pretty well, but it does not reach the global maximum well.
Figure 51: Surface temperature of three days at the north study plot location: measured temperature at the YCN weather station (YCN-Measured) along with numerical results using YCN/AS for the YCN model and AS for the mountain-scale model. The AS/Pio result is consistent with the overall temperature trend, but magnitude for global temperature maximums and minimums is too small. Also, the AS/Pio data does not converge well on local maximums and minimums. The YCN data, however, follows the measured trend very well and includes the local temperature perturbations pretty well. However, it also does not reach the global maximum well.
The most interesting part of the results for this data comes from looking at the temperature profiles with depth. It was not expected that a positive knee-shaped gradient, (colder to warmer to colder from the surface), would be seen anywhere but on south facing slopes. Three days of depth profiles for the north site using 1 m data with YCN/AS weather inputs can be seen in Figure 52-Figure 54. For these temperature profiles, none of them show a strong, positive, knee-shaped gradient in the first two or three centimeters indicative of the RR. There is subsurface warming seen on January 2 (Figure 53) but not a total knee-shaped gradient.
Figure 53: North facing depth vs. temperature results for January 2, the day the radiation recrystallization event was seen. There is no knee gradient even though the entire snow pack does trend toward warming.

Figure 54: North facing depth vs. temperature results. For January 3, the day after the RR event was recorded. Again, no positive knee-shaped gradient seen.
Figure 55-Figure 57 shows the depth vs temperature for the south facing study plot using 1 m YCS/AS data. Figure 55 and Figure 57 show the day before and after the event was recorded. On these days, there is no persistent positive knee-shaped gradient. On the day of the RR event, there is a very strong positive knee-shaped gradient that appears. The maximum temperature gradient between the surface and the apex of the knee is 5.2 °C. This apex occurs at about 3 cm below the surface of the snowpack. This equates to a gradient of about 1.7°C/cm, which exceeds the minimum required to see radiation recrystallization, which is 0.1°C/cm (class 4).

![Graph showing depth vs temperature](image)

Figure 55: 1M YCS/AS depth vs temperature graph for January 2, the day before the RR event was recorded. During the day, there is no significant positive knee-shaped temperature gradient even though one does see a small near-surface knee at the end of the day. It does not persist and thus is not pertinent.
Figure 56: 1 M YCS/AS depth vs temperature graph for January 3, the day that the RR event was recorded. A positive knee-shaped gradient can be seen near the surface at noon, the gradient is even larger at 2:00 PM, and then the gradient begins to reduce in the evening.

Figure 57: 1M YCS/AS depth vs temperature graph for January 4, the day after the RR event was recorded. Again, there is no persistent positive knee-shaped temperature gradient seen.
This event was also modeled using the 30 m RTRT platform and only American Spirit data. In Figure 58-Figure 60, the depth profile for this data was plotted along with the YCS/AS 1M data, seen above, for comparison. On January 2, Figure 58, the temperature profiles have a similar shape with pretty similar temperatures. On January 3, the knee-shaped temperature seen for the YCS/AS data is also seen for the AS 30 m data. With similar accuracy the north site, Figure 61, does not show a strong gradient within the top two centimeters for either the YCN/AS data nor the AS data.

Figure 58: South facing small scale, 1 m, (YCS) and large scale, 30 m, (Pio) depth versus temperature profiles on January 2.
Figure 59: South facing small scale, 1 m, (YCS) and large scale, 30 m, (Pio) depth versus temperature profiles on January 3. The large scale result does not show the positive knee-shaped temperature gradient that is seen in the small scale.

Figure 60: South facing small scale, 1 m (YCS), and large scale, 30 m (Pio), depth versus temperature profiles on January 4.
4.5.3 Spatial Variation

The results from four different locations on the south study plot can be seen in Figure 62. Results from Location 1, on the left between the cliff and the trees, show a much higher surface temperature on the afternoon of the third, Figure 63. Locations 2 and 3 also have temperature spikes above Location 4 during that afternoon. Location 4 is just past a small roll-over in the slope and seems to be partially shaded from the low south-west sun.
Figure 62: RTRT rendering showing 4 elements chosen to show 1 m scale spatial variability.
Figure 63: Spatial surface temperature results for south study plot.

Figure 64 shows the temperature vs depth of the four locations in Figure 62 at 2:00 PM on January 3. Locations 1 and 4 show the largest positive knee-shaped gradients at 1.6 °C/cm and 1.0 °C/cm. Locations 2 and 3, in the trees, have negligible gradients relative to locations 1 and 4.
Surface temperature and temperature versus depth plots were also demonstrated at 6 locations on the mountain, Figure 64. The surface temperatures at all location on January 2 are the very similar over the entire day, and on January 3, the surface temperature only varies by a maximum of 1.5 °C. This does not seem like very good variation, but when one considers the associated depth profiles (Figure 68) the results show better variation with the maximum temperature gradient in the first two centimeters being about 3 °C/cm at Location 3 and the minimum gradient being about 1.5 °C/cm at Location 5. Even these smaller gradients may indicate an event where one would not necessarily expect one.
Figure 65: Locations of RTRT results shown below.
Figure 66: Surface temperature of six locations on the mountain scale.

Figure 67: Depth vs temperature profile of 6 locations on Pioneer Mountain. These results are for January 2, at 2 PM. They show no spatial variation greater than 1/100 °C.
Figure 68: Depth vs temperature profile of 6 locations on Pioneer Mountain. These results are for January 3, at 2 PM. Despite not seen variation at the surface, there is strong differences in the gradients here.

4.5.4 Mass Flux

Surface mass flux was also numerically modeled. When this was modeled, RTRT assumes that the surface of the snowpack is at water vapor saturation so that what it calculated was the effect of the temperature difference between the surface and the air in conjunction with relative humidity. Seen in Figure 69 and Figure 70 there does not seem to be a large flux in either direction, but it is not yet known exactly what magnitude nor time period is necessary to result in surface hoar or significant snow water equivalent loss. Despite these unknowns, it is still important to consider for weak layer formation.
Figure 69: 1 m YCS surface mass flux.

Figure 70: North facing mass flux for 1 m YCN data.
The surface temperature results show reasonable accuracy with respect to the
general trends. The RTRT global maximums and minimums both tended to run below
the measured results. The 1 m RTRT platform with the local weather station data at the
respective sites showed better results than the 30 m RTRT platform with the AS data,
which was to be expected based on the averaging used for the weather data, the time step
used within RTRT, and the larger area covered. The temperature vs depth results clearly
show a positive, knee-shaped temperature gradient for the day that the event was
recorded, and it did not show any knee-shaped gradient on the days on either side. This
was true for both the 1 m results as well as the 30 m. There were also good spatial results
on both the 1 m and 30 m.
5. CONCLUSIONS AND RECOMMENDATIONS

The scope of this work was to take the existing RTRT model, improve on the shortwave radiation absorption component, and then check and discuss the results with respect to lab experiments and field work using both 1 m LiDAR data and 30 m DEM. The results showed that the shortwave component is a significant contribution to the energy balance solution and density and grains size affect the solution by a few degrees.

In the lab, the surface temperatures of both experiments agreed with the measured results to within about 2°C after they had reached equilibrium. The temperature profiles with respect to depth did not agree as closely, but the thermocouples used to make these measurements carry more error because the metal-halide lamp was heating the thermocouples and the snow differently. Despite this, similar gradients were also shown for both experiments, but the RTRT results ran cooler and with larger positive knee-shaped gradients. The lab work also showed, using hyperspectral imaging, that the RR layer absorbs, reflects, and transmits light differently than the homogenous sifted snow.

The thermal camera was a useful tool to capture spatial variation on the south slope over time. Data from two different seasons were shown and spatial variation was seen to some extent both times. The shadowing from trees drops the temperature by about 5°C, and this was captured with both the 1 m RTRT analysis as well as the thermal imaging. However, spatial temperature variation using the 30 m RTRT results did not show good variation in the spring, but it did show better temperature variation results in the winter.
The winter event model was a good demonstration of RTRT’s ability to model positive radiation recrystallization days. Surface temperature was modelled with good accuracy with the 1 m grid, and useful but less accurate results came from the 30 m grid RTRT results. The temperature vs depth profiles show positive knee-shaped gradients on the days that one would expect, but the accuracy of the temperature magnitudes and differences are unknown due to how difficult it is to measure temperature depth profiles in the field.

The spring event discussed in this document had some really interesting results. The surface temperature profiles for the 1 m RTRT data had good congruency with the measured results just missing the global maximums and minimums by a small margin. The 30 m RTRT results did not do as well with results showing cooler temperature for the global minimums and missing the global maximums by up to 12°C. The 30 m depth vs temperature results and the 1 m depth vs temperature results do not agree as much in this spring example as they did in the winter example. However, they still show the same trend with similar shaped gradients. There were, however, some false positives, which bring into question the importance of absorptivity since this affects both the amount of energy allowed into the snowpack as well as the magnitude of gradients seen.

A high absorptivity was chosen in this project because previous work (Staples 2008) had shown that this provided better results. The absorptivity, however, does change the magnitude of the positive knee-shaped gradients shown, and it would behoove one to do more work on seasonal absorptivity values. The spring result also showcases the importance of material properties.
The 1 m data in spring, for surface temperature, looked even better than the 1 m data for the winter. Part of this is due to the weather inputs chosen. The winter site used the longwave and shortwave measurements from AS for both the 1 m and the 30 m input. However, the spring used the longwave and shortwave results from YCS and YCN, respectively, for the 1 m input.

Mass flux results were shown for the lab, the 1 m field data, and the 30 m field data. This flux has important implications for weak layer formation. Sublimation and condensation could affect the magnitude of the RR events, but this flux needs to be looked into this further. As it is, the mass flux calculates sublimation and deposition. It has been suggested that this parameterizes surface hoar events. Though this effect was not shown in this project, it has been suggested in the past (Staples 2008).

Both 1 m and 30 m data have their uses. The 1 m data does a better job with spatial variability and local perturbations in weather. However, the 30 m data is more accessible because the DEMs are free, fewer weather stations are required, and more terrain is covered. In the spring, the 30 m data does not give very good spatial results in that the aspect and elevation in and of itself is not sufficient to significantly affect the results over terrain at a macro level in the spring. However, the winter results showed a different result with good spatial variation seen with both the 1 m grid size as well as at the 30 m grid size. With regard to these two seasonally dependent results, it would be interesting to see if one could use weather input data from one location and consistently scale it to be the equivalent to input data at other mountain locations. For example, American Spirit data could be scaled up to be equivalent to measurements taken from the
south study plot which in turn could be scaled down to be equivalent to measurements taken at the north study plot.

This work improved the RR results with the ability to control how the solar radiation is absorbed. This is particularly important for the surface as well as the first 2-3 cm of the snowpack. The thermal imaging was a good tool to compare spatial surface temperatures, and it would be worthwhile to continue this work. The coldest months were modeled more accurately than the warmer months of winter, and more time should be spent on spring material properties. However, a few important components were lost when the material properties per layer were added in, and these should be added back in to really improve results.

When the solar absorption component was added into RTRT, the ability to make different snow part types became unavailable due to limited access to code while adding snow layer capabilities. This is a very important component to the model. The snowpack depth varies substantially over a small meadow let alone an entire mountain. Being able to input more than one part as snow and include the density, grain size, and layer thickness of each piece would greatly improve the results and usability of this project. In conjunction with multiple snow parts, it would also be good to write the grain size so that it is also a function of the part type as well as the layer thickness.

The scope of this project ends at identifying the parameters for radiation recrystallization and, as done in previous work, surface hoar. The next logical step in this work is to identify the parameters that result in sustained weak layers and include this within the snow pack layer data. This would add a usability factor as well to the data set.
One thing that would really improve the usability of this work would be to be able to map the temperature gradient of the first 2 centimeters below the surface. This would give a good spatial sense of a RR event, and would allow an RTRT user to quickly assess the aspect and extent of an event.

Another piece that would really improve analysis, is for several seasons, the YC Ski Patrol maintained daily logs for MSU and recorded any events that they witnessed as well as their location and extent. These results and the often accompanying photos were invaluable in studying and discussing RTRT results. Recently, however, the program has lagged. It would be extremely valuable to reinstate this as well as ensuring an open and working dialogue about caring for the weather stations between weekly visits by MSU students and faculty.

Overall, the new absorption parameters in RTRT improve results due to being able to control the density, layer thickness, and grain size of the snowpack. Future work needs to be done on spring parameters and usability of the model. This should include maintaining a layer history in the snow pack, looking at near surface temperature gradients spatially, and more thorough field observations.

-the end-
REFERENCES CITED
ACME Mapper, 2013. ACME. Available at: http://mapper.acme.com/.


Bader, H. & Kuroiwa, D., 1962. *The physics and mechanics of snow as a material*, Hanover NH.


Colbeck, S., 1982. *Growth of faceted crystals in a snow cover*,


Jordan, R., 1991. *A One-Dimensional Temperature Model for a Snow Cover: Technical Documentation for SNTHERM.89*, Available at:  


Labs, O. & Holzer, S., 2008. Surfex. *Surfex*. Available at:  


APPENDICES
APPENDIX A

LIDAR DATA PROCESSING FOR RTRT USING ARCGIS
This appendix provides the basic steps to process Lidar data for use in RadThermRT (RTRT). It assumes that the raster size is 1m x 1m for both sets of data, and that all coordinate systems will be UTM 1983 zone 12. For other resolutions, the basic process is the same. However, in some cases data will have to be resampled to a higher resolution, the area of interest cut, and then the data resampled to the original or a lower resolution in order to be viable. Also, make sure to read the metadata to find vegetation, size, and spatial referencing information.

To begin processing, first consider the two types of data needed, then the steps within ArcMap to process data. One either needs filtered and unfiltered LiDAR for 1 m resolution, or an elevation and vegetation DEM for lower resolution work.

**LiDAR Data**

LiDAR data needed for RadthermRT comes from two sources in this work. First, unfiltered data was uploaded from NCalm <ncalm.org> for the Big Sky area. This data includes a coordinate system and spatial reference.

However, the unfiltered data came from the Brian McGlynn in the LRES department at MSU. This LiDAR was old, and it involved extensive processing before it could be used.

The first step is open ESRI’s ArcMap 10.1, Figure 71. Older versions will also work for this basic process. Begin with a blank map, Figure 72. The map can be saved, but it is not necessary because all that is needed is the layers created within the program.
Next, data needs to be added. First, click the add data icon \( \text{add data icon} \), and then connect to folder, Figure 73. Repeat for all folders containing needed data. These do not need to be the same folders.
Figure 73: Connect to folders containing wanted data.

Figure 74: Expected spatial error for unfiltered data.

The unfiltered data is in an old surfer file format, which is completely antiquated and almost nothing recognizes or uses it anymore, so the first step is to convert the data into a .flt format, Figure 74.

To do this, there are many freeware converters. I used, GridConvert 1.0 <http://gridconvert1-0.software.informer.com/1.0/>. However, this data is missing its spatial reference, but it can be opened in ArcGIS, Figure 75.
Figure 75: Unfiltered data without spatial reference

Use ArcGIS toolbox to apply a spatial reference, Figure 76:

Figure 76: Finding ArcToolbox icon.

- Arc Toolbox, Fig 6.
  - Projections and Transformations
    - Define Coordinate System
    - Select Projected Coordinate System
    - UTM
    - NAD 1983
    - NAD 1983 UTM Zone 12N.pr
Figure 77: Define projection in GUI.
Now that both sets of data are in the same spatial reference system and can
be opened in ArcGIS, Figure 78.

Figure 78: Data with new spatial reference applied

This is a good time to check that your spatial reference system is correct. A good
tool for this is Google Earth. It is quite user friendly and interfaces nicely to ESRI’s
ArcGIS. Convert each data set of LiDAR to a .kmz, and view in Google Earth. To do this, use the ArcToolbox again:

- ArcToolbox
  - Conversion Tools
    - To KML
    - Layer to KML

Choose the layer you wish to check, and save. Then to go to the folder that the file was saved, and open it. Usually, the default will automatically open in Google Earth.

If your area of interest occurs within two sets of data, several rasters can be converted into one:

- ArcToolbox
  - Data Management Tools
    - Raster
      - Raster Dataset
        - Mosaic to New Raster

After you make sure that the spatial system is correct and that the area of interest is in one location, the data will need to be clipped down:

- ArcToolbox
  - Data Management Tools
    - Raster
      - Raster Processing
        - Clip
This may take a few tries to get the hang of, and this is also a good place to double check the location with Google Earth. After one data set is clipped to the desired size and location, the other data set can be clipped to the extent, so that nothing has to be re-measured, by simply following the “clipped” instructions (above) and for Input Raster put the data that has not been clipped and under Output Extent use the layer that has already been dimensioned, Figure 79.

![GUI to clip or clip to extent.](image)

**Figure 79**: GUI to clip or clip to extent.

**NOTE**: Even if you use “clip extent”, it is important to double check that the two layers are exactly the same size and that they line up perfectly. To do this, right click on each layer created, then properties, then source.

Now that the data is fit to size, geographic features need to be processed. To do this, you want to use the raster calculator to subtract the filtered data from the unfiltered data—this is later referred to as the difference layer.

**NOTE**: The spatial analyst is not always turned on. To check go to the working toolbar:

- Customize
Toolbars

- Spatial Analyst, Figure 80.

Figure 80: Turning on Spatial Analyst Toolbar if necessary.

Now that the Spatial Analysis Toolbar is on, calculate the difference between the filter and unfiltered data to create terrain types:

- ArcToolbox
  - Spatial Analyst Tools
    - Map Algebra
  - Raster Calculator

Use the Raster Calculator again to write a conditional statement from the difference layer created for the parameters you would like to use for trees and shrubs.

For example, if you want any difference between the filtered and unfiltered data that is
greater than one 1 meter to be considered a tree and everything else to be snow, (Cliffs and rocks will be discussed later), write a conditional statement as follows:

   Con('name of file'>=1,1,2)

This statement defines different terrain types numerically. The conditional statement reads “if file is greater than one meter, then label terrain type as 1 else label terrain 2. Later, in RTRT, the numerical terrain types will be given meaningful names.

NOTE: In RTRT, 0 is not a recognized part number.

For cliffs, use the filtered data layer. Turn the layer into a slope raster, choose degrees:

   - ArcToolbox
     - Spatial Analyst Tools
       - Surface
       - Slope

Then use another conditional statement to delineate the angle you wish to use to assume snow will not stick and you will have a cliff formation, i.e. con(cliff>=50=1,2).

Then to put the vegetation layer together and the cliff layer together a third conditional statement is needed:

   con(cliffcon==1,3,vegetation con)

So if the layer that is the cliff conditional is 1, then call it 3, otherwise, input the vegetation layer. The result is a layer that has three part numbers.
Now all that needs to be done is change the final layers into a format that RTRT understands, .asc. Two layers are needed to input into RTRT: the unfiltered clipped layer, and the final conditional statement. To do this:

- ArcToolbox
  - Conversion Tools
    - From raster
      - Raster to ASCII

NOTE: The default here is not to .asc, and this needs to be input manually making sure to delete the default .txt, Figure 81.

Figure 81: Saving raster data set to .asc not .txt. After using “save in folder” icon, make sure to delete the “.TXT”.

Place files in directory used for RTRT.
APPENDIX B

BUILDING RTRT TDF TO RUN NUMERICAL MODEL
A Linux server was used to run RTRT. The following discusses how RTRT was set up in order to run the numerical model.

To begin, run RTRT and then in working toolbar:

- **File**
  - **Open**—change “File type” to Digital Elevation Map Files (*.asc)
    - Elevation (unfiltered) .asc
    - Parts
  
  Nb! The order matters here.

Figure 82: RTRT GUI units setting. All work was done in meters.

- **OK**

In Editor tab, rename parts so that they are meaningful and set material properties.

Example: YCN Site

Part 1-Snow

Parts
Part Type= Terrain
Type= Fresh
Condition= Undisturbed
Depth= 2000.0000 (mm)
Core Temperature (C)= -1.000

Part 2-Trees

Parts

Part Type= Terrain
Type= Foliage
Foliage Type= Trees, Coniferous
Growth Factor= Dormant
Cover Factor= Intermediate
Solar Absorptivity= Modeled
Core Temp (C)= -1.000
Surface Moisture= Dry
Bulk Moisture= Dry

Environment

Natural (Weather)

Natural Environment Parameters

Global Position (from bottom left?)

Latitude (deg) = 45.23
Longitude (deg) = 111.45

Nb! RTRT is a west-centric program. West is positive.
TimeZone (hrs) = 7
Ground Elevation (m) = 2530

Solar Data = Measured
Sky Data = Measured
Model Weather = Use ground level weather

Ground Level Weather
Standard

File: wddmmyywhateverelse.txt

Note: The “w” must be there to signify weather file.

Default Terrain

Edit

Type = Snow
Type = User Specified Absorptivity = 0.1000
Condition = Undisturbed
Depth (mm) = 600.00
Core Temperature (C) = -1.000

Scenario
Model Type
Other

Analyze
Params
Start Time

At least one day (24 hrs) was run before the beginning of cycle of interest.

Step Size (min) = Value = 30 min
NOTE: If the step size is not the same as the weather file, RTRT will either iterate between the step sizes or will skip step sizes.

Write Frequency = Value = 1

Convergence Criteria

Tolerance

Tolerance (C) = 1

Maximum # Iterations = 300

Initialization

Initialization Method = Use Part Initial Temperatures

Advanced

Relaxation Parameters

Solution Relaxation=Adaptive**

Fluid Relaxation=1.001**

Solid Parts Relaxation 0.3**

**These are defaults and should not really matter for terrain Analysis

Ray Pattern = Circular

NOTE: This is really important. The helical option will cause RadThermRT crash.

Accuracy Settings

View Factor Rays= middle

Maximum=5

View Factor Subdivisions = middle

Apparent Area = Middle

Output

Nodes to Save Results For = All Nodes
Do not have checked “Save extra data to allow transient initialization”

Save Extra Terrain Results

To change node part type

   Select Element tool in work bar

   Select all nodes to change

   R-click- Assign Part

   Make sure the Editor Part Selector has the part you want to change to selected.

Analyze

   Params

   RUN!

Use the RTRT GUI to set up material properties for the part created. Also, parts can be edited within RTRT to fix errors.
APPENDIX C

RUNNING RTRT WITH LAYER THICKNESS AND DENSITY PROFILE
To run a weather file, use an RTComp text file in conjunction with the make file RTComp.pl. Within the RTComp text file, the domain, the input version, the output version, the RTRT file containing the complex topography, the weather file, start and end date, density profile, and thickness profile can all be controlled. This file can be named anything, the naming scheme shown is just for the version control scheme I chose. To run these files, use the make file. For example, in the command line: RTComp.pl RTCom-1elem.txt, will run version 9.2.1a-c2 and output it as version 9.2.1a-c3 using the RTRT model file, Test Elem1.tdf, etc., etc. To open this file within the RTRT GUI, run in the command line: rtrt TestElem1.tdf.
Figure 83: Text file for running RadThermRT with layer thickness and density profiles.
APPENDIX D

ATTENUATION CODE IN RTRT BACKGROUND
Please note: A copy of this code is on file and available at MSU. Contact Ladean McKittrick. This code is written in background.cpp. in RTRT version 9.2.1a-c2.

/*===================================================================================================================================

*  Snow::setLayerSolarFactors
*/

/*!
* \brief Sets the solar absorption factors for the snow terrain layers
*
* The solar absorption factors is the fraction of the solar load on the
* surface of the terrain that is absorbed by each snow layer. The
* sum of the layerFractions MUST be 1.0.
*
* \sa createThermalNodes()
*/

void Snow::setLayerSolarFactors()
{
    #ifdef Debug
        printf("Called Snow::setLayerSolarFactors()\n");
    #endif

    /*
* Create all the variables needed in the method*******************************************************************************/

    // The number of snow layers that absorb solar MUST match the
const int nLayersAbsorbed = nLayers; // MUST be <= nLayers

const int nBins = 8;

double *binKice = new double[nBins];

double *binWeight = new double[nBins];

double **solarK = new double*[nLayersAbsorbed];

double dGrain = (float)1000 * pow(10, -6);

double rhoi = (float)917.0; // [kg/m^3] density of ice

double binFrac; // Amount absorbed by [i] each layer and [j] each bin
double **binFrac = new double*[nLayersAbsorbed];

// Fraction Remaining at the Bottom of Each Layer (Total - binFrac per layer)
double **binFracLeft = new double*[nLayersAbsorbed];

double *layerFraction = new double[nLayersAbsorbed];

double totalAbsorbed = 0.0;

/*
 * Initialize the variables
*********************************************************
* Initailize the variables
*********************************************************/

// Attenuation coefficient of ice from Warren and Me
binKice[0]= 0.008226; //[1/m]
binKice[1]= 0.48971; //[1/m]
binKice[2]= 10.03; //[1/m]
binKice[3]= 60.87;//[1/m]
binKice[4]= 124.02; //[1/m]
binKice[5]=151.58; //[1/m]
binKice[6]=160.28; //[1/m]
binKice[nBins-1]=195.8; //[1/m]

// binWeight is the percentage available to be absorbed for each span of wavelengths.
binWeight[0]=0.010; //[%]
binWeight[1]=0.037; //[%]
binWeight[2]=0.123; //[%]
binWeight[3]=0.148; //[%]
binWeight[4]=0.229; //[%]
binWeight[5]=0.191; //[%]
binWeight[6]=0.149; //[%]
binWeight[nBins-1]=0.113; //[%]

// Initialize binFrac, binFracLeft, solarK, and layerFraction to 0
for (int i=0; i<nLayersAbsorbed; i++) {
    binFrac[i] = new double[nBins];
    binFracLeft[i] = new double[nBins];
    solarK[i] = new double[nBins];
    layerFraction[i] = 0.0;
    for (int j=0; j < nBins; j++) {
        binFrac[i][j] = 0.0;
        binFracLeft[i][j] = 0.0;
        solarK[i][j]=0.0;
    }
}

/*
This is the guts of the calculation

NOTES: i = Layers from top to bottom
j = spans of wavelengths from short to long (left to right)

1. If the snow is layered on top of asphalt or concrete, then we calculate
   this in LayeredTerrain::setLayerSolarFactors(). [This is skipped]
2. Calculate the amount absorbed in the first layer because of center
differencing. (Only .5 layer below, not .5 layer above, and .5 below)
3. Calculate the amount remaining for lower layers to absorb.
4. For all layers below the first, calculate the amount absorbed and the
   amount remaining.

// 1. Skip if on asphalt or concrete.
if (!hasLayerBelow()) {
    // printf("Layer\tvThick\tBin\tbinFrac binFracLeft\n");
    // 2. Calculate the amount absorbed in the first layer.
    // 2.1. For each wavelength bin calculate the amount absorbed.
    // printf("0%f\t", vThickness[0]);
    for (int j=0; j < nBins; j++) {
        // Pat is concerned that the area doesn't add up to 100%. Emil thinks
        // we don't care because Sum(binWeight[*]) = 100%.
        solarK[0][j] = 0.84*sqrt(binKice[j]/dGrain)*vDensity[0]/rhoi;
        // If attenuation is the amount left, then binFrac and binFracLeft are reversed. Does
        // that change the thickness problem? I think it does. 95% sure in fact.
binFracLeft[0][j] = binWeight[j]*(exp(-solarK[0][j]*(vThickness[0]/2)));

// 3. Calculate the amount absorbed.
binFrac[0][j] = binWeight[j] - binFracLeft[0][j];

// Total the binFractions per layer.
layerFraction[0] += binFrac[0][j];

// printf(" %d\t%f\t%f\n", j, binFrac[0][j], binFracLeft[0][j]);
}

printf("\t Sum: %f\n", layerFraction[0]);

// 4. For all layers below the first, calculate the amount absorbed and the
// the amount remaining.
for (int i=1; i < nLayersAbsorbed; i++) {
    // printf("%d\t%f\t", i, vThickness[i]);
    for (int j=0; j < nBins; j++) {
        double topLayer = 0.0;
        double bottomLayer = 0.0;
        // 4.1. Calculate solarK
        solarK[i][j] = 0.84*sqrt(binKice[j]/dGrain)*vDensity[i]/rhoi;
        // 4.2. Calculate binFrac for 1/2 layer above .999
        topLayer = binFracLeft[i-1][j]*(exp(-solarK[i-1][j]*(vThickness[i-1]/2)));
        // 4.3. Add binFracLeft for 1/2 layer below .998
        bottomLayer = topLayer*(exp(-solarK[i][j]*(vThickness[i]/2)));
        binFrac[i][j] = (binFracLeft[i-1][j] - bottomLayer);
        binFracLeft[i][j] = bottomLayer;
// Total the binFractions per layer
layerFraction[i] += binFrac[i][j];

// printf(" %d\t%f \%f\", j, binFrac[i][j], binFracLeft[i][j]);
}
printf("\t Sum: %f\n", layerFraction[i]);

// Be dastardly to our data and shove the remaining minutia into the last layer.
// This removes a very slight rounding error in converting to doubles.
for (int i = 0; i < nLayersAbsorbed; i++ ) {
    totalAbsorbed += layerFraction[i];
    // printf("\t Totalabsorbed: %f\n", totalAbsorbed);
}
if (totalAbsorbed < 1) {
    layerFraction[nLayersAbsorbed-1] += 1 - totalAbsorbed;
    // printf("\t lastlayer: %f\n", layerFraction[nLayersAbsorbed-1]);
}
for (int i=0; i < nLayersAbsorbed; i++ ) {
    // printf("\t Sum: %f\n", layerFraction[i]);
}

// Set the SolarAbsorFactors
SetSolarAbsorFactors(vSolver, vLayerNodeID,
nLayersAbsorbed, layerFraction);

delete[] binKice;
delete[] binWeight;
delete[] binFrac;
delete[] solarK;
delete[] layerFraction;
delete[] binFracLeft;
}

} /* End Snow::setLayerSolarFactors() */
APPENDIX E

MATLAB CODE FOR CHECKING ATTENUATION CODE
%%Qsw--RTRT coding affirmation%%
delz=0.05; % Layer Thickness [m]
alpha=0.1; %Absorptivity [%]
Qsw=695; %Shortwave radiation [W/m^2]
rho=100; % Density of Snow [kg/m^3]
% Attenuation coefficient constants

dGrain=1000*10^-6; % Snow grain diameter [m]
rhoi=917.0; %Density of ice [kg/m^3]
%Attenuation coefficients of ice
binKice(1)=0.008226; %[1/m]
binKice(2)=0.48971; %[1/m]
binKice(3)=10.03; %[1/m]
binKice(4)=60.87; %[1/m]
binKice(5)=124.02; %[1/m]
binKice(6)=151.58; %[1/m]
binKice(7)=160.28; %[1/m]
binKice(8)=195.8; %[1/m]
%binWeight is the percentage available to be absorbed for each span of
%wavelengths
binWeight(1)=0.010; [%]
binWeight(2)=0.037; [%]
binWeight(3)=0.123; [%]
binWeight(4)=0.148; [%]
binWeight(5)=0.229; [%]
binWeight(6)=0.191; [%]
binWeight(7)=0.149; [%]
binWeight(8)=0.113; [%]
%Initialization of layer fraction
layerFraction(1)=0;
layerFraction(2)=0;
layerFraction(3)=0;
layerFraction(4)=0;
layerFraction(5)=0;
layerFraction(6)=0;
layerFraction(7)=0;
layerFraction(8)=0;
layerFraction(9)=0;
layerFraction(10)=0;
layerFraction(11)=0;
layerFraction(12)=0;
layerFraction(13)=0;
layerFraction(14)=0;
layerFraction(15)=0;
layerFraction(16)=0;
layerFraction(17)=0;
layerFraction(18)=0;
layerFraction(19)=0;
layerFraction(20)=0;
%Surface layer
for j=1:8; %spans of wavelength from short to long (left to right)
solarK(1,j)=0.84*sqrt(binKice(j)/dGrain)*rho/rhoi;
binFracLeft(1,j)=binWeight(j)*exp(-solarK(1,j)*delz/2);
binFrac(1,j)=binWeight(j)-binFracLeft(1,j);
layerFraction(1)=layerFraction(1)+binFrac(1,j);
end

%Subsurface layers
for i=2:20; %Layers from top to bottom
for j=1:8; %spans of wavelength from short to long (left to right)
solarK(i,j)=0.84*sqrt(binKice(j)/dGrain)*rho/rhoi;
topLayer=binFracLeft(i-1,j)*exp(-solarK(i-1,j)*delz/2);
bottomLayer=topLayer*exp(-solarK(i,j)*delz/2);
binFrac(i,j)=binFracLeft(i-1,j)-bottomLayer;
binFracLeft(i,j)=bottomLayer;
layerFraction(i)=layerFraction(i)+binFrac(i,j);
end
end

layerFraction(20)=1-sum(layerFraction)+2*layerFraction(20);
layerFraction;

QswFinal=[alpha*Qsw*layerFraction(1);
alpha*Qsw*layerFraction(2);
alpha*Qsw*layerFraction(3);
alpha*Qsw*layerFraction(4);
alpha*Qsw*layerFraction(5);
alpha*Qsw*layerFraction(6);
alpha*Qsw*layerFraction(7);
alpha*Qsw*layerFraction(8);
alpha*Qsw*layerFraction(9);
alpha*Qsw*layerFraction(10);
alpha*Qsw*layerFraction(11);
alpha*Qsw*layerFraction(12);
alpha*Qsw*layerFraction(13);
alpha*Qsw*layerFraction(14);
alpha*Qsw*layerFraction(15);
alpha*Qsw*layerFraction(16);
alpha*Qsw*layerFraction(17);
alpha*Qsw*layerFraction(18);
alpha*Qsw*layerFraction(19);
alpha*Qsw*layerFraction(20)];

QswFinal